



South Pacific Form Seven Certificate

# MATHEMATICS WITH CALCULUS SYLLABUS

2023



## GENERAL INFORMATION

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The Office of the Educational Quality and Assessment Programme (EQAP) is located at:

3 Luke Street, Nabua  
Suva  
Fiji

Tel: (679) 3315600  
Email: eqap@spc.int  
Web: www.spc.int / www.eqap.org.fj

Any correspondence regarding this syllabus should be addressed to:

The Director EQAP  
Pacific Community (SPC)  
Private Mail Bag  
Suva  
Fiji

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# SOUTH PACIFIC FORM SEVEN CERTIFICATE

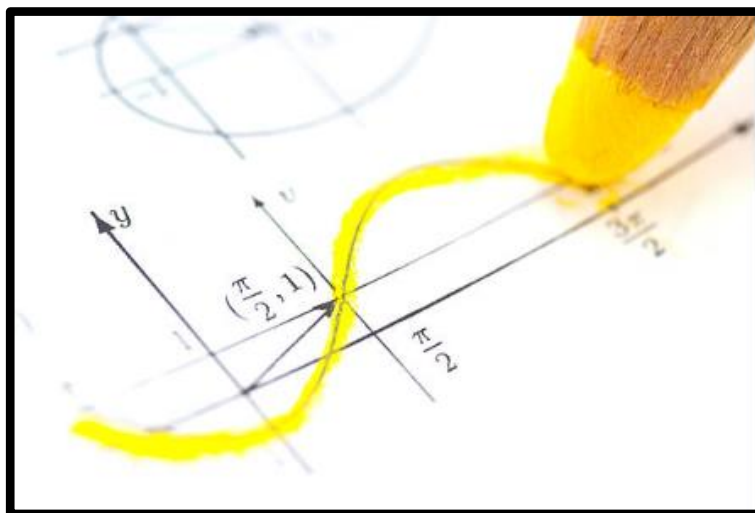
## MATHEMATICS with CALCULUS

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# MATHEMATICS with CALCULUS

## 1.0 PREAMBLE AND RATIONALE

This syllabus defines the requirements for the South Pacific Form Seven Certificate Mathematics (with Calculus) qualification.

There are explanatory notes provided below the table of specific learning outcomes in some Strands. The specific learning outcomes in these Strands are to be read in conjunction with these notes. Students also require knowledge and understanding of outcomes from the National Year 12 or Form 6 qualification or its equivalent, which are related to the specific outcomes of SPFSC Mathematics Calculus.

This syllabus subsumes/replaces all previous EQAP Mathematics with Calculus syllabus. The content and outcomes of the subject are aligned to the New Zealand National Certificate of Educational Assessment (NCEA) Level 3 Mathematics (with Calculus) Achievement Standards as published by New Zealand Qualifications Authority (NZQA).

The course is designed for students who wish to undertake university studies in mathematics and other related fields.

## 2.0 COURSE AIM

Students undertaking this course are expected to:

- demonstrate mathematical skills, concepts and understanding of the mathematical processes, required for Measurement, Calculus, Trigonometry and Algebra at a level that is equivalent to that required for any Form 7 qualification or its equivalents including the NZ Universities Entrance, NCEA Level 3, University of the South Pacific (USP) Foundation, etc.
- apply these skills, concepts, and understanding to familiar and unfamiliar problems arising in real and simulated situations.
- demonstrate the ability to select and use appropriate mathematical techniques for problem solving.
- demonstrate the ability to reason logically and systematically.
- demonstrate the ability to communicate mathematical ideas.

## 3.0 PREREQUISITES

Students taking this course are expected to have successfully completed the national Year 12 Senior Secondary Certificate Mathematics course or its equivalent.

## 4.0 GENERAL OBJECTIVES

In a range of meaningful contexts, students will be engaged in thinking mathematically. They will solve problems and model situations that require them to:

1. apply algebraic techniques to real and complex numbers.
2. use and manipulate trigonometric functions and expressions.
3. demonstrate knowledge of basic and advanced concepts and techniques of differentiation and integration.

## 5.0 CONTENT COMPONENTS

The content of the SPFSC Mathematics with Calculus course is organised under four Strands and a number of Sub-Strands under each Strand. These are outlined below:

Strand Number	Strand Title/ Major Learning Outcomes	Sub Strand number	Sub-Strand Title / Key Learning Outcomes	External / Internal Assessment
1.0	<b>Algebra</b> <i>Students are able to demonstrate knowledge and critically evaluate problems and model situations that require them to apply algebraic techniques to real and complex numbers.</i>	1.1	Algebra basic skills  <i>Students are able to demonstrate knowledge and critically evaluate basic algebraic skills.</i>	External and Internal
		1.2	Polynomial and non-linear equation  <i>Students are able to demonstrate knowledge and critically evaluate how to form and use polynomial and non-linear equations.</i>	External and Internal
		1.3	Complex numbers  <i>Students are able to demonstrate knowledge and critically evaluate how to use and manipulate complex numbers.</i>	External
2.0	<b>Trigonometry</b> <i>Students are able to demonstrate knowledge and critically evaluate how to use trigonometric functions as well as apply its relationship to solve problems.</i>	2.1	Trigonometry basic skills  <i>Students are able to demonstrate knowledge and critically evaluate solving problems and model situations that require them to use and manipulate trigonometric functions and expressions.</i>	External and Internal
		2.2	Trigonometric Functions to Solve Problems  <i>Students are able to demonstrate knowledge and critically evaluate how to select and form a trig function to solve problems.</i>	External
		2.3	Trigonometric Identities  <i>Students are able to demonstrate knowledge and critically evaluate how to prove trigonometric identities using various formulae.</i>	External and Internal

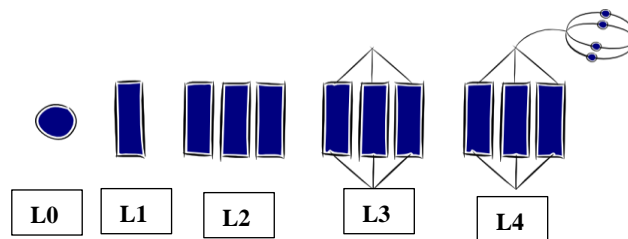
3.0	<b>Differentiation</b> <i>Students are able to demonstrate knowledge and critically evaluate advanced concepts and techniques of differentiation.</i>	3.1	Differentiation Basic skills <i>Students are able to demonstrate knowledge and critically evaluate how to use basic differentiation skills to solve simple problems.</i>	External
		3.2	Discontinuity and Limits of Functions <i>Students are able to demonstrate knowledge and critically evaluate how to identify discontinuity and limits of functions.</i>	External
		3.3	Application of Differentiation <i>Students are able to demonstrate knowledge and critically evaluate how to apply a variety of differentiation technique to functions and relations.</i>	External and Internal
		3.4	Differentiate by Sketching to Solve Problems <i>Students are able to demonstrate knowledge and critically evaluate how to use differentiation to solve problems involving sketching graphs of polynomials and derivatives.</i>	External
4.0	<b>Integration</b> <i>Students are able to demonstrate knowledge and critically evaluate advanced concepts and techniques of integration.</i>	4.1	Integration Basic Skills <i>Students are able to demonstrate knowledge and critically evaluate how to use basic integration skills to solve simple problems.</i>	External and Internal
		4.2	Use of variety of integration technique <i>Students are able to demonstrate knowledge and critically evaluate how to apply a variety of integration and anti-differentiation techniques to functions and relations, using both analytical and numerical methods.</i>	External

		4.3	Volumes of Solids of Revolution <i>Students are able to demonstrate knowledge and critically evaluate</i> how to use integration to find the volumes of solids of revolution.	External and Internal
		4.4	Form and interpret solutions <i>Students are able to demonstrate knowledge and critically evaluate</i> how to form differential equations and interpret the solutions.	External

## 6.0 UNPACKING LEARNING OUTCOMES

In this syllabus, Learning Outcomes are stated at three levels of generality: Major Learning Outcomes (MLOs) are stated at the Strand level, Key Learning Outcomes (KLOs) are stated at the Sub-Strand level, and Specific Learning Outcomes (SLOs) are unpacked from the Key Learning Outcomes. Each SLO is a combination of a cognitive skill and a specific content component. Each SLO is given a skill level, (1, 2, 3 or 4) between, and this skill level results from the categorisation of the cognitive skill that is embedded in the SLO using the SOLO taxonomy<sup>1</sup>.

The SOLO taxonomy provides a simple, reliable and robust model for three levels of understanding – surface, deep, and conceptual (Biggs and Collis 1982).



At the **Prestructural** level (L0) of understanding, the task is inappropriately attacked, and the student has missed the point or needs help to start. The next two levels, unistructural and Multistructural are associated with bringing in information (surface understanding).

At the **Unistructural** level (L1), one aspect of the task is picked up, and student understanding is disconnected and limited. The jump to the Multistructural level is quantitative.

At the **Multistructural** level (L2), several aspects of the task are known but their relationships to each other and the whole are missed. The progression to relational and extended abstract outcomes is qualitative.

<sup>1</sup> Structure of Observed Learning Outcomes by Biggs and Collis (1982)

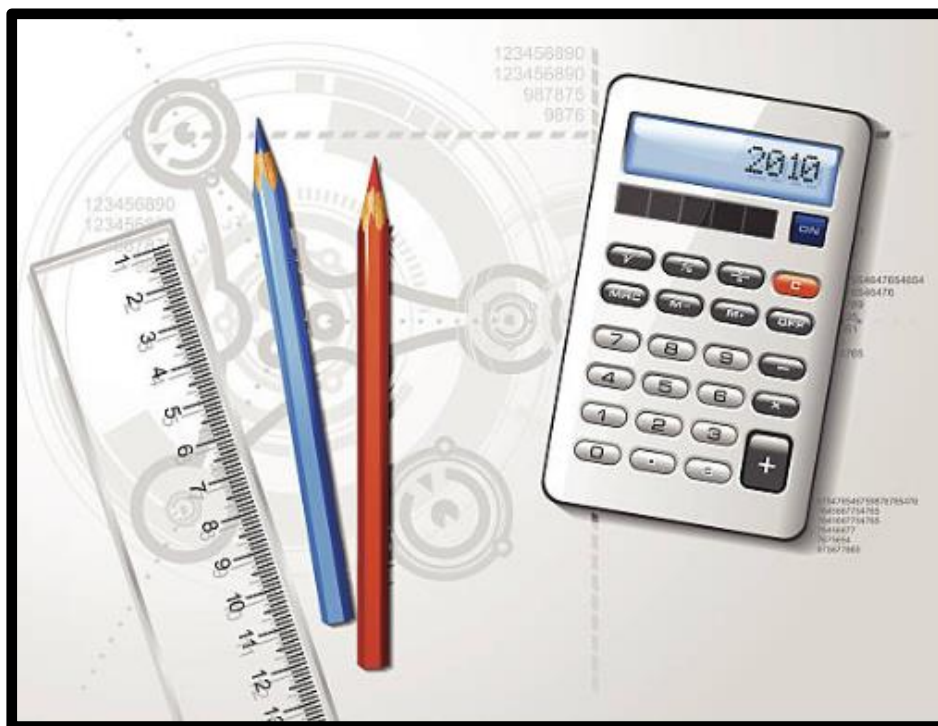


At the **Relational** level (L3), the aspects are linked and integrated, and contribute to a deeper and more coherent understanding of the whole.

At the **Extended Abstract** level (L4), the new understanding at the relational level is re-thought at another conceptual level, looked at in a new way, and used as the basis for prediction, generalisation, reflection, or creation of new understanding (adapted from Hook and Mills 2011). [[http://pamhook.com/solo-taxonomy/..](http://pamhook.com/solo-taxonomy/)]

The progression from Level 1 to Level 4 is exemplified in the progression from *define* → *describe* → *explain* → *discuss* with each succeeding level indicating a *higher level of understanding*, as follows:

- **define** – to state a basic definition of a concept [Unistructural or L1]
- **describe** – to give the characteristics of, or give an account of, or provide annotated diagrams. [Multistructural or L2]
- **explain** – to provide a reason for a relationship – an event and its impact, a cause and an effect, as to *how* or *why* something occurs. [Relational or L3]
- **discuss** – this means *linking ideas* (descriptions, explanations) to make generalisations or predictions or evaluations. It may involve relating, comparing, analysing, and justifying.
- **solve/calculate/compute** – to carry out a series of algorithms to arrive at a solution [Multistructural (L2) or Relational (L3) or even Extended (L4) depending on the complexity of the algorithm]. If there are two ‘loadings’ in the calculations (a standard problem) then skill level would be L2, if three ‘loadings’ (a complex problem) then L3 and four loadings (a more complex problem) for L4.



## 7.0 STRANDS, SUBSTRANDS AND LEARNING OUTCOMES

### STRAND 1.0 ALGEBRA

#### Major Learning Outcome

Students are able to demonstrate knowledge and critically evaluate problems and model situations that require them to apply algebraic techniques to real and complex numbers.

### SUB-STRAND 1.1 Algebra Basic Skills (EA & IA)

#### Key Learning Outcome

Students are able to demonstrate knowledge and critically evaluate basic algebraic skills.

