No. 108/3

EDUCATIONAL QUALITY AND
ASSESSMENT PROGRAMME

Scoring Rubric
2022

South Pacific Form Seven Certificate

M A T H S with C A L C.

3 Luke Street, Nabua, Private Mail Bag, Suva, Fiji.
Telephone: (679) 3370733 Fax: (679) 3370021

SCORING RUBRIC FOR MATHEMATICS with CALCULUS

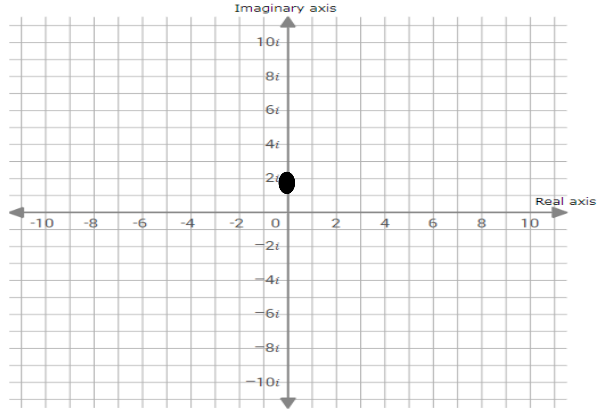
Item	Skill level	Evidence (expected answers)	Level 1 (Unistructural)	Level 2 (Multistructural)	Level 3 (Relational)	Level 4 (Extended Abstract)
1.1	1	$\frac{2y}{3} - \frac{y}{4} = \frac{8y-3y}{12}$ $= \frac{5y}{12}$	Correct answer $\frac{5y}{12}$ *or* $0.42y$ OR Finds LCD = 12 'Allow for slips'			
1.2	1	Use elimination method: $2x - y = 7$ $+\ 3x + y = 13$ $5x \quad = 20$ $x \quad = 4$ Find y: $2x - y = 7$ $2(4) - y = 7$ $8 - y = 7$ $-y = -1$ $y = 1$	Correct answers $x = 4$ $y = 1$ OR Any correct value of x or y	Alternative: Substitution Method 2x – y = 7……. y = 2x – 7 …….3x + (2x – 7) = 13 ….3x + 2x = 13 + 7 5x = 20 x = 4		
1.3	1	$T = a + (n-1)d$ $T - a = (n-1)d$ $\therefore d = \frac{T-a}{(n-1)}$	Correct answer OR Any correct step of minus *a* or divide by (*n-1*).			

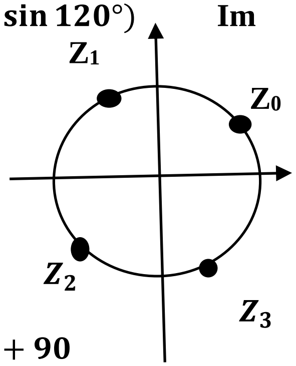
1.4	1	$Factorise\ x^2 + 9x - 70$ Factors of -70 = 14 x -5 9 = 14 + -5 $(x+14)(x-5)$	Correct two factors $(x+14)(x-5)$ OR Any one correct factor	• Alternative: **Quadratic Equation** $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ $= \frac{-(9) \pm \sqrt{(9)^2 - 4(1)(-70)}}{2(1)}$ $= \frac{-(9) \pm \sqrt{361}}{2} = \frac{-(9) \pm 18}{2}$ $= -14, 5 \rightarrow (x+14)(x-5)$		
1.5	1	Solve $log_{16}\, x = \frac{3}{2}$ In base-index form: $16^{(3/2)} = x$ $\therefore x = 64$	Correct answer $x = 64$ OR Writes the correct expression in Base-index form. $16^{(3/2)} = x$			
1.6	1	Simplify $\frac{24x^4y^{11}z}{3x^2yz^7}$ $= 8x^{4-2}.y^{11-1}.z^{1-7}$ $= 8x^2y^{10}z^{-6}$ or $\frac{8x^2y^{10}}{z^6}$	Correct answer OR Has shown any correct use of law of indices – subtracting powers			
1.7	1	Simplify $3log4 - 2log2$ $= log4^3 - log2^2$ $= log64 - log4$ **Alternative Solution** $3log4 - 2log2$ $= 3log4 - log2^2$ $= 3log4 - log4$	Correct answer $log16$ OR			

