No. 108/3

EDUCATIONAL QUALITY AND ASSESSMENT PROGRAMME

Scoring Schedule 2021

MATHS with CALC.

South Pacific Form Seven Certificate

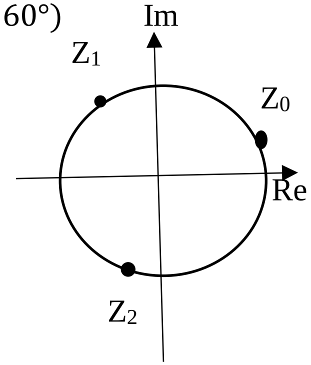
SCORING RUBRIC FOR MATHEMATICS with CALCULUS

Item	Skill level	Evidence (expected answers)	Level 1 (Unistructural)	Level 2 (Multistructural)	Level 3 (Relational)	Level 4 (Extended Abstract)
1.1	1	$2x + y = 4 \rightarrow (1)$ $y = x - 5 \rightarrow (2)$ Substitute (2) into (1) $2x + (x - 5) = 4$ $3x - 5 = 4$ $3x = 9$ $\boldsymbol{x = 3}$ Find y $y = x - 5$ $y = 3 - 5$ $\boldsymbol{y = -2}$ $\therefore$ ***coordinates of P is*** $(\boldsymbol{3, -2})$	Correct answer $(\boldsymbol{3, -2})$ OR Able to get $x = 3$ *or* $y = -2$	<u>Alternate Solution</u>: <u>Elimination Method</u> $2x + y = 4 \rightarrow (1)$ $2x + y = 4$ $y = x - 5 \rightarrow (2)$ $\underline{-(-x + y = -5)}$ $3x = 9 \quad \therefore x = 3$		
1.2	1	$3(2 - x) \leq -18$ $6 - 3x \leq -18$ $-3x \leq -18 - 6$ $-3x \leq -24$ $\boldsymbol{x \geq 8}$	Correct answer $\boldsymbol{x \geq 8}$ OR Sign reversed to $\geq$	<u>Alternate Solution</u>: DON'T EXPAND. TAKE "3 ACROSS" $3(2 - x) \leq -18 \rightarrow (2 - x) \leq -\frac{18}{3} \rightarrow (2 - x) \leq -6$ $\rightarrow -x \leq -8 \quad \therefore x \geq 8$		
1.3	1	$y = \frac{3x}{2} + h$ $y - h = \frac{3x}{2}$ $2(y - h) = 3x$ $\therefore \boldsymbol{x = \frac{2(y-h)}{3}}$ or $\boldsymbol{x = \frac{2y-2h}{3}}$ or $\boldsymbol{x = \frac{2(h-y)}{-3}}$	Correct answer OR Any correct two steps.			
1.4	1	*Factorise* $3x^2 + 11x + 10$ Factors of 30 = 6 x 5 11 = 6 + 5 $(3x^2 + 6x) + (5x + 10)$ $3x(x + 2) + 5(x + 2)$ $(\boldsymbol{3x + 5})(\boldsymbol{x + 2})$	Correct two factors $(\boldsymbol{3x + 5})(\boldsymbol{x + 2})$ OR Any one correct factor	Some might use the Quadratic Equation to get the factors: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(11) \pm \sqrt{(11)^2 - 4(3)(10)}}{2(3)} = \frac{-(11) \pm \sqrt{1}}{6}$ $\therefore x = \frac{-11 + 1}{6} = \frac{-10}{6} = -\frac{5}{3} \rightarrow (3x + 5)$ $x = \frac{-11 - 1}{6} = \frac{-12}{6}$ $x = -2 \rightarrow (x + 2)$		