SLO No.	Specific Learning Outcomes Students are able to:	Skill Level	SLO Code
1	<b>simplify</b> linear equations eliminating fractional terms. e.g. $\frac{x}{7} + \frac{x}{2}$	1	Cal1.1.1.1
2	<b>apply</b> laws of indices to simplify exponential expressions. (Any two laws used) e.g. $\frac{x^2 \cdot x^6}{x^3}$ , $2xy^3 \times (x^2y)^3$	1	Cal1.1.1.2
3	<b>apply</b> laws of logarithms to simplify logarithmic expressions (Any two laws used) e.g. $\log 4^2 + \log 4$ , $\log 3 + \log 4 - \log 2$	1	Cal1.1.1.3
4	<b>solve</b> straightforward logarithmic equations. e.g. $\log_3(x) = 4$	1	Cal1.1.1.4
5	<b>divide</b> a polynomial by $(x \pm a)$ .	1	Cal1.1.1.5
6	<b>find</b> the remainder to a function for $x = a$ where $(x - a)$ is not a factor.	1	Cal1.1.1.6
7	<b>complete</b> the square of reducible quadratics of the form $ax^2 + bx + c$ . (The value of 'a' or coefficient of $x^2$ is 1)	1	Cal1.1.1.7
8	<b>simplify</b> sums, differences, and products of surds.	1	Cal1.1.1.8
9	<b>solve</b> two linear equations simultaneously. -(both equations are linear) - The values of x and y are whole numbers.	2	Cal1.1.2.1
10	<b>rearrange</b> a formula to obtain the correct subject. (Note. Subject appears TWICE.)  e.g. Make y the subject in $\frac{4x+y}{m+zy} = 6t$  $= \frac{4x+y}{m+zy} = 6t \rightarrow 4x + y = 6t(m + zy) - 1. (form a linear eqn)$  $= 4x + y = 6tm + 6tzy - 2. (Expand bracket)$  $y - 6tzy = 6tm - 4x - 3. (Collect terms that have y)$  $y(1 - 6tz) = 6tm - 4x - 4. Factorize "y"$  $\therefore y = \frac{6tm - 4x}{(1 - 6tz)} - 5. y the subject$	4	Cal1.1.4.1
11	<b>solve</b> quadratic equations by factorisation	2	Cal1.1.2.2
12	<b>factorise</b> a cubic function using factor theorem  Method: 1 1. Apply long division	4	Cal1.1.4.2

	2. Answers in the long division will be a quadratic. 3. Factorise to get two other factors. 4. Write ALL 3 factors.  Method: 2 1. identify the constant in the cubic function. 2. Find factors of the constant. 3. Substitute each factor into the function. 4. The factors that give Zero as answers are the factors. Write ALL 3 factors.		
13	<b>complete</b> the square of quadratics of the form $ax^2 + bx + c$ . (The value of 'a' or coefficient of $x^2 > 1$ ) e.g. Complete the square for $2x^2 + 4x - 3$	2	Cal1.1.2.3
14	<b>solve</b> straightforward surd equations and check solutions by substitution. e.g. $\sqrt{2x - 1} = x - 2$	2	Cal1.1.2.4
15	<b>simplify</b> quotients of surds (including rationalizing). e.g. Simplify $\frac{2}{3 + \sqrt{2}}$  $= \frac{2}{3 + \sqrt{2}} \times \frac{3 - \sqrt{2}}{3 - \sqrt{2}} - 1. (Conjugate)$ $= \frac{2(3 - \sqrt{2})}{(3^2) - (\sqrt{2})^2} - 2. Difference of Two Squares (denominator)$  $= \frac{2(3 - \sqrt{2})}{9 - 2} - 3. Using algebraic skills$ $= \frac{(3 - \sqrt{2})}{4} - 4. Cancelling & showing denominator is rationalized$	4	Cal1.1.4.3
16	<b>simplify</b> products of surds by expanding the brackets (apply distributive law)	2	Cal1.1.2.5
17	<b>express</b> a single algebraic fraction as a sum of its partial fractions where the denominator includes two linear factors.	2	Cal1.1.2.6
18	<b>solve</b> linear inequations. i.e. $\frac{x - 3}{4} + 1 < \frac{x + 1}{-3}$	3	Cal1.1.3.1
19	<b>express</b> a single algebraic fraction as a sum of its partial fractions where denominators of fractions include a non-linear function or an irreducible quadratic function. (Algebraic Fractions can be LINEAR, REPEATED or QUADRATIC.)	3	Cal1.1.3.2
20	<b>find</b> specific terms in expansions $(a \pm b)^n$ where n is greater than 4 using the Binomial theorem where a or b is a fraction. e.g. $(2x + \frac{1}{x^2})^5$ . Method: - Identify 'a' and 'b' and write the fraction in base-index form. - Writes the General term formula or Binomial theorem. - Substitutes the values of r, a, b and n into either formula. - Using algebraic skills to manipulate the formula for the $n^{\text{th}}$ term.	4	Cal1.1.4.4
21	<b>prove</b> a given mathematical statement is true by using the method of mathematical induction, whereby the variable "n" in the statement provided is not a power. e.g. $5 + 10 + 15 + 20 + \dots + 5n = \frac{5n(n+1)}{2}$	3	Cal1.1.3.3
22	<b>solve</b> quadratic equations using the quadratic formula. Quadratic equation cannot be factorised.	3	Cal1.1.3.4
23	<b>use</b> Binomial Theorem to expand and simplify expressions of the form $(x \pm y)^n$ for $n = 4$ or $5$ .	3	Cal1.1.3.5
24	<b>find</b> the coefficients or constant term, or the term "independent of x" in expansions where n is greater than 4, using the Binomial theorem.	4	Cal1.1.4.5
25	<b>prove</b> a given mathematical statement is true by using the method of mathematical induction, whereby the variable "n" in the statement to be proved, is a power (the laws of indices is applied). e.g. $2^2 + 2^3 + 2^4 + \dots + 2^{n+1} = 2^2(2^n - 1)$	4	Cal1.1.4.6

	Method: <ul style="list-style-type: none"> <li>- Prove that it is true for <math>n = 1</math></li> <li>- Assume that it is true for <math>n = k</math></li> <li>- Prove that it is also true for <math>n = k + 1</math>. (Laws of indices is applied)</li> <li>- Writes the conclusion.</li> </ul>		
26	<b>express</b> a single algebraic fraction as a sum of its partial fractions where one Type is merged into another Type. Example; includes a quadratic factor which is repeated. i.e. $\frac{3x^4 + 4x^2 + 2x + 2}{(x^2 + 1)^2} = \frac{Ax + B}{(x^2 + 1)} + \frac{Cx + D}{(x^2 + 1)^2}$	4	Cal1.1.4.7

## SUB-STRAND 1.2 Polynomial and Non- Linear Equations (EA & IA))

### Key Learning Outcome

*Students are able to demonstrate knowledge and critically evaluate how to form and use polynomial and non-linear equations.*

SLO No.	Students are able to:	Specific Learning Outcomes	Skill Level	SLO Code
1	<b>simplify</b> exponential equations that include expressing negative powers with positive exponents. e.g. $\left(-\frac{1}{3}\right)^{-2}$		1	Cal1.2.1.1
2	<b>solve</b> linear equations. e.g. $\frac{x}{7} + 2 = -3$		1	Cal1.2.1.2
3	<b>solve</b> exponential equations That can be expressed with the same base but different powers e.g. $2^{2x} = 4^{x+1}$		2	Cal1.2.2.1
4	<b>use</b> Remainder and Factor theorems involving straight forward substitution method.		2	Cal1.2.2.2
5	<b>solve</b> problems that require translating word problems into simple mathematical expressions, involving two variables and two equations.		3	Cal1.2.3.1
6	<b>analyse</b> the existence of solutions in the context of a simple situation involving word problems.		3	Cal1.2.3.2
7	<b>solve</b> simultaneous equations, where one equation is linear and the other a quadratic or cubic function		3	Cal1.2.3.3
8	<b>solve</b> logarithmic equations involving all the application of log rules. e.g. $2 \log(x) = \log(2x) + \log 3$		3	Cal1.2.3.4
9	<b>solve</b> rational equations. e.g. $\frac{x+2}{5} = x - 1$ or $\frac{x+1}{x-2} = 3$ or $\frac{x}{5} - \frac{x+2}{3} = 7$		3	Cal1.2.3.5
10	<b>use</b> Remainder and Factor theorems to find unknowns in a polynomial equation e.g. A polynomial is given as $g(x) = x^3 + 2x^2 + kx + m$ . When $g(x)$ is divided by $(x + 1)$ the remainder is 3 and when $g(x)$ is divided by $(x - 1)$ the remainder is -1. Determine the values of $k$ and $m$ . Method: <ul style="list-style-type: none"> <li>- Let the two divisors = 0, then solve for <math>x</math>.</li> <li>- Substitute the two <math>x</math> values and remainders.</li> <li>- Form two linear equations in terms of <math>k</math> and <math>m</math>.</li> <li>- Solve the two equations simultaneously to find <math>k</math> and <math>m</math>.</li> </ul>		4	Cal1.2.4.1

11	Solve a simultaneous system involving a hyperbolic function and a linear function	4	Cal1.2.4.2
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### SUB-STRAND 1.3 Complex numbers (EA)

#### Key Learning Outcome

Students are able to demonstrate knowledge and critically evaluate how to use and manipulate complex numbers.

SLO No.	Specific Learning Outcomes Students are able to:	Skill Level	SLO Code
1	<b>simplify</b> sums, differences, and products of complex numbers expressed in rectangular form.	1	Cal1.3.1.1
2	<b>simplify</b> powers of imaginary number $i$ , where $i^2 = -1 \rightarrow i = \sqrt{-1}$	1	Cal1.3.1.2
3	<b>simplify</b> products and quotients of complex numbers expressed in rectangular form.	2	Cal1.3.2.1
4	<b>simplify</b> products and quotients of complex numbers expressed in polar form.	2	Cal1.3.2.2
5	<b>convert</b> between rectangular ( $a + ib$ ) and polar ( $r\text{cis } \theta$ ) forms of a complex number.	2	Cal1.3.2.3
6	<b>use</b> graphical representations of complex numbers, using polar and rectangular form on an Argand diagram. These problems include <u>a point</u> on the complex plane can be written in rectangular or polar form.	2	Cal1.3.2.4
7	<b>determine</b> solutions of quadratic equations with discriminant less than 0. (There are complex roots)	3	Cal1.3.3.1
8	<b>use</b> graphical representations of complex numbers, using polar and rectangular form on an Argand diagram. These problems include regions that are shaded on a line ( $a \leq \text{Re}/\text{Im}(Z) \leq b$ ) that can be linked back to its rectangular or polar form.	3	Cal1.3.3.2
9	<b>find</b> powers of complex numbers using De Moivre's Theorem. i.e. $z^n = r^n(\cos n\theta + i \sin n\theta)$	3	Cal1.3.3.3
10	<b>Use</b> factor theorem to factorise a polynomial where there are complex factors.	3	Cal1.3.3.4
11	<b>Solve</b> quadratic equations by completing the square when there are complex roots.	3	Cal 1.3.3.5
12	<b>find</b> roots of a complex number given in polar form $z = r(\cos \theta + i \sin \theta)$ or $z = r\text{Cis}\theta$ and represent the solutions on an argand diagram.	4	Cal1.3.4.1
13	<b>find</b> roots of a complex number given in rectangular form. i.e. $z = a + bi$ Method: - Find modulus of $z$ , $r$ - Find argument of $z$ , $\theta$ - Use De Moivre's theorem. - Use algebraic skills to manipulate the equation.	4	Cal1.3.4.2
14	<b>use</b> graphical representations of complex numbers, using polar and rectangular form on an Argand diagram. (These problems include a region involving a circle that can be linked back to its rectangular/polar form. E.g. $-2 \leq  z  < 1$ )	4	Cal1.3.4.3
15	<b>find</b> roots over the complex number system for polynomial equations with real coefficients, including the special case of the $n$ th roots of that	4	Cal1.3.4.4

come from solving equations of the form $z^n = a$ , making links with their graphs.		
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### Explanatory Notes

The outcomes listed above require that students

- have an understanding of the need and relevance of different number systems leading to the development of  $i$  and complex numbers
- carry out mathematical operations on expressions incorporating  $i$ , e.g. powers of  $i$
- convert between rectangular and polar form
- carry out operations on  $rcis\theta$
- use de Moivre's theorem to solve equations of the form  $z^n = a + ib$  and display these solutions

## STRAND 2.0 TRIGONOMETRY

### Major Learning Outcome

Students are able to demonstrate knowledge and critically evaluate how to use trigonometric functions as well as apply its relationship to solve problems.

### SUB-STRAND 2.1 Trigonometry Basic Skills (EA & IA)

### Key Learning Outcome

Students are able to demonstrate knowledge and critically evaluate solving problems and model situations that require them to use and manipulate trigonometric functions and expressions.