		$= log\frac{64}{4}$ $= log16$ $= 2log4$ $= log4^2$ $= log16$	Has shown any correct use of logarithmic laws. $log4^3$or $log2^2$ or $log\frac{64}{4}$			
1.8	1	$2x^3 + 5x - 6$ $divide\ by\ (x - 1)$ By Remainder Theorem = f (1) = remainder Let $f(x) = 2x^3 + 5x - 6$ $f(1) = 2(1)^3 + 5(1) - 6$ $f(1) = 1 \rightarrow remainder$ Alternative solution: Long Division	Correct answer Remainder = 1 OR Finds f (1)			
1.9	1	Expand and simplify $(1 - 2x)^4$ $\binom{4}{0}(1)^4(-2x)^0$ $= 1.1.1 = 1$ $\binom{4}{1}(1)^3(-2x)^1$ $= 4.1.(-2x)$ $= -8x$ $\binom{4}{2}(1)^2(-2x)^2$ $= 6.1.4x^2$ $= 24x^2$	Correct answer 'Allow for slips' OR Writes the binomial theorem correctly for a = 1, b = -2x and n = 4. OR Writes any correct term in the expansion.			

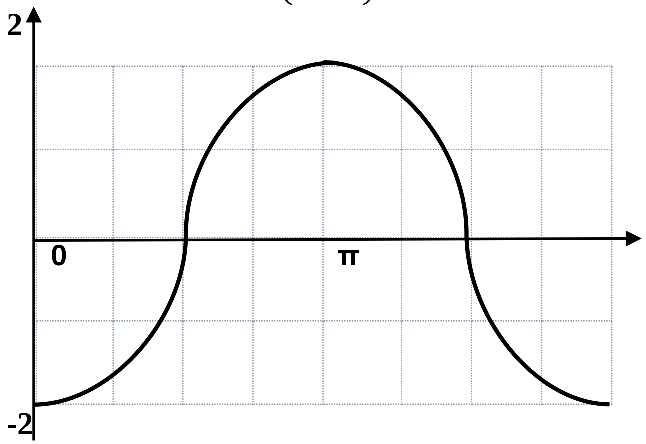
		$\binom{4}{3}(1)^1(-2x)^3$ $= 4.1.-8x^3$ $= -32x^3$ $\binom{4}{4}(1)^0(-2x)^4$ $= 1.1.16x^4$ $= 16x^4$ $1 - 8x + 24x^2 - 32x^3 + 16x^4$				
1.10	**1**	$\sqrt{8} + \sqrt{32}$ $= \sqrt{4 \times 2} + \sqrt{16 \times 2}$ $= \sqrt{4}.\sqrt{2} + \sqrt{16}.\sqrt{2}$ $= 2\sqrt{2} + 4\sqrt{2}$ $= 6\sqrt{2}$	**Correct answer** $6\sqrt{2}$ **OR** **Uses rules of surds to simplify** $\sqrt{8} = \sqrt{4 \times 2}$ *or* $\sqrt{32} = \sqrt{16 \times 2}$			
1.11	**1**	$\frac{1+x}{8} = \frac{2+x}{4}$ $4(1+x) = 8(2+x)$ $4 + 4x = 16 + 8x$	**Correct answer** **'Allow for slips'** **OR**			

		$4x - 8x = 16 - 4$ $-4x = 12$ $x = -3$	**Correct expansion on both sides** $4 + 4x = 16 + 8x$			
1.12	**1**	$2^{x-3} = 32$ $2^{x-3} = 2^5$ **Equate the powers** $x - 3 = 5$ $x = 8$ **Alternative Solution** $\ln 2^{x-3} = \ln 32$ $(x-3)\ln 2 = \ln 32$ $x - 3 = \frac{\ln 32}{\ln 2}$ $x - 3 = 5$ $x = 8$	**Correct answer** **OR** **Takes '*ln*' or 'log' on both sides.** **OR** **Writes 32 = 2^5**			
1.13a	**1**	$z \times w = 2i(3 + i)$ $= 6i + 2i^2$ $= 6i + 2(-1)$ $= 6i - 2 \ or -2 + 6i$	**Correct answer** **OR** **Any substitution of -1 into i^2** **OR** **Has $6i + 2i^2$**			

