Pacific Community
Communauté du Pacifique

EDUCATIONAL QUALITY AND ASSESSMENT PROGRAMME

EQAP

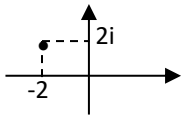
MATHS WITH CALCULUS

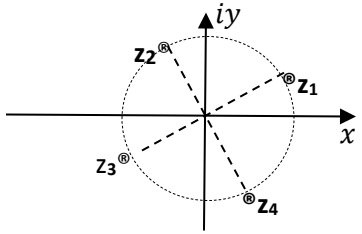
Scoring Rubric 2019

South Pacific Form Seven Certificate

© Educational Quality and Assessment Programme, 2019
3 Luke Street, Nabua, Private Mail Bag, Suva, Fiji.
Telephone: (679) 3370733 Fax: (679) 3370021
All rights reserved. No part of this publication may be reproduced by any means without the prior permission of the Educational Quality and Assessment Programme.

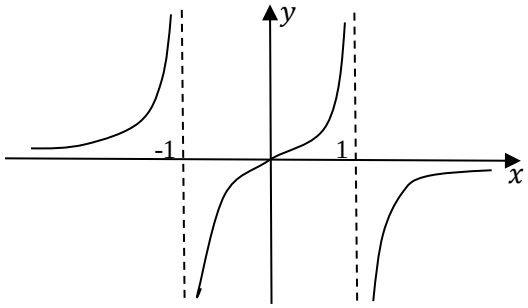
SCORING RUBRIC : Mathematics with Calculus 2019

Item	Band	Solution/Evidence	4	3	2	1	0
1.1	1	$\frac{x}{2} - x < 1 \rightarrow -x < 2 \rightarrow x > -2$	• *x2 throughout* $* \frac{-x}{2} < 1 * 2 \times 1$			*Correct ans. Allow for 'slips'*	*No merit at all in answer*
1.2	1	$y - 3x = 2 \ \& \ y + x = 4 \rightarrow (4 - x) - 3x = 2$ $-4x = -2 \rightarrow x = \frac{1}{2} \rightarrow y = 3\frac{1}{2}$	*Consider elimination method and award any creditable work e.g. subtracting correctly*			*correct x and y values, or one value correct.*	*No merit at all in answer*
1.3	1	$(2a^2b)^3 \div \left(\frac{4a^5}{b^{-3}}\right) \rightarrow (2a^2b)^3 x \frac{b^{-3}}{4a^5} = \frac{2a}{1} = 2a$	*Correct expansion to power of 3 OR x reciprocal of divisor*			*Correct ans. Look for 2a, whatever*	*No merit at all in answer*
1.4	1	$x^2 - 7x + 10 = 0 \rightarrow (x - 5)(x - 2) = 0$ $\rightarrow x = 2, 5$	*Accept solution by quadratic formula. Accept just one solutioneither -6 or 13*			*Correct ans. Or correct factors*	*No merit at all in answer*
1.5	1	$2x^3 + 4x^2 - 2x + 3 \div (x - 2)$ $= 2x^2 + 8x + 14 + \frac{31}{(x+2)}$	• If using long division accept '$2x^3$' • Accept R=31 or just 31			*Correct answer or remainder, or correct quotient*	*No merit at all in answer*
1.6	1	$(\frac{a}{\sqrt{r}} - b)^2 = c \rightarrow \frac{a}{\sqrt{r}} = \sqrt{c} + b \rightarrow r = (\frac{a}{\sqrt{c}+b})^2$	*Credit any correct 2 steps in correct order e.g.* $\sqrt{c}$ *then +b then a divided by* $(\sqrt{c} + b)$ *then sq.*			*Correct ans. or at least 2 correct steps*	*No merit at all in answer*
1.7	1	: $\log_8 x^3 = 2 \rightarrow x^3 = 8^2 = 64 \rightarrow x = 4$	• *Credit any correct log operation* • *3logx or* 8^2			*Correct answer or* $x^3 = 8^2$	*No merit at all in answer*
1.8	1	$\boldsymbol{v} = \sqrt{8}cis\ \frac{3\pi}{4} = \sqrt{8}\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4} = -2 + 2i$	• $\sqrt{8}$ (correct expression evaluated)			*Correct answer OR one correct*	*No merit at all in answer*
1.9	1	$\boldsymbol{v} = -2 + 2i$ 	• *Observe consistency with 1.8 above*			*Correct plot only*	*No merit at all in answer*
1.10	1	$\boldsymbol{z} = \boldsymbol{u} - \boldsymbol{w} = 2 - i - (4 + 3i) = -2 - 4i$	• *Credit any correct real or imaginary part*			*Correct exp. allow for 'slips'*	*No merit in response*
1.11	1	$(3\sqrt{2} - \sqrt{3})(3\sqrt{2} + \sqrt{3}) = 18 - 3 - 3\sqrt{6} + 3\sqrt{6} = 15$	• *Allow for minor errors in the working*			*correct answer only*	*no merit in response*