SLO No.	Specific Learning Outcomes Students are able to:	Skill Level	SLO Code
1	find exact values of trig expressions using special triangles. e.g. $\sin 75^\circ$ , $\cos 15^\circ$	1	Cal2.1.1.1
2	find values of reciprocal trig functions from a given right-angled triangle. e.g., use of $\sec \theta$ , $\csc \theta$ and $\cot \theta$	1	Cal2.1.1.2
3	solve problems that involve manipulating trig expressions using trigonometric forms of Pythagoras theorem.	2	Cal2.1.2.1
4	solve problems that involve a simple manipulation of trig expressions using compound angle formula.	2	Cal2.1.2.2
5	solve problems that involve a simple manipulation of trig expressions using double angle formula	2	Cal2.1.2.3
6	sketch trig function of the form $y = A \frac{\sin}{\cos} (Bx)$ (involves no shifting)	2	Cal2.1.2.4
7	solve trigonometric equations using algebraic manipulation.	2	Cal2.1.2.5
8	use addition formulae to write a sum of sine and cosine as either a sine or cosine function. e.g. Express $y = a\cos\theta \pm b\sin\theta$ in terms of $y = r \frac{\sin}{\cos} (\theta \pm \alpha)$	3	Cal2.1.3.1
9	solve problems that involve a complex manipulation of trig expressions using compound angle formula.	3	Cal2.1.3.2
10	solve problems that involve a complex manipulation of trig expressions using double angle formula.	3	Cal2.1.3.3
11	solve problems that involve a complex manipulation of trig expressions using sums and products	3	Cal2.1.3.4

12	<b>sketch</b> trig function of the form $y = A \frac{\sin}{\cos} (Bx \pm C)$ (involves horizontal shifting only)	3	Cal2.1.3.5
13	<b>sketch</b> the graphs of inverse and/or reciprocal trigonometric functions ( $\sin x$ , $\cos x$ , $\tan x$ ) with $x$ in radians or degrees showing the main features of the graphs. Consideration of restrictions on the domain of a function so that its inverse is also a function is required.	4	Cal2.1.4.1
14	<b>solve</b> trigonometric equations using identities or by factorising.	4	Cal2.1.4.2
15	<b>sketch</b> trig function of the form $y = A \frac{\sin}{\cos} (Bx \pm C) \pm K$ (involves both horizontal and vertical shifting)	4	Cal2.1.4.3
16	<b>find</b> the maximum/minimum points of a trig. function given in the form: $y = a \cos \theta \pm b \sin \theta$	4	Cal2.1.4.4
17	<b>solve</b> a trigonometric function given in the form: $y = a \cos \theta \pm b \sin \theta$	4	Cal2.1.4.5

**Explanatory Notes:**

A simple manipulation of trig expressions involves a two-step procedure to arrive at a possible solution, where the equation is equal to 0,  $\frac{1}{2}$  or 1. E.g. Solve  $\sin A$  or  $\cos A = 0$ , or  $\sin A$  or  $\cos A = \frac{1}{2}$  and  $\sin A$  or  $\cos A = 1$ . The angle values should be recognisable by sight at Year 13 level.

A complex manipulation of trig expressions involves a three step procedure to arrive at a possible solution. The equation to be solved is equal to a value other than 0,  $\frac{1}{2}$  or 1 and thus need the manipulation of the inverse trig function first in order to arrive at possible angles.

**SUB-STRAND 2.2: Trigonometric functions to solve problems (EA)**

**Key Learning Outcome**

*Students are able to demonstrate knowledge and critically evaluate how to select and form a trig function to solve problems.*

SLO No.	Specific Learning Outcomes Students are able to:	Skill Level	SLO Code
1	<b>solve</b> straightforward problems with models involving trigonometric functions of the form; $y = A \sin B (x + C) + D$ where C or D may be zero	3	Cal2.2.3.1
2	<b>solve</b> straightforward problems with models involving trigonometric functions of the form; $y = A \cos B (x + C) + D$ where C or D may be zero	3	Cal2.2.3.2
3	<b>solve</b> straightforward problems with models involving trigonometric functions of the form; $y = A \tan B (x + C) + D$ where C or D may be zero	3	Cal2.2.3.3
4	<b>solve</b> straightforward problems with models involving trigonometric functions of the form; $y = A \tan B (x + C) + D$ .	4	Cal2.2.4.1
5	<b>solve</b> straightforward problems with models involving trigonometric functions of the form; $y = A \sin B (x + C) + D$ .	4	Cal2.2.4.2
6	<b>solve</b> straightforward problems with models involving trigonometric functions of the form; $y = A \cos B (x + C) + D$ .	4	Cal2.2.4.3
7	<b>form</b> an equation for a model and use the model to solve problems; $y = A \sin B (x + C) + D$ .	4	Cal2.2.4.4
8	<b>form</b> an equation for a model and use the model to solve problems $y = A \cos B (x + C) + D$ .	4	Cal2.2.4.5

### Explanatory Notes

The above outcomes refer to trigonometric functions where C or D may be zero. Solutions of the problems may require knowledge of amplitude, period and frequency.

### SUB-STRAND 2.3: Prove Trigonometric identities (EA & IA)

#### Key Learning Outcome

Trigonometric Identities

*Students are able to demonstrate knowledge and critically evaluate how to prove trigonometric identities using various formulae.*

SLO No.	Specific Learning Outcomes Students are able to:	Skill Level	SLO Code
1	<b>prove</b> trigonometry identities (involving <b>direct substitution</b> of reciprocal relationships $\cot\theta$ , $\operatorname{cosec}\theta$ and $\sec\theta$ ), by solving equations to provide a general solution or a solution within a specified domain.	1	Cal2.3.1.1
2	<b>prove</b> trigonometry identities (involving reciprocal relationships but involves <b>algebraic working</b> ), by manipulating equations to provide a general solution or a solution within a specified domain.	2	Cal2.3.2.1
3	<b>prove</b> trigonometry identities (involving Pythagorean identities), by manipulating equations to provide a general solution or a solution within a specified domain.	2	Cal2.3.2.2
4	<b>prove</b> trigonometry identities (involving compound angle formulae), by manipulating equations to provide a general solution or a solution within a specified domain.	3	Cal2.3.3.1
5	<b>prove</b> trigonometry identities (involving double angle formulae), by manipulating equations to provide a general solution or a solution within a specified domain.	3	Cal2.3.3.2
6	<b>prove</b> trigonometry identities (involving sum and product formulae), by manipulating equations to provide a general solution or a solution within a specified domain.	4	Cal2.3.4.1

$$\tan \frac{x}{2} = \frac{1 - \cos x}{1 + \cos x}$$

$$= \frac{2 \sin^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}}$$

$$= \tan^2 \frac{x}{2}$$



## STRAND 3.0 DIFFERENTIATION

### Major Learning Outcome

*Students are able to demonstrate knowledge and critically evaluate advanced concepts and techniques of differentiation.*

### SUB-STRAND 3.1 Differentiation Basic Skills (EA)

### Key Learning Outcome

Differentiation Basic skills

*Students are able to demonstrate knowledge and critically evaluate how to use basic differentiation skills to solve simple problems.*

SLO No.	Specific Learning Outcomes Students are able to:	Skill Level	SLO Code
1	<b>identify</b> features of a piecewise function of $f(a)$ .	1	Cal3.1.1.1
2	<b>identify</b> features of a piecewise function if the function is discontinuous.	1	Cal3.1.1.2
3	<b>identify</b> features of a piecewise function if the limit exists.	1	Cal3.1.1.3
4	<b>identify</b> features of a piecewise function if the function is differentiable.	1	Cal3.1.1.4
5	<b>find</b> limits of a piecewise function.	1	Cal3.1.1.5
6	<b>differentiate</b> sums of functions.	2	Cal3.1.2.1
7	<b>differentiate</b> quotients, where $f$ and $g$ are both singular functions.	2	Cal3.1.2.2
8	<b>differentiate</b> products, where $f$ and $g$ are both singular functions.	2	Cal3.1.2.3
9	<b>differentiate</b> composite functions (chain rule), where $f$ and $g$ are both singular functions.	2	Cal3.1.2.4
10	<b>differentiate</b> parametric functions.	3	Cal3.1.3.1
11	<b>finding</b> the second derivatives of a given function.	3	Cal3.1.3.2
12	<b>use</b> composite function (Chain Rule) to differentiate complex functions (functions include exponential, trig and log ).	3	Cal3.1.3.3
13	<b>differentiate</b> to find the points of inflection.	3	Cal3.1.3.4
14	<b>use</b> the first principles to differentiate a function (only for polynomials of degree $\leq 3$ ) using $\lim_{h \rightarrow 0} \left( \frac{f(x+h) - f(x)}{h} \right)$ .	4	Cal3.1.4.1
15	<b>solve</b> problems by finding the maxima or minima with proof for polynomial and rational functions.	4	Cal3.1.4.2
16	<b>use</b> Quotient Rule to differentiate complex functions (functions include exponential, trig and log ).	4	Cal3.1.4.3
17	<b>use</b> Product Rule to differentiate complex functions (functions include exponential, trig and log ).	4	Cal3.1.4.4

### Explanatory Notes:

Singular functions include  $\sin ax$ ,  $\cos ax$ ,  $\tan ax$ ,  $ax^n$ ,  $e^{ax}$ , etc.

For level 2 SLOs, exact simplification is presumed to be a possessed skill in year 13.

## SUB-STRAND 3.2 Discontinuity and limits of functions (EA)

### Key Learning Outcome

#### Discontinuity and Limits of Functions

*Students are able to demonstrate knowledge and critically evaluate* how to identify discontinuity and limits of functions.

SLO No.	Students are able to:	Specific Learning Outcomes	Skill Level	SLO Code
1	<b>identify</b> discontinuities algebraically.		1	Cal3.2.1.1
2	<b>find</b> limits algebraically, and numerically by considering behaviour of a function as $x$ approaches a specific value from above and below.		2	Cal3.2.2.1
3	<b>find</b> limits algebraically and numerically by considering behaviour as $x$ approaches $+\infty$ or $-\infty$ .		2	Cal3.2.2.2
4	<b>link</b> features of graphs with the limiting behaviour of functions for either a cubic or quadratic function.		3	Cal3.2.3.1
5	<b>use</b> limiting features of functions to sketch piecewise and rational graphs.		3	Cal3.2.3.2
6	<b>link</b> features of graphs with the limiting behaviour of functions for either a hyperbolic or polynomial function.		4	Cal3.2.4.1
7	<b>use</b> limiting features of functions to sketch complex graphs.		4	Cal3.2.4.2
8	<b>demonstrate</b> knowledge and application of continuity at a point (the limit as $x$ tends to $a$ of $f(x) = f(a)$ ).		4	Cal3.2.4.3
9	<b>link</b> informally concepts of continuity and differentiability.		4	Cal3.2.4.4

## SUB-STRAND 3.3 Application of differentiation technique (EA & IA)

### Key Learning Outcome

#### Application of Differentiation

*Students are able to demonstrate knowledge and critically evaluate* how to apply a variety of differentiation technique to functions and relations.

SLO No.	Students are able to:	Specific Learning Outcomes	Skill Level	SLO Code
1	<b>use</b> a range of differentiation techniques to solve optimization of simple functions (maxima, minima or point of inflection) linked to only <b>one</b> differentiation, $f(x) \rightarrow f'(x)$ .		2	Cal3.3.2.1
2	form equations based on the conditions in optimization problems and reduce to one variable equations.		2	Cal3.3.2.2
3	<b>use differentiation to solve simple</b> problems in kinematics, e.g. in relation to projectile motions etc. where time, distances and velocity are to be found for initial and stationary (at rest) conditions. One level of differentiation involved e.g. from $d(t) \rightarrow v(t)$ , and substitution of time may not be necessary.		2	Cal3.3.2.3
4	<b>use</b> implicit differentiation or parametric differentiation to differentiate simple functions (functions include polynomials, and trigonometric functions).		3	Cal3.3.3.1
5	<b>use</b> a range of differentiation techniques to find an equation of a tangent		3	Cal3.3.3.2
6	<b>use</b> a range of differentiation techniques to solve optimization problems (maxima, minima or point of inflection) that are linked to <b>two</b> differentiations, $f(x) \rightarrow f'(x) \rightarrow f''(x)$		3	Cal3.3.3.3

7	<b>use</b> differentiation to find the rates of change, using the Chain rule with two variables involved. The relationship of both variables is given in the problem.	3	Cal3.3.3.4
8	<b>use differentiation to solve</b> problems in kinematics, e.g. in relation to projectile motions etc. where descriptions of motion are involved, as in $d(t) \rightarrow v(t)$ or $v(t) \rightarrow a(t)$ and values of time are substituted	3	Cal3.3.3.5
9	<b>use</b> implicit differentiation or parametric differentiation to differentiate complex functions (functions include: $Ae^{px}$ , $\ln(ax + b)$ , power functions such as $Ap^x$ )	4	Cal3.3.4.1
10	<b>use</b> a range of differentiation techniques to find an equation of a normal	4	Cal3.3.4.2
11	<b>use</b> a range of differentiation techniques to solve optimization problems (maxima, minima or point of inflection) that involve many overlapping equations or equations to be differentiated are not obviously stated. Method: <ul style="list-style-type: none"> <li>- Write the formula for the quantity, <math>Q(x)</math> to be maximised or minimised.</li> <li>- Find an expression for the <math>Q(x)</math> in one variable.</li> <li>- Differentiate to get <math>Q'(x)</math>, equate to zero and solve for the maximum/minimum values.</li> <li>- Determine whether it is a maximum or a minimum.</li> </ul>	4	Cal3.3.4.3
12	<b>use</b> differentiation to find the rates of change, that uses the Chain rule with two variables involved. The relationship of both variables is <b>not</b> given in the problem. Method: <ul style="list-style-type: none"> <li>- Identify the known rate of change and the rate of change to be found.</li> <li>- Write an equation that relates the quantities identified in step 1.</li> <li>- Take the derivative with respect to time of both sides of the equation.</li> <li>- Solve for the unknown rate of change.</li> </ul>	4	Cal3.3.4.4
13	<b>apply</b> differentiation to problems in kinematics in which “2 derivatives” or 2 <sup>nd</sup> differentiation applies, e.g. $d(t) \rightarrow \text{find } a(t)$ , and values of t are to be substituted.	4	Cal3.3.4.5

### SUB-STRAND 3.4 Differentiate to solve problems by sketching graphs (EA)

<b>Key Learning Outcome</b>
Differentiate by Sketching to Solve Problems <i>Students are able to demonstrate knowledge and critically evaluate</i> how to use differentiation to solve problems involving sketching graphs of polynomials and derivatives.