1.13b	1	Plot $z = 2i$ on the Argand Diagram  	Correct plotting of z			
1.14	2	Let $f(x) = x^3 + 2x^2 - 5x - 6$ By Factor Theorem: $f(-1) = (-1)^3 + 2(-1)^2 - 5(-1) - 6 = 0 \rightarrow (x+1) \textit{ is a factor}$ $f(2) = (2)^3 + 2(2)^2 - 5(2) - 6 = 0 \rightarrow (x-2) \textit{ is a factor}$ $f(-3) = (-3)^3 + 2(-3)^2 - 5(-3) - 6 = 0 \rightarrow (x+3) \textit{ is a factor}$ OR	Any one correct factor OR Uses factor theorem: f (a) = 0, (x – a) is a factor f (-a) = 0, (x + a) is a factor.	Correct answer Three correct factors $(x+1)(x-2)(x+3)$ OR Two correct factors **Alternate Method:** **Using coefficients only:** -1 -1 6 -1 1 2 -5 -6 1 1 -6 0 $x^2 + x - 6$ Remainder (x +3) (x – 2) (x + 1) is a factor Factors: $(x+1)(x-2)(x+3)$		

		To find the other factors; $x^2 + x - 6$ $x + 1 \overline{) x^3 + 2x^2 - 5x - 6}$ $\underline{-x^3 + x^2}$ $x^2 - 5x$ $\underline{-x^2 + x}$ $-6x - 6$ $\underline{- - 6x - 6}$ . . **Factors are:** $(x+1)(x^2 + x - 6)$ $(x+1)(x-2)(x+3)$	**OR** **Has** **(x + 1)(x^2 + x – 6)** **(x – 2)(x^2 + 4x + 3)** **(x + 3)(x^2 – x - 2)**			
1.15	**4**	$Z^4 = 256\,(\cos 120° + i \sin 120°)$ **Angles:** $\theta = \frac{360°}{4} = 90°\ apart$ $\theta_0 = \frac{120°}{4} = 30°$ **Re** $\theta_1 = 30 + 90 = 120°$ $\theta_2 = 120 + 90 = 210°$ $\theta_3 = 210 + 90 = 300°$  $r = 256^{\frac{1}{4}} = 4$ $z_0 = 4cis30°$ $= 4cis\frac{\pi}{6}$ $= 3.46 + 2i$	**Has the correct value of r = 4** **OR** **Uses De Moivre's theorem.** $r^{\frac{1}{4}}\left(cis\frac{\theta + 360k}{4}\right)$ **OR** **Any correct argument, θ**	**Has only one root correct without the Argand diagram.** **OR** **Writes only the angles.**	**Has only two roots correct without the Argand diagram.** **OR** **Has all the roots correct except for R.**	**Has all the 4 roots correct and represented on the Argand diagram.** **Or** **All 3 roots either in rectangular form or polar form.** **Or** **At least 3 roots correct.**

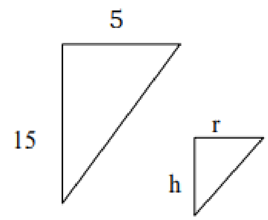
		$z_1 = 4cis120°$ $= 4cis\frac{2\pi}{3}$ $= -2 + 3.46i$ $z_2 = 4cis210°$ $= 4cis\frac{7\pi}{6}$ $= -3.46 - 2i$ $z_3 = 4cis300°$ $= 4cis\frac{5\pi}{3}$ $= 2 - 3.46i$				
2.1	1	$\cot\theta = \frac{1}{\tan\theta}$ $\cot\left(\frac{\pi}{4}\right)$ $= \frac{1}{\tan\left(\frac{\pi}{4}\right)}$ $\cot\left(\frac{\pi}{4}\right) = \frac{1}{1}$ $= 1$	Correct answer OR Substitutes the reciprocal identities $cot\ \theta = \frac{1}{\tan\theta}$			
2.2	2	$(1 + \cot^2\theta)(1 - \cos^2\theta) = 1$ LHS: $(1 + \cot^2\theta)(1 - \cos^2\theta)$ $(\csc^2\theta)(\sin^2\theta)$ $\left(\frac{1}{\sin^2\theta}\right)(\sin^2\theta)$	Has shown that $(1 + \cot^2\theta) = \csc^2\theta$ OR $(1 - \cos^2\theta) = \sin^2\theta$	Correct steps shown to get 1 on the LHS.		