Item	Band	Solution/Evidence	Skill Level/Score				
			4	3	2	1	0
1.12	1	$(2x-3)^3 = 1.(2x)^3(-3)^0 + 3.(2x)^2(-3)^1 + 3(2x)^1(-3)^2 + 1.(2x)^0(-3)^3 = 8x^3 - 36x^2 + 54x - 27$	• *Allow for one correct coefficient in the expansion*			*Correct ans. or correct coeffic. Or any 1 correct terms*	*Attempts, no meaningful result*
1.13	1	$x^2 - 10x + c = x^2 - 10x + (\frac{1}{2}of(-10))^2 \rightarrow c = 25$	• *Allow for minor error in working*			*Correct answer only*	*no merit in response*
1.14	1	$f(-2) = -2(-2)^3 + 3(-2)^2 - 5(-2) - 4 = 34$	• *Allow for minor errors in working*			*correct answer or f(-2)*	*no merit in response*
1.15	2	$f(x) = x^3 - 2x^2 - 5x + 6 \rightarrow \pm 3, \pm 2, or \pm 6, \pm 1$ $f(-2) = -8 - 8 + 10 + 6 = 0 \rightarrow (x+2)\ a\ factor$ $f(3) = 27 - 18 - 15 + 6 = 0 \rightarrow (x-3)\ a\ factor$ $\rightarrow f(x) = (x+2)(x-3)(x-1)$	• $f(\pm 1,2,3)$ *for SL '1'* • *Consider the long division method and credit '1' for correct division*		*Correct answer or two correct factors*	*one correct factor or correct method*	*Attempts with no meaningful result*
1.16	4	$z^4 - 1 - i = 0 \rightarrow z^4 = \sqrt{2}cis\frac{\pi}{4} = \sqrt{2}cis(\frac{\pi}{4} \pm 2n\pi)$ $z^4 = \sqrt{2}cis\left(\frac{\pi + 8n\pi}{4}\right) \rightarrow z = (\sqrt{2})^{1/4}cis\left(\frac{\pi + 8n\pi}{16}\right)(A)$ $\rightarrow z_1 = (\sqrt{2})^{1/4}\ cis\left(\frac{\pi}{16}\right) = 1.07 + 0.21i$ [n=0] $z_2 = (\sqrt{2})^{1/4}cis\left(\frac{9\pi}{16}\right) = -0.21 + 1.07i$ [n=1] $z_3 = (\sqrt{2})^{1/4}cis\left(\frac{17\pi}{16}\right) = -1.07 - 0.21i$ [n=2] $z_4 = (\sqrt{2})^{1/4}cis\left(\frac{25\pi}{16}\right) = 0.21 - 1.07i$ [n=3] 	*Plots solutions on Argand plane and shows symmetry of points*	*Finds at least 2 correct solutions in polar or rectangular form*	*Expresses the polar form with general argument Or applies De Moivres Theorem correctly to get (A) or has 1 correct solution OR* $(\sqrt{2})^{\frac{1}{4}}$ *OR divide argument by 16*	*Finds the polar form of z correctly or has 1 + i in RHS, or has* $\frac{\pi}{4}$ *or* $\sqrt{2}$ *or shows correct method. Look for n=0,1,2 etc and credit '1'*	*no merit in response*