SLO No.	Specific Learning Outcomes Students are able to:	Skill Level	SLO Code
1	<b>find</b> features of given function, $f(x)$ . (e.g. Intervals of concavity, turning points, points of inflection).	2	Cal3.4.2.1
2	<b>sketch</b> the graph of a derived function, $f^1(x)$ from a given graph $f(x)$ .	3	Cal3.4.3.1
3	<b>identify more than two</b> features of given graphs (selection from limits, differentiability, discontinuity, gradients, concavity, turning points, points of inflection).	3	Cal3.4.3.2
4	<b>sketch</b> graphs of polynomials of degree 3 by using differentiation to identify features such as turning points, point of inflection and region of concavity.	4	Cal3.4.4.1
5	<b>sketch</b> graphs/derivatives to demonstrate knowledge of the above features.	4	Cal3.4.4.2

## STRAND 4.0 INTEGRATION

### Major Learning Outcome

Students are able to demonstrate knowledge and critically evaluate advanced concepts and techniques of integration.

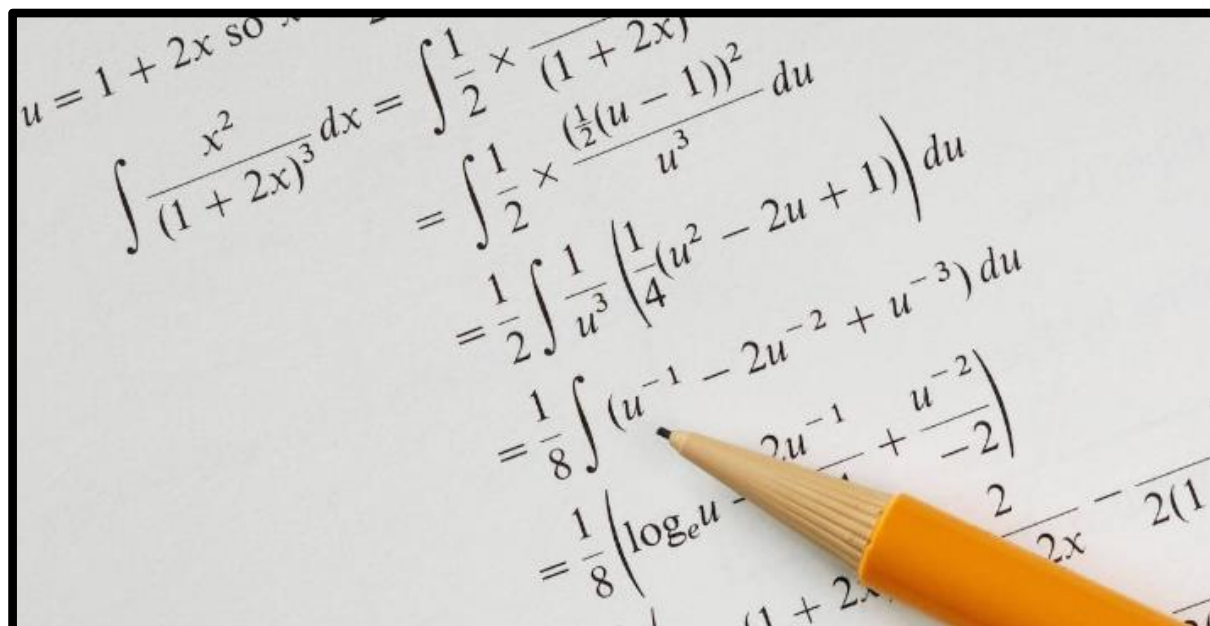
### SUB-STRAND 4.1: Integration Basic Skills (EA & IA)

### Key Learning Outcome

Integration Basic Skills

Students are able to demonstrate knowledge and critically evaluate how to use basic integration skills to solve simple problems.

SLO No.	Students are able to:	Specific Learning Outcomes	Skill Level	SLO Code
1	<b>integrate</b> functions of $ax^n$ , where $n \neq -1$		1	Cal4.1.1.1
2	<b>integrate</b> functions with negative or fractional powers. e.g. $x^{-2}$ , $x^{1/2}$		1	Cal4.1.1.2
3	<b>integrate</b> exponential functions of the form $Ae^{f(x)}$ , where $f(x) = ax + b$		1	Cal4.1.1.3
6	<b>find</b> definite integrals in a simple integration. e.g. $\int_1^2 (x^2 + 2x - 1) dx$		2	Cal4.1.2.1
7	<b>use</b> integration to <b>solve</b> 'straightforward' problems involving areas, where there is a singular function, and the limits of integration are obvious in the problem.		2	Cal4.1.2.2
4	<b>integrate</b> product of two trigonometric functions.		3	Cal4.1.3.1
5	<b>integrate</b> rational functions of the type $y = \frac{ax+b}{cx+d}$ .		3	Cal4.1.3.2
8	<b>use definite</b> integration to <b>solve</b> problems involving areas (the limits of integration are not obvious and may have to be obtained from a sketch of the function/s).		3	Cal4.1.3.3



## SUB-STRAND 4.2: Use of variety of integration techniques (EA)

### Key Learning Outcome

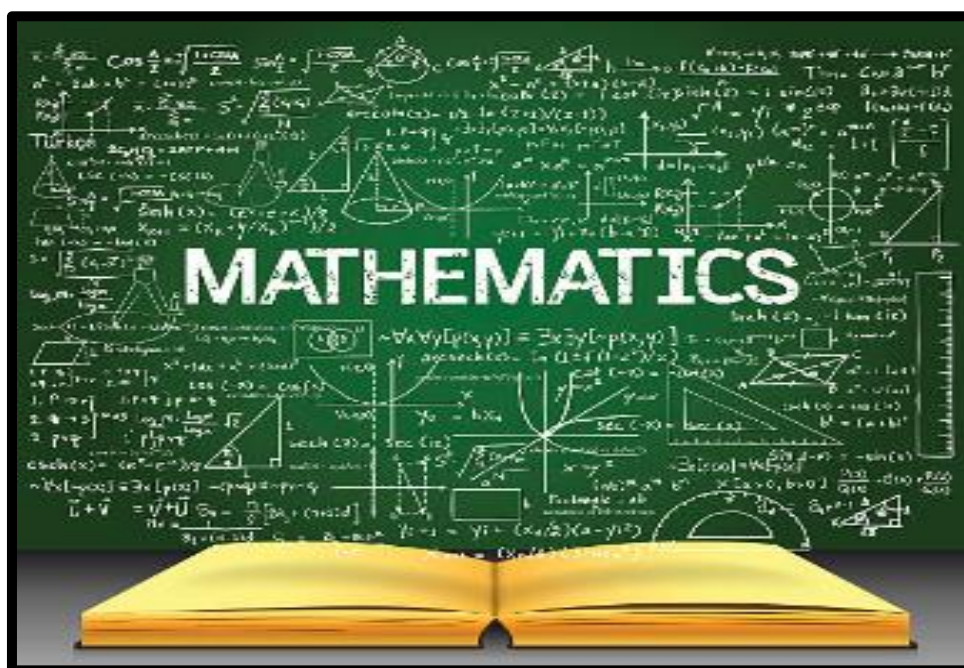
Use of variety of integration technique

*Students are able to demonstrate knowledge and critically evaluate how to apply a variety of integration and anti-differentiation techniques to functions and relations, using both analytical and numerical methods.*

SLO No.	Students are able to:	Specific Learning Outcomes	Skill Level	SLO Code
1	<b>apply</b> anti-differentiation to problems in kinematics. Any question that only involves substitution of “time”, but no integration, i.e. substituting “ $t$ ” into the formula. Initial ( $t = 0$ ) or $t = a$ .		2	Cal4.2.2.1
2	<b>use</b> and apply definite integral to find areas bounded by the curve and the x axis. (curves functions include polynomials, trig exponential and root graphs.)		3	Cal4.2.3.1
3	<b>apply</b> anti-differentiation to solve word problems in kinematics, involving the application of one level of integration, e.g. $a(t) \rightarrow v(t)$ or $v(t) \rightarrow d(t)$		3	Cal4.2.3.2
4	<b>use</b> and apply definite integral to find areas bounded by the curve and the y axis (curves functions include polynomials, trig exponential and root graphs)		4	Cal4.2.4.1
5	<b>use</b> and apply definite integral to find areas <i>between</i> two curves including linear, polynomials, trig, exponential and root graphs.		4	Cal4.2.4.2
6	<b>apply</b> anti-differentiation to solve word problems in kinematics, involving the application of two levels of integration e.g. $a(t) \rightarrow$ find $d(t)$		4	Cal4.2.4.3

### Explanatory Notes:

1. integration by substitution is restricted only to those involving simple algebraic substitutions
2. integration using partial fraction is to be used as an alternative method of integration for students to use where appropriate, but are not tested directly in this syllabus



### SUB-STRAND 4.3: Integration: Volumes of solids of revolution (EA & IA)

#### Key Learning Outcome

Volumes of Solids of Revolution

*Students are able to demonstrate knowledge and critically evaluate how to use integration to find the volumes of solids of revolution.*

SLO No.	Students are able to:	Specific Learning Outcomes	Skill Level	SLO Code
1	use of integration to <b>find volumes</b> of revolution of simple function (such as $y = ax^n$ ) and around the x axis or y axis.		3	Cal4.3.3.1
2	use of integration to <b>find volumes</b> of revolution of complex function (such as exponential or trig function) and around an axis parallel to the x or y axis, and limits of integration are not obvious in the problem.		4	Cal4.3.4.1
3	use of integration to <b>find volumes</b> of revolution of area between two functions (linear, trigonometric, quadratic, exponential, logarithmic, hyperbola), and in which the limits of integration are obvious in the problem.		4	Cal4.3.4.2

#### Explanatory notes

1. Simple functions do not involve sums, products, or combinations of the standard functions other than expressions such as  $ax^2 + bx + c$  or  $A - e^{ax}$ .
2. Complex functions are those that do involve sums, products and combinations of the standard functions.
3. For areas between two function (as in SLO3 above, L4), the limits of integration are obvious in the problem.

### SUB-STRAND 4.4: Form and interpret solutions of differential equations (EA)

#### Key Learning Outcome

Form and interpret solutions

*Students are able to demonstrate knowledge and critically evaluate how to form differential equations and interpret the solutions.*

SLO No.	Students are able to:	Specific Learning Outcomes	Skill Level	SLO Code
1	use anti-differentiation to form and solve differential equations of first order only with variables easily separated (functions include: polynomials, $e^x$ , $\ln(x)$ , trig functions, $x^n, n \in \mathbb{R}$ )		2	Cal4.4.2.1
2	use anti-differentiation to form and solve simple differential equations with the rate of change directly or inversely proportional to the variable of interest (functions include polynomials, $e^x$ , $\ln(x)$ , trig functions, $x^n, n \in \mathbb{R}$ )		3	Cal4.4.3.1
3	use anti-differentiation to form and solve complex differential equations with the rate of change directly or inversely proportional to the variable of interest (functions include: polynomials, $e^x$ , $\ln(x)$ , trig functions, $x^n, n \in \mathbb{R}$ ) e.g. $\frac{dx}{dt} = kx$ Method:		4	Cal4.4.4.1

	<ul style="list-style-type: none"> <li>- Solve the equation by separating the variables.</li> <li>- Integrate both sides of the equation and introduce a constant as part of the solution.</li> <li>- Use the information given to evaluate the constant and substitute back to get a particular solution.</li> <li>- Use the particular solution to answer the question.</li> </ul>		
4	use anti-differentiation to form and solve differential equations of first and second order with variables easily separated (functions include: polynomials, $e^x$ , $\ln(x)$ , trig functions, $x^n, n \in \mathbb{R}$ )	4	Cal4.4.4.2

### Explanatory notes

Student applies boundary or initial conditions to solutions of differential equations. Student are expected to distinguish between families of solutions and exact solutions using given boundary or initial conditions and interprets these solutions.