		1 = RHS				
2.3	1	$\sqrt{2}\sin\theta = 1$ $\sin\theta = \frac{1}{\sqrt{2}}$ θ *falls on quadrant I and II* $\theta = 45°, 135°$ $\theta\epsilon\{45°, 135°\}$ or $\theta\epsilon\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$	Correct answer OR One correct angle either in degrees or radians.			
2.4	1	$A = 2$ $Period = \frac{2\pi}{1}$ $= 2\pi\ (360°)$  	Correct –ve cosine curve shape OR Correct amplitude OR Correct period			

2.5	2	**Compound Angle Formula:** $\sin A\cos B + \cos A\sin B = \sin(A+B)$ $\sin 10°\cos 80° + \cos 10°\sin 80° = \sin(10° + 80°)$ $= \sin 90°$ $= 1$	**Writes the correct compound angle formula.** **OR** **Only uses calculator to get 1 or 0.98**	**Correct answer**		
2.6	3	$A = \frac{max - min}{2}$ $= \frac{75-3}{2} = \frac{72}{2}$ $= 36$ $B = \frac{360°}{0.5} = \frac{2\pi}{0.5}$ $= 4\pi\ or\ 720°$ **Vertical shift:** $D = \frac{max + min}{2}$ $= \frac{75+3}{2} = \frac{78}{2}$ $= 39$ $C = 0$ **– no horizontal shift** $\therefore h(t) = 36\ Cos\ 4\pi t + 39$	**Was able to identify the amplitude** **A = 36** **OR** **Has *C = 0*** **OR** **Has *D = 39***	**Was able to find the value of B correctly.** $B = \frac{360°}{\frac{1}{2}} = \frac{2\pi}{\frac{1}{2}}$ $= 4\pi\ or\ 720°$ **OR** **Finds any two from A, C and D correctly.**	**Correct answers.** ***A = 36*** ***B = 4π or 720° or 12.57*** ***C = 0*** ***D = 39***	
3.1	1	**g(x) is discontinuous at:** $x = 2$	**Correct answer**			

3.2	2	$\lim_{x\to -3}\frac{x^2+5x+6}{x+3}=\frac{(-3)^2+5(-3)+6}{(-3)+3}=\frac{0}{0}$ $\rightarrow$ ***limit may exist*** $\lim_{x\to -3}\frac{(x+3)(x+2)}{x+3}$ **L'Hopital's Rule:** $\lim_{x\to -3} x+2$ $\lim_{x\to -3}\frac{2x+5}{1}$ $=(-3)+2$ $=2(-3)+5$ $=-1$ $=-1$	**Was able to factorise the numerator into** $(x+3)(x+2)$ **OR** **Substitutes -3 into the expression** **OR** **Differentiates the numerator or denominator.**	**Correct answer**		
3.3	2	$\lim_{x\to\infty}\frac{5x(x-2)}{3x^2-2x+1}$ $\lim_{x\to\infty}\frac{5x^2-10x}{3x^2-2x+1}$ $\lim_{x\to\infty}\frac{\frac{5x^2}{x^2}-\frac{10x}{x^2}}{\frac{3x^2}{x^2}-\frac{2x}{x^2}+\frac{1}{x^2}}$ $\lim_{x\to\infty}\frac{5-\frac{10}{x}}{3-\frac{2}{x}+\frac{1}{x^2}}$ $=\frac{5-0}{3-0+0}$	**Expand** $5x(x-2)$ **Correctly to get** $5x^2-10x$ **OR** **Identifying the variable with the highest power as** x^2	**Correct answer** **Alternative Method:** **Some students might only use the highest POWER:** $\lim_{x\to\infty}\frac{5x^2}{3x^2}=\frac{5}{3}$		