Item	Band	Solution/Evidence	Skill Level/Score				
			4	3	2	1	0
2.1	2	$cot\theta = \frac{1}{tan\theta} = \frac{1}{\frac{x}{\sqrt{1-x^2}}} = \frac{\sqrt{1-x^2}}{x}$	• $\sqrt{1-x^2}$ *for '1'* • $\frac{1}{tan\theta}$ *for '1'*		*Has final correct expression*	*Has* $cot\theta = \frac{1}{tan\theta}$ *or has* $\sqrt{1-x^2}$	*Meaningless response*
2.2	2	$\tan 15 = \frac{\sin(2x15)}{1+\cos(2x15)}$ $= \frac{1/2}{1+\frac{\sqrt{3}}{2}} = \frac{1}{2+\sqrt{3}}$ [0.267949…]	• *2 x 15* • *Uses sohcahtoa to find values with 30º*		*Rearranges to have final form of the expression.*	*has* sin 30 *or 2x15 0r cos 30 or substitutes values of sin and cos of 30º correctly.*	*Meaningless response*
2.3	3	$h = Asin\,B(x+C)+D \;\; where\; B = \frac{2\pi}{T} = \frac{2\pi}{4\pi} = \frac{1}{2}$ $A = \frac{max-min}{2} = \frac{6-0}{2} = 3;\; C = 0 \;\&\; D = +3$ $\rightarrow h = 3\sin(\frac{1}{2}x) + 3$		*Has required expression. Allow for minor errors*	*Works out B or D correctly.*	*Has correct amplitude or has correct C or writes 'sin' or 'cos' or has correct period of 4π s.*	*No merit in response*
2.4	3	$h(t) = A\cos B(t-C) + D;$ $A = \frac{2.6}{2} = 1.3\ cm;\; D = 0;\; C = 0;\; B = \frac{2\pi}{T} = \frac{2\pi}{20} = \frac{\pi}{10}$ $\rightarrow h(t) = 1.3cos\frac{\pi}{10}t$ $h(8) = 1.3cos\frac{\pi}{10}(8)$ $h(8) = -1.05$ (1.05 cm below the central horizontal axis, i.e. 2.35 cm from the top)	*Look for consistency in substituting t = 8s for SL '2'*	*Writes relationship correctly. Allow for 'slips' OR*	*Finds correct value of B* ***or*** *C=0, OR D=0* ***OR*** *look for consistency as in h(8)*	*Has correct value of A or has (max±min)/2 or has 'cos'*	*Meaningless response*

Item	Band	Solution/Evidence	Skill Level/Score				
			4	3	2	1	0
3.1	1	x = 1				*Has x = 1, no other*	*Meaningless result*
3.2	2	$f(x) = \frac{3x+5}{(x-2)(x+7)} \quad \rightarrow x = 2, -7$	*Credit SL '1' if denominator = 0*		*Has 2 correct x-values*	*Has one correct answer*	*Meaningless result*
3.3	2	$\lim_{x \to -\infty} \frac{3x^4+2}{x^4} = \lim_{x \to -\infty} 3 + \frac{2}{x^4} = 3$	*Credit SL '1' for differentiating numerator & denominator*		*Has correct limit of 3. Allow for slips*	*Divides into numerator*	*Meaningless response*
3.4	2	$d = 6t + 5t^2$ $\rightarrow v = \frac{dd}{dt} = 6 + 10t = 42$ $\rightarrow t = 3.6\ s$			*Has the correct answer. Allow for minor 'slips'*	*Differentiates correctly OR finds the derivative OR dd/dt = 42*	*Meaningless response*
3.5	3	$V = \pi r^2 h \rightarrow \frac{dV}{dh} = \pi r^2 \quad since\ r\ is\ constant;$ $\frac{dV}{dt} = 20\ \frac{cm}{s} \rightarrow \frac{dh}{dt} = \frac{dh}{dV}\ x\ \frac{dV}{dt} = \frac{1}{\pi r^2}\ x\ 20$ $Now\ \frac{dh}{dt} = \frac{1}{\pi(5)^2}\ x\ 20 = 0.25\ \frac{cm}{s}\ [= \frac{4}{5\pi}]$	*Note that the answer is also* $\frac{1000}{3926}$	*Substitutes correctly and has correct answer*	*Has correct expression for* $\frac{dh}{dt}$ *or differentiates to have* $\frac{dh}{dV} = \frac{1}{\pi r^2}$	*correct Vol. formula, or has* $\frac{dV}{dh}$, *or* $\frac{dh}{dt}$ *or* $\frac{dV}{dt} = 20$,	*No meaningful response*
3.6	3	$e^y + x^2 = 4 \rightarrow e^y y' + 2x = 0 \rightarrow y' = \frac{-2x}{e^y}$ $\rightarrow y' = m = \frac{-2(-3)}{e^0} = 6$ at (-3, 0) $\rightarrow \frac{y-0}{x-(-3)} = 6 \rightarrow y = 6x + 18$	*Consider consistency as shown by arrow for SL '1'*	*Uses method to find equation. Allow for minor errors*	*Finds "m" of tangent or has* $m = y' = \frac{-2x}{e^y}$ *or y=mx+c*	*Shows intention to find dy/dx or has one derived term correct*	*No meaningful response*