#### Possible context elaborations

$$\frac{dy}{dx} = y \sin x$$

$$\frac{d^2y}{dx^2} = \text{polynomial}$$

$$\frac{dy}{dx} = e^x$$

$$\frac{dy}{dx} = \frac{1}{x}$$

Situations which students will be expected to form a model for and solve include:

Growth and decay  $\frac{dy}{dx} = ky$

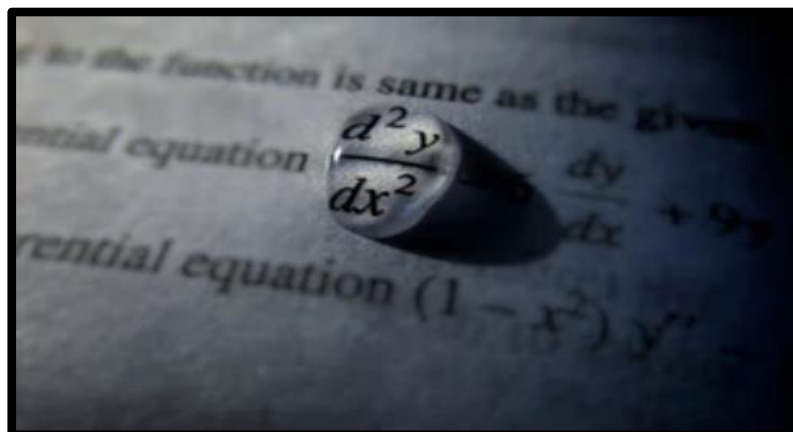
Newton's law of cooling  $\frac{dT}{dt} = k(T - T_0)$

Kinematics

### Explanatory Notes

Once solutions to differential equations are available students are required to:

- make predictions about a situation based on the solution
- make further calculations with particular solution of differential equation to check accuracy of model
- comment on limitations of their model in relation to uncontrolled factors, etc.



## 8.0 ASSESSMENT

Assessment in Mathematics Calculus course is in two parts:

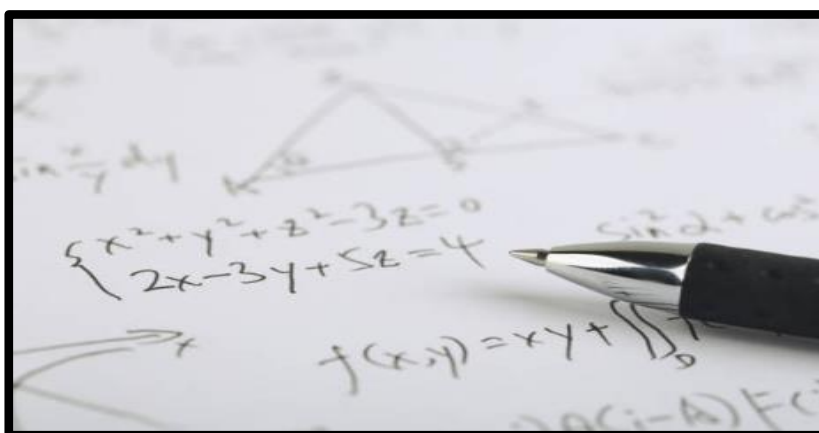
1. External Assessment (EA) is worth 60.
2. Internal Assessment (IA) is worth 40.

The Principal, or his/her nominee, will certify a proposed IA programme for the course. This is to be submitted to EQAP early in the year.

The Principal, or his nominee, will also certify that the internal assessment requirements have been fulfilled.

### 8.1 Suggested Teaching Time and Weightings

Outcomes		External / internal	Approximate weighting	Suggested Time
1	<b>Algebra:</b> Students will be able to apply algebraic techniques to real and complex numbers.	<i>External</i>	20	8 weeks
		<i>Internal</i>	10	
2	<b>Trigonometry:</b> Students will be able to demonstrate basic skills and use trigonometric functions to solve simple problems. Prove trigonometric identities	<i>External</i>	10	5 weeks
		<i>Internal</i>	10	
3	<b>Differentiation:</b> Students will be able to demonstrate basic skills of differentiation and solve problems by sketching.	<i>External</i>	15	8 weeks
		<i>Internal</i>	10	
4	<b>Integration:</b> Students will be able to demonstrate basic skills and use a variety of integration technique to solve problems	<i>External</i>	15	7 weeks
		<i>Internal</i>	10	
<b>Total</b>			<b>100</b>	<b>28 weeks</b>





## 8.2 Assessment Blueprint

Strand	Assessment Type	SKILL LEVEL/ SCORE				Weight
		Level 1 SS = 1	Level 2 SS = 2	Level 3 SS = 3	Level 4 SS = 4	
1. Algebra	EA	7	1	1	2	20
	IA	-	-	2	1	10
2. Trigonometry	EA	1	3	1	-	10
	IA	-	1	-	2	10
3. Differentiation	EA	1	3	-	2	15
	IA	-	-	2	1	10
4. Integration	EA	1	2	2	1	15
	IA	-	-	2	1	10
<b>Total number of items</b>		<b>10</b>	<b>10</b>	<b>10</b>	<b>10</b>	<b>100</b>
<b>Total skill score</b>		<b>10</b>	<b>20</b>	<b>30</b>	<b>40</b>	

## 8.3 External Assessment

The syllabus contains the Major Learning Outcome (MLO), the Key Learning Outcome (MLO) and the Specific Learning Outcomes. Examination questions, which require specific mathematical knowledge, will be based on these specific outcomes.

The External Assessment will contribute 60% of the final grade. The table below gives an approximate weighting for each major learning outcome in the examination. This is based on the distribution of learning outcomes for each Strand.

The four Strands in the syllabus are reflected in the Examination paper, and students will be given **3 hours** (plus 10 minutes of reading time) for the examination.

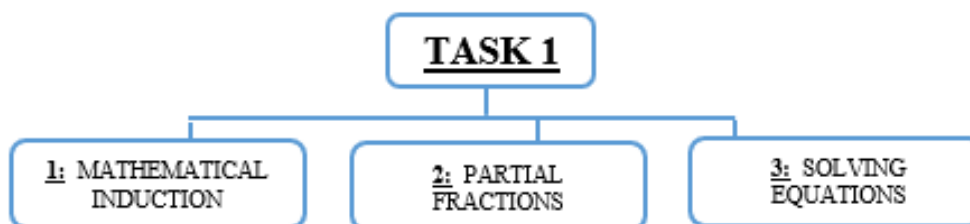
Strand	Outcomes	Percentage per section	Suggested Exam Time
	<i>In a range of meaningful contexts, students are able to solve problems and model situations that require them to:</i>		
1	Algebra:	20%	60 mins
	Apply algebraic techniques to real and complex numbers.		
2	Trigonometry	10%	30 mins
3	Differentiation:	15%	45 mins
	Demonstrate knowledge of advanced concepts and techniques of differentiation.		
4	Integration:	15%	45 mins
	Demonstrate knowledge of advanced concepts and techniques of integration.		

## 8.4 Internal Assessment

The four IA tasks are Common Assessment Tasks (CATs), and they assess the specific learning outcomes (SLOs) indicated. The task instructions for each task are provided below.

TOPICS	Task No.	Assessment Type	SOLO LEVELS				Weighting
			Level 1	Level 2	Level 3	Level 4	
ALGEBRA	TASK 1	INDIVIDUAL ASSIGNMENT			2 6	1 4	10
TRIGONOMETRY	TASK 2	GROUP PRESENTATION TASK		1 2		2 8	10
DIFFERENTIATION	TASK 3	PRACTICAL ACTIVITY			2 6	1 4	10
INTEGRATION	TASK 4	GROUP TASK			2 6	1 4	10

### 8.4.1 IA TASK 1 Instructions



The concepts of solving equations, Mathematical Induction and Partial Fractions are often given a lot of emphasis by teachers and students. This emphasis reflects a similar level of emphasis given to these in the Year 13 modules in the USP Foundation programme. The inclusion of a Task on these three concepts gives recognition to this emphasis. The inclusion of an IA task on Strand 1 also gives recognition to the large number of SLOs within this Strand.

#### TASK DESCRIPTION

This task is an **INDIVIDUAL ASSIGNMENT** and is worth **10%**. Students can discuss the content in class or in groups, but each student must submit their own answers to the teacher by the deadline. The idea is for teachers to act as facilitators for collaborative and interactive discussions to ensure that constructive discussions are taking place.

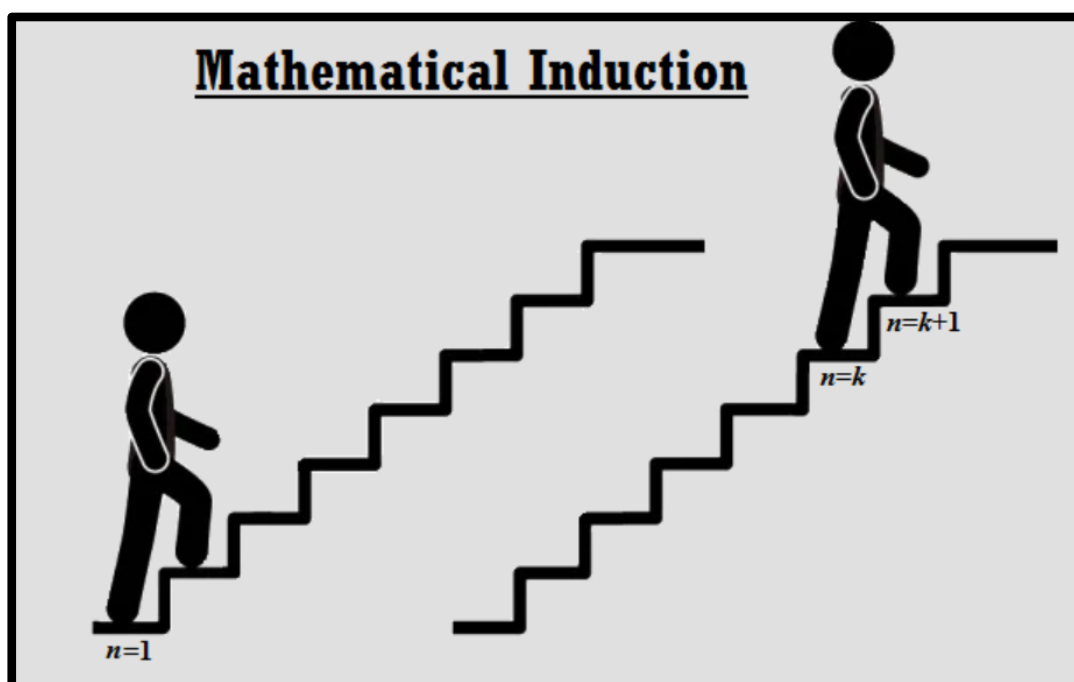
This gives students the opportunity to learn from each other and from their teachers. This idea focuses on **peer tutoring**, where students find ways to answer task questions themselves, rather than relying on teachers as a source of information. Teachers should refrain from providing answers directly to students, and also ensure that students do not plagiarize or copy other people's work.

The questions in this task are based on Strand 1 - Algebra. The task focuses on solving basic algebraic expressions using concepts on proofs by Mathematical Induction, Partial Fractions and solving Word Problems. It is recommended that a maximum of 5 - 6 lessons are dedicated to this task, and that the teacher collects student worksheets at the end.

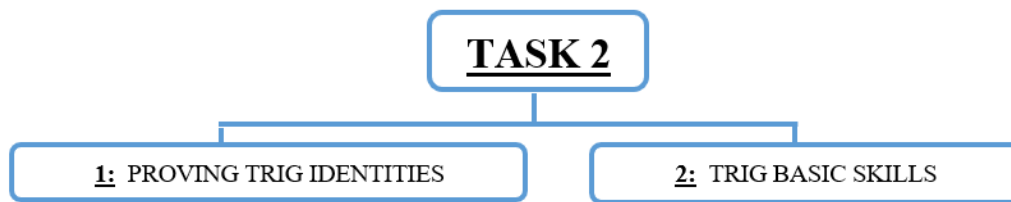
The task document itself along with the scoring rubric will be made available at the appropriate time by EQAP, so the teacher will be responsible for following the specifications provided above and scoring. However, teachers should prepare worksheets based on the learning outcomes of this task for students to work on to give them the experience and skills needed for the actual task, as well as for the final examination.

There are 3 tagged Specific Learning Outcomes for this Task.

	Specific Learning Outcome	Skill Level	SLO Code
1	prove a given mathematical statement is true by using the method of mathematical induction, whereby the variable “n” in the statement to be proved is a power (the laws of indices is applied)	4	Cal1.1.4.6
2	express a single algebraic fraction as a sum of its partial fractions where denominators of fractions include a non-linear function or an irreducible quadratic function	3	Cal1.1.3.2
3	<b>solve</b> simultaneous equations, where one equation is linear and the other a quadratic or cubic function	3	Cal1.2.3.3



## 8.4.2 IA TASK 2 Instructions



**Task 2 is worth 10%. The questions in this task are based on Strand 2 - Trigonometry. The task focuses on proving trigonometric identities, solving and sketching trigonometric functions.**

*The key learning outcome that is assessed in this task is the student's ability to demonstrate and critically evaluate basic trig skills as well as proving trigonometric identities.*

The Specific Learning Outcomes for Task 2 are taken from Strand 2, Sub-Strands 2.1 and 2.3. This task is provided by EQAP at the appropriate time of the year, along with a rubric of assessment. The teacher is responsible for implementation within the specifications shown below and for scoring student responses. The scoring rubric is also provided by EQAP.