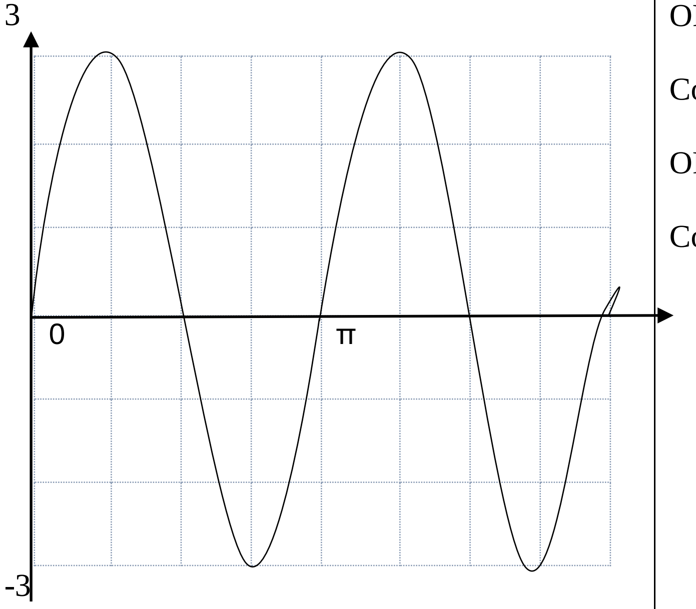
1.5	1	$Solve\ \ x^2 - 7x - 44 = 0$ Factors of -44 = -11 x 4 -7 = -11 + 4 $(x-11)(x+4) = 0$ $x = 11 \quad x = -4$ $\boldsymbol{x \in \{-4, 11\}}$	Correct answer $x = 11 \quad x = -4$ OR Any one correct value of x	Some might use the Quadratic Equation to get the factors: $\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-44)}}{2(1)} = \frac{7 \pm \sqrt{225}}{2}$ $\therefore x = \frac{7+15}{2} = \frac{22}{2} = 11 \qquad x = \frac{7-15}{2} = \frac{-8}{2} = -4$		
1.6	1	$(xy^2)^4 \times (3x^2y)^2$ $= x^4y^8 \times 3^2x^4y^2$ $= x^4y^8 \times 9x^4y^2$ $= \boldsymbol{9x^8y^{10}}$	Correct answer OR Has shown any correct use of law of indices.			
1.7	1	Simplify $\frac{3log2 + log4}{log8}$ $= \frac{3log2 + log2^2}{log2^3}$ $= \frac{3log2 + 2log2}{3log2}$ $= \frac{5log2}{3log2}$ $= \boldsymbol{\frac{5}{3}}$	Correct answer OR Has shown any correct use of logarithmic laws.	Alternate Solution: $\frac{3log2 + log4}{log8} = \frac{log2^3 + log4}{log8} = \frac{log8 + log4}{log8}$ $= \frac{log32}{log8} = \frac{5}{3}$		

1.8	1	$Divide\ x^3+4x^2-x+3\ by\ (x+2)$ x^2+2x-5 $x+2\,\overline{)\,x^3+4x^2-x+3}$ $\underline{-x^3+2x^2}$ $2x^2-x$ $\underline{-2x^2+4x}$ $-5x+3$ $\underline{--5x-10}$ 13 $\Rightarrow \boldsymbol{x^2+2x-5+\frac{13}{x+2}}$	Correct answer OR Any two correct terms OR Remainder = 13	Alternate Solution: (x + 2) =0 x = -2 -2 -4 10 -2 1 4 -1 3 1 2 -5 **13** $\boldsymbol{x^2+2x-5+\frac{13}{x+2}}$		
1.9	1	$x^3+3x^2-x+7\ divide\ by\ (x-2)$ Let $f(x)=x^3+3x^2-x+7$ By remainder theorem: $f(2)=remainder$ $f(2)=(2)^3+3(2)^2-(2)+7$ $=8+12-2+7$ $= \boldsymbol{25 \ \Rightarrow remainder}$	Correct answer OR Substitutes $x=2$	Some might use LONG DIVISION to get the factors: (x - 2) =0 x = 2 2 10 18 2 1 3 -1 7 1 5 9 **25** $\boldsymbol{x^2+5x-5+\frac{25}{x-2}}$		
1.10	1	$3\sqrt{2}(\sqrt{2}-\sqrt{8})$ $=3\sqrt{4}-3\sqrt{16}$ $=3(2)-3(4)$ $=6-12$ $= \boldsymbol{-6}$	Correct answer			