Item	Band	Solution/Evidence	Skill Level/Score				
			4	3	2	1	0
3.7	3	$y = \frac{-6x}{(x+1)(x-1)} \quad \rightarrow \quad asymptotes = \pm 1$ $x \rightarrow 1^+, y \rightarrow -\infty; \quad x \rightarrow 1^-, y \rightarrow +\infty$; (0,0) intercepts $x \rightarrow -1^+ \; y \rightarrow -\infty; \; x \rightarrow -1^-, \; y \rightarrow +\infty;$ $x \rightarrow \infty \; y \rightarrow \frac{-6}{+\infty} \rightarrow 0^-; \; x \rightarrow -\infty \; y \rightarrow \frac{-6}{-\infty} \rightarrow 0^+$ 	*If one draws a graph in the correct section of the grid then SL '1'.*	*Correct behaviour near the $x = +1$ or $x = -1$ asymptotes or has correct sketch of f(x)*	*Correct intercept at (0,0) or 2 correct behaviours as $x \rightarrow \pm\infty$ in the correct section*	$x = \pm 1$ *or has one correct limiting behaviour near the asymptotes or at infinity or testing for positive or negative in various sections of grid*	*No meaningful response*
3.8	4	$V = x^2h \ldots (I) \quad S = 4xh + x^2 \ldots (II) \quad V = 512\ cm^3$ *Use (I) to substitute for h in (II)* $h = \frac{V}{x^2} \quad \rightarrow S = \frac{4V}{x} + x^2$ $\rightarrow S' = \frac{-4(512)}{x^2} + 2x = 0$ $\rightarrow 2x^3 = 4(512) \rightarrow x = 10.08\ cm.$ $h = \frac{512}{(10.08)^2} \quad \rightarrow h = 5.04\ cm.$	*Solves equation correctly and has correct values for 'x' and 'h'*	*Finds S' correctly and equates to '0'*	*Has correct expression for S in terms of 'x' only OR attempts to find S' OR equates S'=0*	*Has either $4xh$ or x^2h or $V = 512$ or rewrites expressions for V and S (as given in question)*	*No meaningful response*