Proving identities is a high level mathematical skill that involves manipulation of multiple possibilities; therefore, students are required to present their solutions to the teacher as a group at the end of their discussion. This will build confidence and better understanding of the concepts of proving trig. Identities and sketching trig. Graphs.

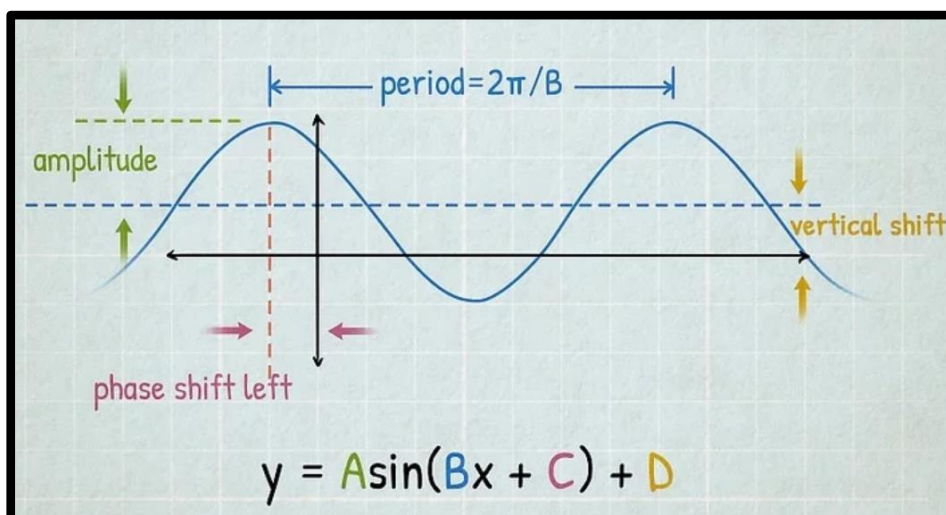
### **TASK SPECIFICATIONS:**

1. This task is a group task, and the teacher is to allocate students into their respective **groups of two or three**. A group of 4 may be necessary in some situations but it will end up being a large group, so teachers are cautioned about organising students into groups of 4s.
2. The actual task document as well as the scoring rubric will be sent to schools from EQAP at the appropriate point in the year. This is very important for teachers to note.
3. There will be **ONE common version of Task 2** and teachers are to give this to each group. Students are to be encouraged to work collaboratively in solving the task problems; however, students can submit their responses to the task as a group to the teacher for scoring after their presentation.
4. Students are to be given time in-class to work on their task. At the end of each class time, the teacher is to collect the students' work and return the same to them to work on during the next class time. In this way, the efforts of students are more meaningful and fairer. Teachers are to make independent decisions about allocation of scores for students who are absent during lessons in which groups are working on their tasks.

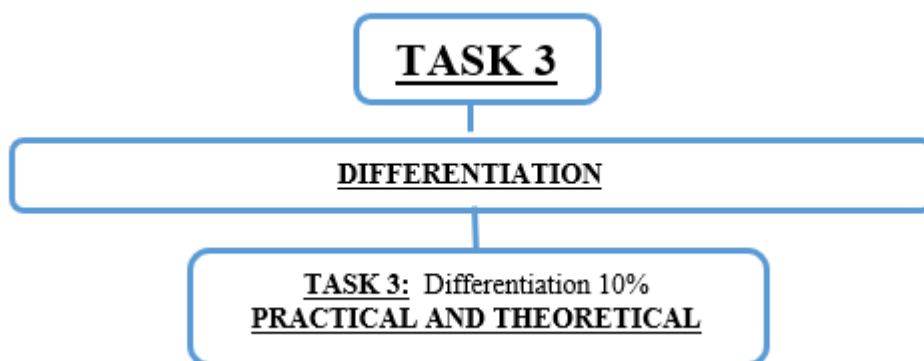
5. Each group to present their solutions to the three questions tagged for this task to their teacher.
6. It is expected that students spend about 5 - 6 lessons on this task.
7. Teachers are to refer to the scoring rubric provided from EQAP and follow this closely when scoring students' responses to Task 2.

There are 3 tagged Specific Learning Outcomes for this Task.

	Specific Learning Outcome	Skill Level	SLO Code
1	Sketch trig function of the form: $y = A \begin{matrix} \sin \\ \cos \end{matrix} (Bx \pm C) \pm K$	4	Cal2.1.4.3
2	Find the maximum/minimum points of a trig. function given in the form: $y = A\cos\theta \pm B\sin\theta$	4	Cal2.1.4.4
3	prove trigonometry identities (involving reciprocal relationships but involves algebraic working), by manipulating equations to provide a general solution or a solution within a specified domain.	2	Cal2.3.2.1



## 8.4.3 IA TASK 3 Instructions



**Task 3** is worth **10%**. The task is based on Strands 3 and is divided into two parts – **Part 3A-Practical** and **Part 3B-Theoretical** aspects of Differentiation.

### **Task 3A - PRACTICAL.**

This part is designed for students to see how differentiation is used in real life application problems in the area of optimization. It is based on the Learning Outcomes in Strand 3 (Differentiation), Sub-strand 3.3 (Application of Differentiation), SLO code: Cal3.3.4.2 - **use** a range of differentiation techniques to solve optimization problems (maxima, minima or point of inflection) that involve many overlapping equations or equations to be differentiated are not obviously stated.

The key idea of this task is to contextualise the task activity so students can relate to a real life application of differentiation in optimisation of quantities. In our daily lives, we benefit from the application of Mathematical Optimization algorithms.

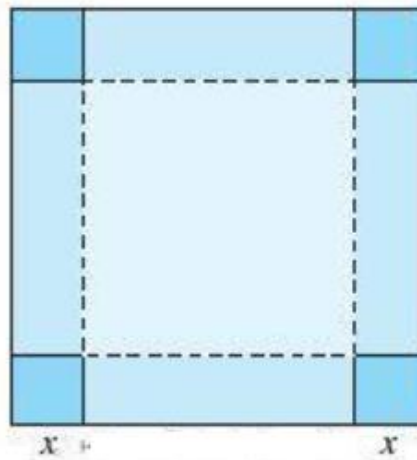
They are used, for example, by GPS systems, by shipping companies delivering packages to our homes, by financial companies, airline reservations systems, etc. Mathematical Optimization, also known as Mathematical Programming, is a discipline that solves a great variety of applied problems in diverse areas: medicine, manufacturing, transportation, supply chain, finance, government, physics, economics, artificial intelligence, etc.

### **Task Activity**

Students are to construct a box out of a sheet of cardboard in order to maximise the volume of the box.

### **Task Instructions**

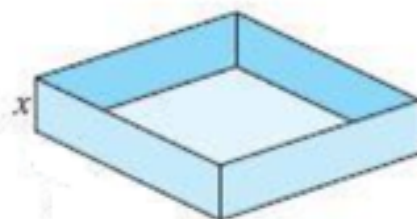
1. Students will be divided into groups.
2. Each group will be given a rectangular sheet of vanguard or cardboard sheet of different lengths and widths in centimetres.



3. To construct a box, you will need to cut out equal squares at the corners as shown in the figure above. **Hint:** Let the side of each square be  $x$  cm long.
4. Measure the length ( $L$ ) and the width ( $W$ ) of the rectangular cardboard.
5. Before you cut out the squares you will need to apply the concept of optimisation to find  $x$  (length of the square) so that the volume of the box is maximised.

The following steps can be used:

- Write down the equation for the volume of the box with the corners cut out in terms of  $x$ . Hint:  $V(x) = L \times W \times h$
  - $V(x) = (L - 2x)(W - 2x) x$
  - Differentiate  $V(x)$  to get  $V'(x)$
  - Let  $V'(x) = 0$
  - Solve for  $x$
  - Substitute  $x$  into  $V(x)$  to find out which  $x$  value gives a maximum volume.
6. Cut out the squares with length equal to  $x$  that gives a maximum volume and fold along the dotted lines to form a rectangular box.



The key idea is to **contextualise the task activity**, so students can relate to a real life application of mathematics in the form of differentiation related to optimisation. Task 3 is a school-based task with a common evaluation framework. A common framework is that the activity selected is about optimization, but the actual size of the optimised volume of the box depends on the size of the rectangular cardboard

sheet given to the group. They are to be given 2 - 3 lessons to complete the practical part. Detailed task instructions and the scoring rubric will be provided from EQAP at the appropriate point in the year.

**TASK 3B - THEORETICAL**

Task 3B is worth 10%. For this part, students will use their knowledge of the content to derive their answers. This second part will be provided from EQAP at the appropriate time in the year. The questions in this task are based on Strand 3 - Differentiation. It assesses the students’ understanding of concepts of Differentiation and its application.

This part of the task can be done in-groups where students are allowed to discuss amongst themselves. Each group has to submit their responses to the task at the end. Maximum of 4 -5 lessons should be given to the students to complete the task.

Task 3B will be provided from EQAP at the appropriate time in the year. Other details for the task are to be included in the IA Program that the teacher is to send to EQAP for approval.

The exact day and week of the administration of Task 3 is suggested in the ‘Calendar’, and ‘IA Due Dates’ documents from EQAP.

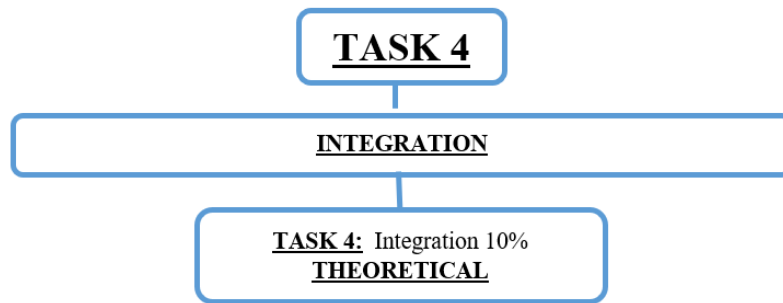
There are 3 tagged Specific Learning Outcomes for this Task.

	Specific Learning Outcome	Skill Level	SLO Code
1	use a range of differentiation techniques to solve optimization problems (maxima, minima or point of inflection) that involve many overlapping equations or equations to be differentiated are not obviously stated.	4	Cal3.3.4.3
2	use implicit differentiation or parametric differentiation to differentiate simple functions (functions include polynomials, and trigonometric functions )	3	Cal3.3.3.1
3	use differentiation to find the rates of change, using the Chain rule with two variables involved. The relationship of both variables is given in the problem.	3	Cal3.3.3.4

$$\frac{d(\text{cube})}{d(\text{sphere})} = \frac{d(\text{cube})}{d(\text{cube})} \times \frac{d(\text{cube})}{d(\text{sphere})}$$

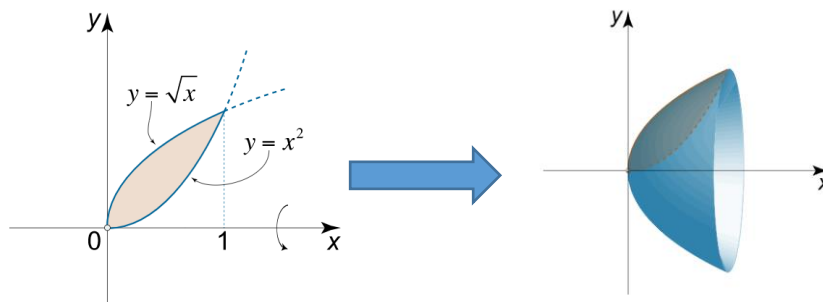


## 8.4.4 IA TASK 4 Instructions



**Task 4 is worth 10%. The task is a Group Task based on Strand 4**

Applying integration is an important part of a mathematical application in a real-world situation. Finding the enclosed area between the boundary and the amount of rotation is such an important application. Task 4 provides an opportunity for students to work collaboratively in order to solve application problems on areas and volumes together.



Teachers are encouraged to provide students with worksheets containing problems that are based on learning outcomes from sub-strands 4.1 and 4.3, upon which they can collaborate, learn together and practice essential skills that relate to the process of finding areas and volumes of revolution.

1. The questions in this task are based on Strand 4 - Integration. The task focuses on integrating rational function, find the area bounded by the curve and the x or y axes and volumes of revolution of area between two functions.
2. This task will be administered as a controlled assignment, and the task paper together with the scoring rubric will be provided by EQAP at the appropriate time in the year. The date for administration of this CAT will be indicated in the IA Due Dates document that will be provided from EQAP at the beginning of the year.
3. Teachers are expected to ensure that the task is administered properly and scored according to the scoring rubric that will be provided. Proper administration includes regular monitoring by teachers to ensure that there is no plagiarism or cheating.
4. Students are to work collaboratively in groups of 2s or 3s, and they are to be encouraged to discuss the various methods of solving each problem. At the end of discussion, each group is to submit their own answer paper.

Maximum of 4-5 lessons should be given to the students to complete the task.

There are 3 tagged Specific Learning Outcomes for this Task.