		$= \frac{5}{3}$				
3.4	2	$f(x) = 4x^3 - 3x + e^x - 1$ $f'(x) = 12x^2 - 3 + e^x$ $f''(x) = 24x + e^x$	Differentiates f(x) correctly. $f'(x) = 12x^2 - 3 + e^x$	Correct answer $f''(x) = 24x + e^x$		
3.5	4	$s(t) = t^3 + 3t^2 + 2t$ 1st Derivative: $v(t) = s'(t) = 3t^2 + 6t + 2$ 2nd Derivative: $a(t) = v'(t) = 6t + 6$ Initial Acceleration: t = 0 s: $a_{t=0} = a(0)$ $\Rightarrow a(0) = 6(0) + 6$ $\Rightarrow \quad a \quad = \ 6\, m/s^2$	Shows any sign of expansion to s(t) OR Shows any sign of differentiation. OR Substitutes t = 0 to any equation.	Was able to differentiate s(t) once correctly $3t^2 + 6t + 2$	Was able to differentiate s(t) twice correctly. $6t + 6$	Correct answer

3.6	4	Given that $\frac{dV}{dt} = 60$, find $\frac{dh}{dt}$ $\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$ Using Similar Triangles: $V = \frac{1}{3}\pi r^2 h$ $V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2$  $V = \frac{\pi}{27}h^3$ $\frac{dV}{dh} = \frac{3\pi}{27}h^2$ $\quad$ $\frac{r}{h} = \frac{5}{15}$ $\frac{dh}{dV} = \frac{9}{\pi h^2}$ $\quad$ $r = \frac{5h}{15} = \frac{h}{3}$ $\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$ $\frac{dh}{dt} = 60 \times \frac{9}{\pi h^2} = \frac{540}{\pi h^2} = \frac{540}{\pi \times 3^2}$ $\frac{dh}{dt} = \frac{60}{\pi}\ cm/s$ or 19.10 cm/s	Writes any of these rates of change: $\frac{dV}{dt}, \frac{dh}{dt}, \frac{dV}{dh}$ *and* $\frac{dh}{dV}$ OR Finds r correctly using similar triangles. OR Shows any sign of differentiation to V. **Alternate method: Implicit Differentiation (NO CHAIN RULE)** $V = \frac{\pi}{27}h^3 \rightarrow \frac{dV}{dt} = \frac{\pi}{27}.3h^2.\frac{dh}{dt} \rightarrow \frac{dV}{dt} = \frac{\pi}{27}.3h^2.\frac{dh}{dt}$ $\rightarrow 60 = \frac{\pi}{9}.(3)^2.\frac{dh}{dt}$ $\rightarrow 60 = \frac{\pi}{9}.9.\frac{dh}{dt} \therefore \frac{dh}{dt} = \frac{60}{\pi}$ OR Writes dV/dt = 60	Has the correct expression for V; $V = \frac{\pi}{27}h^3$ OR Writes the correct Chain rule: $\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$	Has the correct expression for; $\frac{dV}{dh} = \frac{3\pi}{27}h^2$ OR $\frac{dh}{dV} = \frac{9}{\pi h^2}$	Correct answer $\frac{dh}{dt} = \frac{60}{\pi}\ cm/s$ or 19.10 cm/s
4.1	1	$\int x^{1/2}\ dx$ $= \frac{x^{1/2+1}}{\frac{1}{2}+1} + C$	Correct answer Do not penalise if *C* is missing OR			

		$= \frac{x^{3/2}}{\frac{3}{2}} + C$ $= \frac{2}{3}x^{3/2} + C$	Writes $\frac{x^{3/2}}{\frac{3}{2}}$			
4.2	1	$\int 2\, e^{5x}\, dx$ $2\int e^{5x}\ dx$ $= 2.\frac{e^{5x}}{5} + C$ $= \frac{2}{5}e^{5x} + C$	Correct answer Do not penalise if *C* is missing OR Has $\frac{e^{5x}}{5}$			
4.3	2	$\int_{\pi/6}^{\pi/2} 2\cos x\ \ dx$ $= [2\sin x]_{\pi/6}^{\pi/2}$ $= \left[2\sin\frac{\pi}{2}\right] - \left[2\sin\frac{\pi}{6}\right]$ $= [2] - [1]$ $= 1$	Integrates $2\cos x$ Correctly to get $2\sin x$ OR Shows the Fundamental Theorem of Calculus: $F\left(\frac{\pi}{2}\right) - F(\frac{\pi}{6})$ OR Converts $\frac{\pi}{2}$ radians	Correct answer		