1.11	1	$\frac{12x-4}{5} = 3x+1$ $12x-4 = 5(3x+1)$ $12x-4 = 15x+5$ $12x-15x = 5+4$ $-3x = 9$ $x = -\mathbf{3}$	Correct answer 'Allow for slips'			
1.12	1	$\binom{3}{0}(2x)^3y^0 = 1.2^3x^3.1 = 8x^3$ $\binom{3}{1}(2x)^2y^1 = 3.2^2.x^2.y = 12x^2y$ $\binom{3}{2}(2x)^1y^2 = 3.2^1.x.y^2 = 6xy^2$ $\binom{3}{3}(2x)^0y^3 = 1.1.y^3 = y^3$ $= \mathbf{8x^3 + 12x^2y + 6xy^2 + y^3}$	Correct answer 'Allow for slips'			
1.13a	1	$z + \bar{w} = (-1+i) + (2+i)$ $= (-1+2) + (1+1)i$ $= \mathbf{1 + 2i}$	Correct answer			
1.13b	1	$z = -1 + i$ Using the calculator; $r = \sqrt{2}\ ,\ \theta = 135° \ or\ \frac{3\pi}{4}$	Correct answer OR Finds r or θ			

		Polar form: $z = \sqrt{2}(\boldsymbol{cos 135°} + \boldsymbol{isin 135°})$ ***or*** $\boldsymbol{z = \sqrt{2}\, cis 135°}$				
1.13c	2	$\frac{z}{w} = \frac{(-i+1)}{(2-i)} \times \frac{(2+i)}{(2+i)}$ $= \frac{-2-i+2i+i^2}{4+2i-2i-i^2}$ $= \frac{-2+i+(-1)}{4-(-1)}$ $= \frac{-3+i}{5}$ $= -\boldsymbol{\frac{3}{5}} + \boldsymbol{\frac{1}{5}i}$	Multiplied by the conjugate of *2 – i* OR Substitutes $i^2 = -1$	Correct answer $-\frac{3}{5} + \frac{1}{5}i$ OR $\frac{-3+i}{5}$		
1.14	4	$z^3 = 64\,(\cos 60° + i \sin 60°)$ Angles: $\theta = \frac{360°}{3} = 120°\ apart$ $\theta_1 = \frac{60°}{3} = 20°$ $\theta_2 = 20 + 120 = 140°$ $\theta_3 = 140 + 120 = 260°$ $r = 64^{\frac{1}{3}} = 4$ $\boldsymbol{z_0 = 4cis 20° = 4cis\frac{\pi}{9} = 3.76 + 1.37i}$ $\boldsymbol{z_1 = 4cis 140° = 4cis\frac{7\pi}{9} = -3.06 + 2.57i}$ $\boldsymbol{z_2 = 4cis 260° = 4cis\frac{13\pi}{9} = -0.69 - 3.94i}$ 	Has the correct value of r = 4 OR Uses De Moivre's theorem.	Has only one root correct without the Argand diagram.	Has only two roots correct without the Argand diagram.	Has all the 3 roots correct and represented on the Argand diagram. All 3 roots either in rectangular form or polar form.