	Specific Learning Outcome	Skill Level	SLO Code
1	<b>integrate</b> rational functions of the type $y = \frac{ax+b}{cx+d}$	3	<b>Cal4.1.3.2</b>
2	<b>use definite</b> integration to <b>solve</b> problems involving areas (the limits of integration are not obvious and may have to be obtained from a sketch of the function/s)	3	<b>Cal4.1.3.3</b>
3	<b>use</b> of integration to <b>find volumes</b> of revolution of area between two functions (linear, trigonometric, quadratic, exponential, logarithmic, hyperbola), and in which the limits of integration are obvious in the problem.	4	<b>Cal4.3.4.2</b>

### 8.4.5 Internal Assessment (IA) Program Approval

At the beginning of each year, each school presenting students for the South Pacific Form Seven Certificate Mathematics (with Calculus) assessment must complete an Internal Assessment Program Proposal and forward to EQAP by the date set down by the Director. The proposal must make clear a number of things. These include The time (start and completion date) for each task. In the case of the CATs, the start and end date is the same as it is a test. Refer to the IA Due Dates document sent from EQAP to ensure that the start and end dates that is proposed for the school fall within or very close to the dates indicated in the IA Due Dates document. Also, the details for each task that teachers and students are to make decisions about, etc.

Since the syllabus is a new revision for 2022, and all IA tasks are new, all schools are expected to submit a full IA Program proposal. A copy of the IA Program proposal template is provided as Appendix 2 and all teachers are urged to complete the details for the IA proposal for each subject, in compliance with the requirements stipulated in the template. Completed IA program proposals are to be submitted to EQAP for approval by the stipulated due dates.

The IA Program and copies of all assessment tasks and assessment schedules used, as well as a sample of candidate responses to all internal assessment work undertaken, must be available for verification during the IA verification visit.

The moderation of Internal Assessment will be done in accordance with EQAP policy as specified from time to time.

It is recommended that at the start of the year students are given copies of the learning outcomes and the task descriptions or the IA Programme.

## 9.0 APPENDICES

### 9.1 Appendix 1: Sample TASK Scoring Rubrics

#### Task 1: ALGEBRA (10%)

#	SLO Code	Skill Score	Evidence of correct response	Student Response Level			
				1	2	3	4
1	Cal1.1.4.6	4					
2	Cal1.1.3.2	3					
3	Cal1.2.3.3	3					

#### Task 2: TRIGONOMETRY (10%)

#	SLO Code	Skill Score	Evidence of correct response	Student Response Level			
				1	2	3	4
1	Cal2.1.4.3	4					
2	Cal2.1.4.4	4					
3	Cal2.3.2.1	2					

#### Task 3: DIFFERENTIATION (10%)

#	SLO Code	Skill Score	Evidence of correct response	Student Response Level			
				1	2	3	4
1	Cal3.3.4.3	4					
2	Cal3.3.3.1	3					
3	Cal3.3.3.4	3					

#### Task 4: INTEGRATION (10%)

#	SLO Code	Skill Score	Evidence of correct response	Student Response Level			
				1	2	3	4
1	Cal4.1.3.2	3					
2	Cal4.1.3.3	3					
3	Cal4.3.4.2	4					
5							

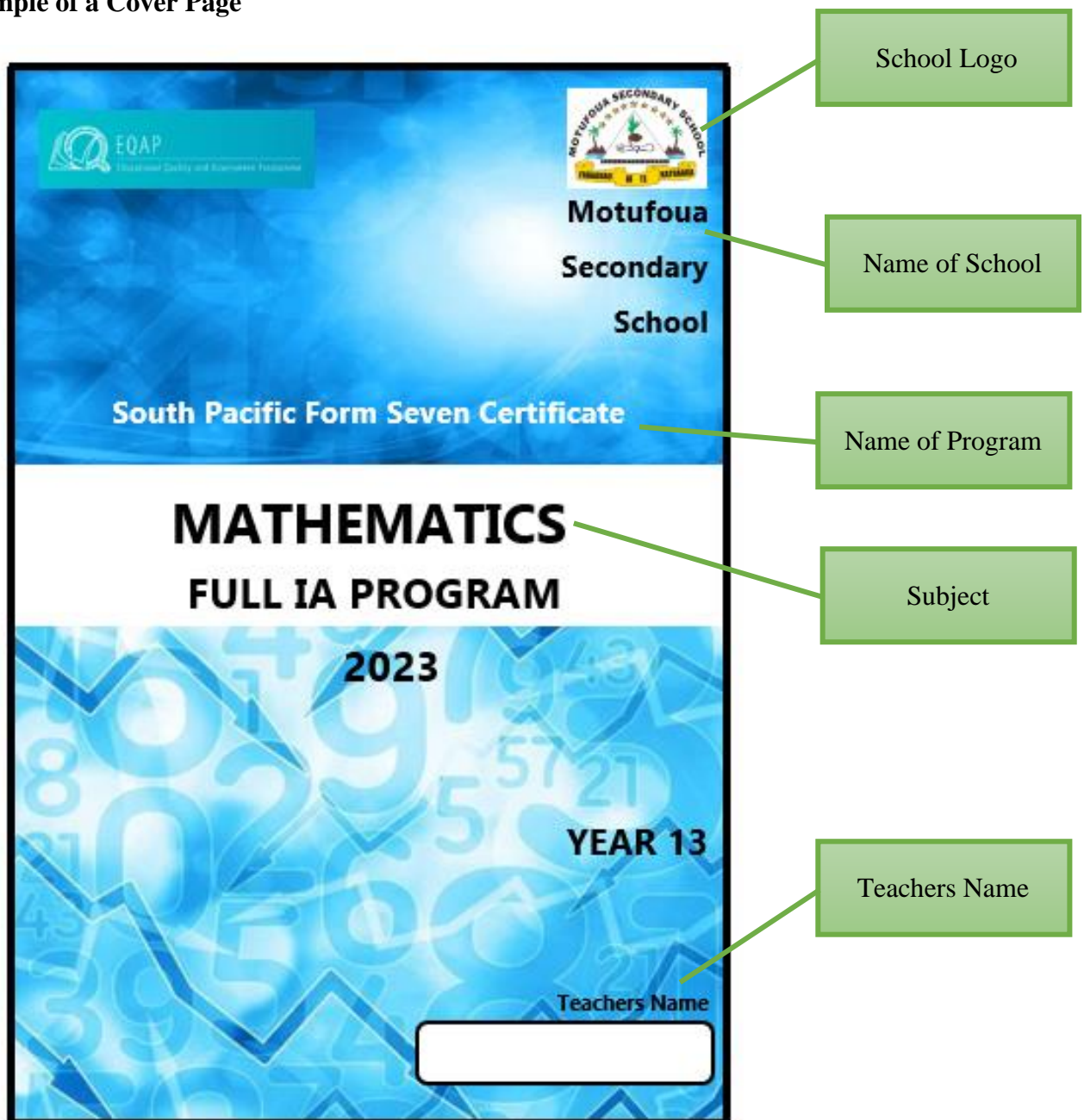
## 9.2 Appendix 2: IA Program Proposal Template

### Page 1 : Cover Page

The Cover Page will have the name of the:

- School
- Subject : FULL IA PROGRAM
- Teachers Name:

### An Example of a Cover Page



## Page 2 : IA SUMMARY FORM

The IA Summary Form must have the following:

- Number of Tasks
- Brief Description of the Tasks
- Start and End Dates
- Signature of Principal and Teacher
- School Stamp/Date

### An Example of an IA Summary Form

**South Pacific Form Seven Certificate**  
**IA Summary Form**  
 2023

**MATHEMATICS**

Task	Brief Description of Tasks	Start Date	End Date	Date to EQAP	Weighting
1. Individual Assignment	Algebra: This Task is an individual Assignment but can be discussed in class.	13/04	17/04		10%
2. Group Task	Trigonometry: This is Group Task that can be done in Groups of 2 or 3.	29/06	06/07		10%
3. Practical	Differentiation: Students will be divided into Groups and construct a box and calculate its volume.	14/08	14/08		10%
4. Group Task	Integration: This is another Group Task where students calculate Volume & Area.	21/09	22/09		10%
<b>TOTAL</b>					<b>40%</b>

**Verification and Endorsement of IA Program**

Principal's Name	Teachers Name	School Stamp
Mr. Ulisesse KAINANO	Mrs. Olimeti Lui	
Signature 	Signature 	
Date: 23/05/2023	Date: 23/05/2023	

**Annotations:**

- School Logo**: Located at the top right of the form.
- Number of Tasks & Brief Description of Each Task**: Points to the first two columns of the task table.
- Shows START date; END date; Date due to EQAP**: Points to the last three columns of the task table.
- Weighting for each Task**: Points to the Weighting column of the task table.
- Signed by Principal**: Points to the Principal's Name and Signature fields.
- Signed by the Teacher**: Points to the Teachers Name and Signature fields.
- Approved by SPFSC Coordinator**: Points to the School Stamp.
- Verified by School Stamp**: Points to the School Stamp.

**1 Task Title: Task 1:** \_\_\_\_\_

The title should be brief and include a reference to the particular syllabus topic or skill which is being assessed by the task.

Example: “*Research Topic – Investigation of a Social Issue.*”

**2 Learning Outcomes: List the Specific Learning Outcomes (SLOs) to be assessed by the task**

These are found in the syllabus and need to be identified before the tasks are constructed.

Example: *Describe a feature of ....*

*(Copy and paste directly from the aligned Syllabus: it must show strand, sub strand and SLOs)*

**3. Assessment/Task:**

**Describe the task as a form of assessment to measure student achievements of the above learning outcomes at different stages of the lesson/task implementation.**

*(Think of what the best types of assessment for the above LOs are so that your students can demonstrate they have achieved the learning outcomes. Also include how you will pre-assess their knowledge at the beginning of the lesson and how you will continuously assess them throughout the strand/topic to monitor their learning progress. The summative assessments are the final IA tasks.)*

e.g. Diagnostic: *(can be oral questions/short tests/ surveys/questionnaires to find out what students already know before the lesson)*

Formative: *1. This is the formative use of the summative assessment such as the drafts submitted, self-assessment, peer assessment, teacher assessment of the drafts and specific feedback provided to improve the task. 2. For CATs – this can be similar items prepared by teachers using the SLOs and given to students for practice. After scoring, the feedback needs to be given to improve learning. If majority students not doing well then re-teach using another strategy, assess and monitor learning.*

Summative: *(these are the final IA tasks or the CATs to measure how much the students have learnt/achieved after the learning period)*

**4 Resources: List materials required for completing the task (for learning & demonstrating the achievement for the SLOs.**

This must specify any material items such as books, documents, maps, stimulus material, equipment required by the task, including use of technology.

**5 Guidelines for the teacher on advance preparation requirements**

- a) **time required** by the student for task completion (monitoring progress)
- b) recommended dates/date range for task completion
- c) organization of room and hardware to facilitate task completion (learning assessment).

(After the task has been completed and scored, teachers will need an IA score capture sheet to record the performance of all students in the class.)

## 6 Guidelines for the teacher on task completion and task control

This must specify:

- the role of the teacher during the period of task completion
- instructions that are to be given by the teacher to the students
- actions that are required of the teacher during task completion

## 7 Preparation by the students beforehand

If students are required to prepare in advance of the task date, preparatory notes must indicate the requirements. For example, students may need to collect support materials for a task that is supervised in a classroom.

## 8 Task outline for the student

This outline is a brief description of the task that the student is to complete. It is a general description without specific detail.

Example: *Your task is to focus on an important social issue. After investigating that issue, you need to process information collected and suggest possible courses of action that authorities could take.*

## 9 Task detail for the student

This must provide a detailed description of the task in the sequence that the student would be expected to follow during task completion. This must clearly state:

- what the student is expected to do
- what the student is expected to record and present for assessment.

## 10. Feedback & Support

Allocate time for:

- i. Student's self-assessment and correction
- ii. Peer assessment, feedback, and time for improvement
- iii. Teacher assessment, feedback, and time for time improvement

(NB: State how this will be carried out)

## 11. Final submission & scoring

State when the final task is due and how it will be assessed. State how the school (HOD/SPFSC Coordinator) will monitor the scoring of the tasks.

## 12 Scoring Rubric

Copy and paste directly from the aligned Syllabus the relevant scoring rubrics

## 13 Assessment score capture sheet for the task

This will be provided by EQAP

**(Repeat 1-13 for other tasks)**

## 9.3 Appendix 3: List of useful formulae and tables

### MATHEMATICS WITH CALCULUS – USEFUL FORMULAE AND TABLES

#### ALGEBRA

##### Quadratic Formula

If  $ax^2 + bx + c = 0$   
 then  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

##### Arithmetic Sequences and Series

$$t_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

##### Binomial Theorem

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + \binom{n}{n}b^n$$

where  $\binom{n}{r} = {}^nC_r = \frac{n!}{(n-r)!r!}$

Some values of  $\binom{n}{r}$  are given in the table below.