Item	Band	Solution/Evidence	Skill Level/Score				
			4	3	2	1	0
4.1	1	$\int \sqrt{x^3}.dx = \int x^{\frac{3}{2}}.dx = \frac{2}{5}x^{\frac{5}{2}} + c$	*Check for consistency in integration. If correct then SL '1' even if exponent not correct*			*Has correct answer.*	*No merit in response*
4.2	1	$\int 5e^{2x-3}dx = 5(\frac{1}{2})e^{2x-3}$	*Any division by 2 for SL '1'.*			*Has correct answer or part correct*	*No merit in response*
4.3	2	$\int_0^{\frac{\pi}{2}} \sin x\, dx = -\cos x \Big|_0^{\frac{\pi}{2}} = -(0-1) = +1$	*Allow for minor errors please*		*Evaluates correctly and has '1'*	*Has '$\pm\cos$' or '0' or '-1' on evaluation*	*No merit in response*
4.4	2	$\frac{dP}{dt} \propto P \ldots (I)$ $\rightarrow \frac{dP}{dt} = (0.1)P \ldots (II)$			*Has (I) or (II) correct or puts* $k = 10\%$ *or* 0.1	*Has* $\frac{dP}{dt}$ *or a proportionality sign, or o.1 or 10%*	*No merit in response*
4.5	3	$\frac{dy}{dx} = x^2y \rightarrow lny = \frac{1}{3}x^3 + c \ldots (I)$ $\rightarrow (0,3) \rightarrow c = \ln 3$ $\rightarrow lny = \frac{1}{3}x^3 + \ln 3 \ldots (III)$ $\rightarrow y = e^{\frac{1}{3}x^3}.e^{\ln 3} \ldots (IV)$ $\rightarrow y = 3e^{\frac{1}{3}x^3} \ldots (V)$	*Look for evidence of integration and award SL '1'*	*Has final correct expression (III) or (IV) or (V)*	*Integrates correctly to have (I) or Uses (0,3) to find 'C'*	*Separates variables correctly to have* $\frac{dy}{y} = x^2dx$ *or shows integration sign*	*No merit in response*

Item	Bad	Solution/Evidence	Skill Level/Score				
			4	3	2	1	0
4.6	3	$V = \int_0^3 \pi(y)^2\,dx = \int_0^3 \pi(e^{2x})^2\,dx$ (I) $= \int_0^3 \pi\{e^{4x}\}dx = \pi(\frac{1}{4}\,e^{4x})\Big\|_0^3$ (II) $= \pi\left(\frac{1}{4}\,e^{12} - \frac{1}{4}\right)$ (III) $= \frac{\pi}{4}(e^{12} - 1) \rightarrow [127\,826.5]$	*Allow for minor errors in working*	*Substitutes correctly to have (III) or last expression.*	*Integrates correctly to have expression (II) OR just* $\frac{1}{4}\,e^{4x}$	*Uses correct expression* $\int_0^3 \pi(y)^2\,dx$ *or has corr. Limits* $\int_0^3$ *or has* e^{4x}	*No merit in response*
4.7	3	$V = \pi\int_0^1[(f(x))^2 - (g(x))^2]dx$ (I) $V = \pi\int_0^1((\sqrt{x})^2 - (x)^2)dx$ (II) $= \pi\int_0^1(x - x^2)dx$ (III) $V = \pi(\frac{1}{2}x^2 - \frac{1}{3}x^3)\Big\|_0^1 = \frac{\pi}{6}\ units^3 \approx 0.52$	*Evaluates to get correct final answer* $\frac{\pi}{6}$*. Accept the answer* $-\frac{\pi}{6}$ *please*	*Has exp (III) or Integrates correctly to have* $\frac{1}{2}x^2 - \frac{1}{3}x^3$	*Works out correct limits 0 and 1 OR substitutes to have correct expression (II)*	*Uses correct expression OR has x or* x^2 *in the working*	*No merit or meaningful response*
4.8	4	$P = no.\,in\ million\ who\ have\ heard$ $\rightarrow (1-P)\ have\ not\ heard \rightarrow \frac{dP}{dt} = k(1-P)$ $\rightarrow -\ln(1-P) = kt + C \rightarrow P = 1 - Ae^{-kt};$ (A) $(0,0) \rightarrow 0 = 1 - Ae^{-k(0)} \rightarrow A = 1 \rightarrow P = 1 - e^{-kt}$ $(1,0.5) \rightarrow 0.5 = 1 - e^{k(1)} \rightarrow k = -ln0.5$ $\rightarrow P(t) = 1 - e^{-(-ln0.5)t} = 1 - e^{(ln0.5)t}$ (B) $\rightarrow P(2) = 1 - e^{(ln0.5)(2)} = 0.75$ m	*Has (B) and evaluates P(2) correctly. OR credit correct method throughout the solution, allowing for minor errors.*	*Has A=1 m or k=0.69 m or Substitutes boundaries into whatever expression is obtained OR substitutes t=2 into whatever final expression is obtained*	*Separates variables or integrates to get (A) OR has P'=k(1-P).*	*Has P and (1-P) in relation or has* $\frac{dP}{dt}$ *or writes* $k(1-P)$	*No merit or meaningful response*

THE END