2.1 a	1	$tan\theta . csc\theta = sec\theta$ LHS: $\frac{\sin\theta}{\cos\theta} \times \frac{1}{\sin\theta}$ $\frac{1}{\cos\theta}$ $sec\theta$ = RHS	Correct steps shown OR Substitutes the reciprocal identities $tan\theta = \frac{sin\theta}{cos\theta}$ $csc\theta = \frac{1}{sin\theta}$			
2.1 b	2	$\sin^2\theta . \cot^2\theta + \sin^2\theta = 1$ LHS: $\sin^2\theta \times \frac{1}{\tan^2\theta} + \sin^2\theta$ $\sin^2\theta \times \frac{1}{\frac{\sin^2\theta}{\cos^2\theta}} + \sin^2\theta$ $\sin^2\theta \times \frac{\cos^2\theta}{\sin^2\theta} + \sin^2\theta$ $\cos^2\theta + \sin^2\theta$ 1 = RHS	Has shown that $\frac{1}{\tan^2\theta} = \frac{\cos^2\theta}{\sin^2\theta}$	Correct steps shown OR Was able to get $\cos^2\theta + \sin^2\theta$ at the end		
2.2	1	$2\cos\theta = -\sqrt{3}$ $\cos\theta = \frac{-\sqrt{3}}{2}$ $\theta\ falls\ on\ quadrant\ II\ and\ III$ $\theta = 150°, 210°$ $\boldsymbol{\theta\epsilon\{150°, 210°\}}$ or $\boldsymbol{\theta\epsilon\left\{\frac{5\pi}{6}, \frac{7\pi}{6}\right\}}$	Correct answer OR One correct angle either in degrees or radians.			

2.3	1	$A = 3$ $Period = \frac{2\pi}{2} = \pi$ 	Correct sine curve shape OR Correct amplitude OR Correct period			
2.4	2	$\tan(A+B) = \frac{\tan(A) + \tan(B)}{1 - \tan(A)tan(B)}$ $= \frac{\frac{1}{5}+\frac{3}{5}}{1-\frac{1}{5}\cdot\frac{3}{5}}$ $= \frac{\frac{4}{5}}{1-\frac{3}{25}}$ $= \frac{\frac{4}{5}}{\frac{22}{25}}$ $= \mathbf{\frac{10}{11}}$	Correct substitution into the compound formula.	Correct answer		

2.5	3	$A = \frac{max - min}{2} = \frac{100 - -100}{2} = \frac{200}{2} = 100$ $B = \frac{360°}{\frac{1}{2}} = \frac{2\pi}{\frac{1}{2}} = 4\pi \; or \; 720°$ $C = 0 \; and \; D = 0$ $\therefore \boldsymbol{V(t) = 100 \; sin4\pi t}$	Was able to identify the amplitude A = 100 OR Has $C = 0$ OR Has $D = 0$	Was able to find the value of B correctly. $B = \frac{360°}{\frac{1}{2}} = \frac{2\pi}{\frac{1}{2}}$ $= 4\pi \; or \; 720°$	Correct answers. *A = 100* *B = 4π or 720°* *C = 0* *D = 0*	
3.1	1	P(x) is not differentiable at: $x = 1, x = 3, x = 5$	Correct answer OR Any correct value given.			
3.2	2	$\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = \frac{3^2 - 9}{3 - 3} = \frac{0}{0} \to limit \; may \; exist$ $\lim_{x \to 3} \frac{(x+3)(x-3)}{x-3}$ L'Hopital's Rule $\lim_{x \to 3} x + 3$ $\lim_{x \to 3} \frac{2x}{1}$ $= 3 + 3$ $= \mathbf{6}$ $= 2(3)$ $= \mathbf{6}$	Was able to use Difference of Two Squares to factorise the numerator into $(x + 3)(x - 3)$ OR Substitutes 3 into the expression	Correct answer		
3.3	2	$\lim_{x \to \infty} \frac{x^2 - 4x^3 + x - 3}{x^3 - 6x}$ $\lim_{x \to \infty} \frac{\frac{x^2}{x^3} - \frac{4x^3}{x^3} + \frac{x}{x^3} - \frac{3}{x^3}}{\frac{x^3}{x^3} - \frac{6x}{x^3}}$	Was able to identify the variable with the highest power, x^3	Correct answer		