##### Logarithms

$$y = \log_b x \Leftrightarrow x = b^y$$

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

$$\log_b(x^n) = n \log_b x$$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

##### Geometric Sequences and Series

$$t_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} \quad r \neq 1$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

##### Binomial Coefficients

$\frac{n}{r}$	0	1	2	3	4	5	6	7	8	9	10
0	1										
1	1	1									
2	1	2	1								
3	1	3	3	1							
4	1	4	6	4	1						
5	1	5	10	10	5	1					
6	1	6	15	20	15	6	1				
7	1	7	21	35	35	21	7	1			
8	1	8	28	56	70	56	28	8	1		
9	1	9	36	84	126	126	84	36	9	1	
10	1	10	45	120	210	252	210	120	45	10	1
11	1	11	55	165	330	462	462	330	165	55	11
12	1	12	66	220	495	792	924	792	495	220	66
13	1	13	78	286	715	1287	1716	1716	1287	715	286
14	1	14	91	364	1001	2002	3003	3432	3003	2002	1001
15	1	15	105	455	1365	3003	5005	6435	6435	5005	3003

##### Complex Numbers

$$z = x + iy = r \operatorname{cis} \theta = r(\cos \theta + i \sin \theta)$$

$$\bar{z} = x - iy = r \operatorname{cis}(-\theta) = r(\cos \theta - i \sin \theta)$$

$$r = |z| = \sqrt{x^2 + y^2}$$

$$\theta = \arg(z) \text{ where } \cos \theta = \frac{x}{r}, \sin \theta = \frac{y}{r}$$

$(r \operatorname{cis} \theta)^n = r^n \operatorname{cis}(n\theta)$  for integer  $n$  (De Moivre's Theorem)



## CALCULUS

### Differentiation

$y$	$\frac{dy}{dx}$
$c$	$0$
$x^n$	$nx^{n-1}$
$\ln x$	$\frac{1}{x}$
$e^{ax}$	$ae^{ax}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$

### Product Rule

$$(f \cdot g)' = f'g + fg'$$

or if  $y = uv$  then  $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$

### Quotient Rule

$$\left(\frac{f}{g}\right)' = \frac{g f' - f g'}{g^2}$$

or if  $y = \frac{u}{v}$  then  $\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

### Integration

$y$	$\int y \, dx$
$k$	$kx + c$
$x^n$	$\frac{1}{n+1} x^{n+1} + c$
$\frac{1}{x}$	$\ln x  + c$
$\frac{f'(x)}{f(x)}$	$\ln f(x)  + c$

### Volume of Revolution

$y = f(x)$  between  $x = a$  and  $x = b$

rotated about the  $x$ -axis.

$$\text{Volume} = \int_a^b \pi y^2 dx$$

### Composite Function or Chain Rule

$$(f(g))' = f'(g) \cdot g'$$

or if  $y = f(u)$  and  $u = g(x)$  then  $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

## COORDINATE GEOMETRY

### Straight Line

$$\text{Gradient } m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Equation } y - y_1 = m(x - x_1)$$

$$\text{Distance } d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$\text{Midpoint } M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

### Circle

$$x^2 + y^2 = r^2 \text{ or } (r \cos \theta, r \sin \theta)$$

has centre  $(0, 0)$  and radius  $r$

$$(x - a)^2 + (y - b)^2 = r^2$$

has centre  $(a, b)$  and radius  $r$

### Parabola

$$y^2 = 4ax \text{ or } (at^2, 2at)$$

### Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ or } (a \cos \theta, b \sin \theta)$$

### Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ or } (a \sec \theta, b \tan \theta)$$

asymptotes  $y = \pm \frac{b}{a} x$

## MEASUREMENT

### Triangle

$$\text{area} = \frac{1}{2} ab \sin C$$

### Trapezium

$$\text{area} = \frac{1}{2} (a + b)h$$

### Circle

$$\text{circumference} = 2\pi r$$

$$\text{area} = \pi r^2$$

### Sector

$$\text{area} = \frac{1}{2} r^2 \theta$$

$$\text{arc length} = r\theta$$

### Cylinder

$$\text{volume} = \pi r^2 h$$

$$\text{curved surface area} = 2\pi r h$$

### Cone

$$\text{volume} = \frac{1}{3} \pi r^2 h$$

curved surface area =  $\pi r l$  where  $l$  = slant height

### Sphere

$$\text{volume} = \frac{4}{3} \pi r^3$$

$$\text{surface area} = 4\pi r^2$$

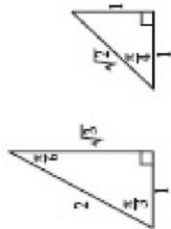
## TRIGONOMETRY

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$



## Double Angles

$$\sin 2A = 2 \sin A \cos A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A \end{aligned}$$

## Products

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2 \cos A \sin B = \sin(A+B) - \sin(A-B)$$

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

## Sums

$$\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$$

$$\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}$$

## General Solutions

If  $\sin \theta = \sin \alpha$  then  $\theta = n\pi + (-1)^n \alpha$

If  $\cos \theta = \cos \alpha$  then  $\theta = 2n\pi \pm \alpha$

If  $\tan \theta = \tan \alpha$  then  $\theta = n\pi + \alpha$

where  $n$  is any integer

## Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

## Cosine Rule

$$c^2 = a^2 + b^2 - 2ab \cos C$$

## Identities

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$$

## Compound Angles

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

## NUMERICAL METHODS

### Trapezium Rule

$$\int_a^b f(x) dx \approx \frac{1}{2}h[y_0 + y_n + 2(y_1 + y_2 + \dots + y_{n-1})]$$

where  $h = \frac{b-a}{n}$  and  $y_r = f(x_r)$

### Simpson's Rule

$$\int_a^b f(x) dx \approx \frac{1}{3}h[y_0 + y_n + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2})]$$

where  $h = \frac{b-a}{n}$ ,  $y_r = f(x_r)$  and  $n$  is even.

## 9.4 Appendix 4: IA Program Summary Form



South Pacific Form Seven Certificate

IA Summary Form  
2023

MAC – IA

### MATHEMATICS WITH CALCULUS

COUNTRY					
SCHOOL					
Task	Brief Description of Tasks	Start Date	End Date	Date to EQAP	Weighting
1. Algebra					10%
2. Trigonometry					10%
3. Differentiation					10%
4. Integration					10%
<b>TOTAL</b>					<b>40%</b>

- Note:**
1. Be specific about dates, not just Week 3 Term 1, etc.
  2. Assessment Schedules/Scoring Rubrics for the tasks will be provided by EQAP.  
Teachers must use these when scoring students' work.
  3. All IA Score Capture Sheets will be provided by EQAP to schools.

### Verification and Endorsement of IA Program

Principal's Name	Teachers Name	School Stamp
Signature	Signature	
Date	Date	

**A full IA program** is to be submitted together with this IA Summary Form.

## 9.5 Appendix 5: Electronic calculators in Examinations

Written or printed materials (including information and routines stored in the programmable memory of calculators) are prohibited.

Due to rapid changes in calculator technology, the Board will regularly review its policy on the use of calculators in examinations. However, every effort is made to ensure that schools are given adequate notice of policy changes.

The following policy aims to compromise between rewarding the appropriate use of technology while giving consideration to associated equity issues. The Board has a responsibility to ensure fairness and equity to all candidates.

Examination setters are aware of calculator technology, and take calculator capability into account in the design of examination questions and marking schedules.

## 9.6 General Policy

The EQAP *Assessment and Certification Rules and Procedures for Secondary Schools* allow candidates to use a calculator in any of its examinations provided that the calculator is silent, hand-held, non-printing and contains its own power source. However, calculators may not be used to pass information to other candidates, bring information into the examination, or as a dictionary/translator.

EQAP encourages examiners to set papers that examine understanding of concepts in such a way that the use of sophisticated calculators is not a significant advantage. Examination questions may require details of working steps to be shown to ensure that candidates understand the key concepts being assessed.

EQAP's policy on calculators in examinations allows the legitimate use of most types of calculator, including graphical and programmable calculators. The intention of the policy is to support the directions of curriculum development and encourage the appropriate use of calculator technology. The policy does not allow the use of calculators to contravene other examination rules and procedures.

The exception to the above paragraph is the use of any calculator that has symbolic algebraic manipulation capability. These will continue to be prohibited in all of the Board's examinations as they may offer candidates who use them a significant advantage over other candidates.

The following models have currently been identified as having this capability:

- Texas Instruments T189
- Texas Instruments T192
- Texas Instruments T192 Plus
- Casio CFX 9970G
- Casio Algebra FX 2.0
- Hewlett Packard HP48G
- Hewlett Packard HP48GX
- Hewlett Packard HP49G

EQAP may from time to time publish more detailed rules for the use of calculators, or further add to the list of prohibited calculators.

## 9.7 Information for Students

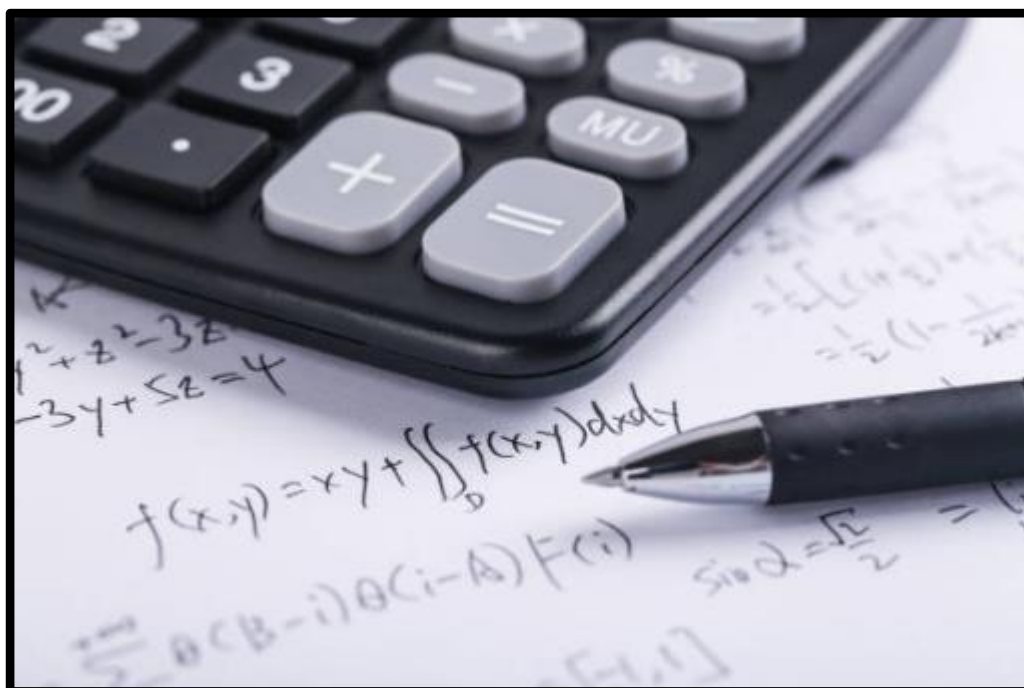
The *Instructions to Candidates* booklet, which is issued to all candidates prior to the examination period, summarises the above rules and procedures and also states:

## 9.8 Calculators

Candidates are recommended to take a calculator into the examination room for subjects where they have used a calculator during the year. For subjects where a scientific calculator has been used during the year, this should be taken to the examination.

Candidates bringing more sophisticated calculators into an examination room may be subjected to additional scrutiny by supervisors.

Any possible misuse of calculators during an examination will be handled through the Board's standard procedures for investigating possible misconduct in examinations.



## 10.0 ADVISORY SECTION

### 10.1 Suggested Texts

This is a list of only some Mathematics texts that are available and have been used for teaching the course for University Bursary Mathematics with Calculus. It is important teachers use this as only a guide and check current book lists available through publishers and book retailers.

#### A. Suggested Text

1. Delta Mathematics - Barton, D.; Johnson, W. & Laird S

#### B. Supplementary Texts

1. Year 13 Study Guide, *Mathematics with Calculus* - Sidebotham, T. ESA
2. Longman Write-on, *Notes – Calculus* - Barton, D
3. Bursary Calculus - Questions from the last 8 bursary papers with suggested answers.  
Really Useful Resources  
Box 19-939  
Woolston  
Christchurch
4. Study Pass reference notes, Year 13 Calculus - [info@studypass.co.nz](mailto:info@studypass.co.nz)
5. Advanced Mathematics - JR Sealy; AW Agnew

### 10.2 Websites

5. Calculus Website
  - i). [www.bbc.uk/education/asguru/maths/intro.shtml](http://www.bbc.uk/education/asguru/maths/intro.shtml) is part of [www.bbc.co.uk](http://www.bbc.co.uk)
  - ii) [www.ies.co.jp/math/java/calc/index/html](http://www.ies.co.jp/math/java/calc/index/html) is part of [www.ies.co.jp/math/indexeng.html](http://www.ies.co.jp/math/indexeng.html)
  - iii) [www.ies.co.jp/math/java/comp/index.html](http://www.ies.co.jp/math/java/comp/index.html) is part of [www.ies.co.jp/math/](http://www.ies.co.jp/math/)
  - iv) [www.ies.co.jp/math/java/conics/index.html](http://www.ies.co.jp/math/java/conics/index.html) is part of [www.ies.co.jp/math/](http://www.ies.co.jp/math/)
  - v) [www.unc.edu/~rowlett/units.html](http://www.unc.edu/~rowlett/units.html) is part of [www.unc.edu/](http://www.unc.edu/)
  - vi) [www.mathforum.org/pow/](http://www.mathforum.org/pow/)
  - vii) [www.ies.co.jp/math/java/misc/index.html](http://www.ies.co.jp/math/java/misc/index.html) is part of [www.ies.co.jp/math/indexeng.html](http://www.ies.co.jp/math/indexeng.html)
  - viii) [www.btinternet.com/~rfbarrow/](http://www.btinternet.com/~rfbarrow/)

THE END