			to 90° or $\frac{\pi}{6}$ to 30°			
4.4	2	$Area$ $= \int_2^3 (x^2 - 4)\, dx$ $= \left[\frac{x^3}{3} - 4x\right]_2^3$ $= \left[\frac{(3)^3}{3} - 4(3)\right] - \left[\frac{(2)^3}{3} - 4(2)\right]$ $= [-3] - \left[-5\frac{1}{3}\right]$ $\therefore Area = 2\frac{1}{3}\ sq.units$	Integrates x^2 or 4 correctly OR Shows the Fundamental Theorem of Calculus: $F(3) - F(2)$	Correct answer		
4.5 a	2	$v(t) = 180 - 10t$ Stone at rest: $v = 0\ m/s$ $0 = 180 - 10t$ $10t = 180$ $t = 18\ s$	Any substitution of $v = 0$ for stone at rest.	Correct answer		
4.5 b	3	$v(t) = 180 - 10t$ Displacement Equation: $h(t) = \int v(t)\, dt$ $= \int (180 - 10t)\, dt$ $= 180t - \frac{10t^2}{2} + C$	Was able to integrate any term correctly. OR Shows any sign of integration.	Uses the initial conditions to find the constant. $C = 20$ OR	Correct answer OR Has the correct expression for *s(t)*: $h(t) = 180t - 5t^2 + 20$	

		$= 180t - 5t^2 + C$ $t = 0,\ h(0) = 20$ $\therefore C = 20$ $h(t) = 180t - 5t^2 + 20$ $h(5) = 180(5) - 5(5)^2 + 20$ $= 900 - 125 + 20$ $\therefore\ h = 795\ m\ from\ the\ ground.$	**OR** **Find** $h(5)$	**Has** $180t - 5t^2$ **without the constant C = 20**		
4.6	**4**	**Let** $N(t)$ = **quantity at time,** t **and** k **be the constant of proportionality** $(k > 0 \Rightarrow growth\ rate)$ $\frac{dN}{dt} \alpha\ N$ $\frac{dN}{dt} = kN$ $\int \frac{dN}{N} = \int k\ dt$ $lnN = kt + C$ $lnN = kt + lnN_0$ **Let** $C = ln\ N_0$ $lnN - lnN_0 = kt$ $ln\left(\frac{N}{N_0}\right) = kt$	**Was able to separate variables:** $\frac{dN}{N} = k\ dt$ **OR** $\frac{dN}{dt} = kN$ **OR** **Integrates on both sides.** **OR**	**Was able to get the general solution:** $N = N_0e^{kt}$ **OR** $N = 1e^{kt}$	**Used the initial conditions to find k:** $ln(6000) = 10k$ **OR** $k = 0.8699$ **or** $k = 0.870$ **or** $k = 0.9$	**Correct answer** **Possible answers will depend on the value of k:** $t = 10.47\ days$ $t_{k=0.87} = 10.47\ days$ $t_{k=0.9} = 10.12\ days$

		$\frac{N}{N_0} = e^{kt}$ $\therefore \boxed{N = N_0 e^{kt}}$ **where** N_0 **= initial value** **Find k:** $N_0 = 1,\ t = 10,\quad N = \frac{1}{2} \times 12000 = 6000$ $6000 = 1e^{k(10)}$ $\frac{6000}{1} = e^{10k}$ $ln(6000) = 10k$ $k = 0.8699 \ldots$ $\therefore N = 1e^{0.8699t}$ N **= ¾ x 12000 = 9000,** $t = ?$ $N = 1e^{0.8699t}$ $9000 = 1e^{0.8699t}$ $\ln 9000 = 0.8699t$ $t = 10.47\ days$ $t_{k=0.87} = 10.47\ days$ $t_{k=0.9} = 10.12\ days$	**Finds half of population to be 6000** **OR** **Finds three-quarter of population to be 9000**			