		$\lim_{x \to \infty} \frac{\frac{1}{x} - 4 + \frac{1}{x^2} - \frac{3}{x^3}}{1 - \frac{6}{x^2}}$ $= \frac{0 - 4 + 0 - 0}{1 - 0}$ $= \frac{-4}{1}$ $= \mathbf{-4}$				
3.4	2	$f(x) = x^7 + 5e^{3x} - 2x^{-2} + x - 17$ $f'(x) = 7.1x^{7-1} + 3.5e^{3x} - 2.-2x^{-2-1} + x^{1-1}$ $\boldsymbol{f'(x) = 7x^6 + 15e^{3x} + 4x^{-3} + 1}$	Any term differentiated correctly.	Correct answer		
3.5	4	$s(t) = -t^3 + 3t^2 - 3t + 12$ **1st Derivative:** $v(t) = s'(t) = -3t^2 + 6t - 3$ **2nd Derivative:** $a(t) = v'(t) = -6t + 6$ **Acceleration at t = 3 s:** $a_{t=3} = a(3)$ $\Rightarrow a(3) = -6(3) + 6$ $\Rightarrow \boldsymbol{a = -12\ m/s^2}$	Shows any sign of differentiation. OR Substitutes t = 3 to an equation which was differentiated incorrectly.	Was able to differentiate s(t) once correctly $-3t^2 + 6t - 3$	Was able to differentiate s(t) twice correctly. $-6t + 6$	Correct answer

3.6	4	$Amount\ of\ fencing\ wire\ used = 3000$ $x + y + x = 3000$ $2x + y = 3000$ $y = 3000 - 2x$ $Area: A = x \times y$ $A = x(3000 - 2x)$ $A = 3000x - 2x^2$ Differentiate A: $A' = 3000 - 4x$ Maximum Area: *A' = 0* $3000 - 4x = 0$ $4x = 3000$ $\therefore \boldsymbol{x = 750\ m}$ Find y: $y = 3000 - 2x$ $= 3000 - 2(750)$ $\boldsymbol{y = 1500\ m}$ $\Rightarrow Maximum\ Area = x \times y$ $= 750 \times 1500$ $= \boldsymbol{1125000\ m^2}$	Was able to write an equation for the amount of fencing wire used: $2x + y = 3000$ OR Making y or x the subject of the formula: $y = 3000 - 2x$ OR Shows any sign of differentiation.	Writes the area equation correctly. $A = x(3000 - 2x)$ or $A = 3000x - 2x^2$ OR Was able to differentiate the area equation correctly. $A' = 3000 - 4x$	Was able to differentiate correctly and equate it to zero: Maximum Area: *A' = 0* OR Has either the value for x or y correctly. $\boldsymbol{x = 750\ m}$ $\boldsymbol{y = 1500\ m}$	Correct answer
4.1	1	$\int \left(12x^5 + \frac{1}{2}x^2 - x\right) dx$ $= \frac{12x^{5+1}}{5+1} + \frac{1}{2}\frac{x^{2+1}}{(2+1)} - \frac{x^{1+1}}{1+1}$ $= \frac{12x^6}{6} + \frac{1}{2}\frac{x^3}{3} - \frac{x^2}{2}$ $= \boldsymbol{2x^6 + \frac{x^3}{6} - \frac{x^2}{2} + C}$	Correct answer OR Any term integrated correctly			

4.2	1	$\int \frac{1}{2} e^{4x+3}\, dx$ $\frac{1}{2}\int e^{4x+3}\, dx$ $= \frac{1}{2} . \frac{e^{4x+3}}{4} + C$ $= \mathbf{\frac{1}{8} e^{4x+3} + C}$	Correct answer Do not penalise if C is missing OR Has $\frac{e^{4x+3}}{4}$			
4.3	2	$\int_{-1}^{4} 3x^2 - 2\, dx$ $= \left[\frac{3x^3}{3} - 2x\right]_{-1}^{4}$ $= [x^3 - 2x]_{-1}^{4}$ $= [(4)^3 - 2(4)] - [(-1)^3 - 2(-1)]$ $= [64 - 8] - [-1 + 2]$ $= 56 - 1$ $= \mathbf{55}$	Any term integrated Correctly. OR Shows the Fundamental Theorem of Calculus: $F(4) - F(-1)$	Correct answer		
4.4	2	$\int 8x\, cos 4x^2\, dx$ $Let\ u = 4x^2$ $\int 8x\, cos\, u\, \frac{du}{8x}$ $\frac{du}{dx} = 8x$ $\int cos\, u\, du$ $dx = \frac{du}{8x}$ $= \sin u + C$ $= \mathbf{sin\ 4x^2 + C}$	Differentiates u correctly. $\frac{du}{dx} = 8x$ OR Has ***sin u*** the integral of *cos u*	Correct answer		

4.5 a	2	$v(t) = 3t^2 - 4t - 8$ Velocity at $t = 3$: $v(3) = 3.(3)^2 - 4(3) - 8$ $= 27 - 12 - 8$ $\boldsymbol{v = 7\ m/s}$	Any substitution of t = 3	Correct answer		
4.5 b	3	$v(t) = 3t^2 - 4t - 8$ Displacement Equation: $s(t) = \int v(t)\, dt$ $= \int (3t^2 - 4t - 8)\, dt$ $= \frac{3t^3}{3} - \frac{4t^2}{2} - 8t + C$ $= t^3 - 2t^2 - 8t + C$ $t = 0, \quad s(0) = 0$ $\therefore C = 0$ $s(t) = t^3 - 2t^2 - 8t$ $s(1) = (1)^3 - 2(1)^2 - 8(1)$ $= 1 - 2 - 8$ $\therefore \quad \boldsymbol{s = -9\ m\ from\ the\ point}.$	Was able to integrate any term correctly. OR Shows any sign of integration. OR Find $s(1)$	Uses the initial conditions to find the constant. $C = 0$ OR Has the correct expression for *s(t)*: $s(t) = t^3 - 2t^2 - 8t$	Correct answer	
4.6	4	Let $P(t)$ = quantity at time, t and k be the constant of proportionality $(k > 0 \Rightarrow growth\ rate)$ $\frac{dP}{dt} \alpha\ P$	Was able to separate variables: $\frac{dP}{P} = k\ dt$ OR	Was able to get the general solution: $\boldsymbol{P = P_0 e^{kt}}$	Used the initial conditions to find k: $ln\left(\frac{5}{3}\right) = k$ or	Correct answer Possible answers will depend on the value of k:

		$\frac{dP}{dt} = kP$ $\int \frac{dP}{P} = \int k\, dt$ $lnP = kt + C$ $lnP = kt + lnP_0$ Let $C = ln\, P_0$ $lnP - lnP_0 = kt$ $ln\left(\frac{P}{P_0}\right) = kt$ $\frac{P}{P_0} = e^{kt}$ $\therefore \boxed{\boldsymbol{P = P_0 e^{kt}}}$ where P_0 = initial value **Find k:** $\boldsymbol{P_0 = 1500}$, $\boldsymbol{t = 1}$, $\boldsymbol{P = 2500}$ $2500 = 1500e^{k(1)}$ $\frac{2500}{1500} = e^{k}$ $ln\left(\frac{5}{3}\right) = k$ $\boldsymbol{k = 0.5108}$ … So $P = 1500e^{0.5108t}$ Population after 3 days: $t = 3$ $P = 1500e^{0.5108(3)}$ $= 1500e^{1.5324}$ $P = \boldsymbol{6944\ mosquitoes}$ $P_{k=0.511} = \boldsymbol{6948\ mosquitoes}$ $P_{k=0.51} = \boldsymbol{6927\ mosquitoes}$	$\frac{dP}{dt} = kP$ OR Integrates on both sides.	OR $\boldsymbol{P = 1500e^{kt}}$	$\boldsymbol{k = 0.5108}$ or $\boldsymbol{k = 0.511}$ or $\boldsymbol{k = 0.51}$	$P = \boldsymbol{6944\ mosquitoes}$ $P_{k=0.511}$ $= \boldsymbol{6948\ mosquitoes}$ $P_{k=0.51}$ $= \boldsymbol{6927\ mosquitoes}$

THE END