

**EDUCATIONAL QUALITY AND ASSESSMENT
PROGRAMME [EQAP]**



Pacific
Community

Communauté
du Pacifique

**SOUTH PACIFIC FORM SEVEN
CERTIFICATE [SPFSC]**

**MATHEMATICS with STATISTICS
SYLLABUS**

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SOUTH PACIFIC FORM SEVEN CERTIFICATE MATHEMATICS with STATISTICS

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MATHEMATICS WITH STATISTICS

PREAMBLE AND RATIONALE

The SPFSC Mathematics with Statistics course has undergone a review and an alignment. Three main factors contributed to this review.

1. The need to consider international trends in the teaching of Statistics and the subsequent expectations of universities in the Oceania region on what statistical understanding students should possess.
2. The need to raise student achievement by “doing less, but doing it better”, which reflects another international trends in education.
3. The influence of changes in technology on the curriculum content.
4. The need for constructive alignment between the intentions and results of teaching and learning.

The main changes made to the syllabus were:

- The introduction of the analysis of bivariate data. It was considered that this was an important skill for further study at university across many fields.
- A focus on understanding and using the statistical investigation process in its complete form, as opposed to assessing small parts of it in isolation. This is in alignment with international trends and the need to have students who are more data literate.
- The introduction of three compulsory internal statistical investigations to acknowledge the three different data analysis methods: Bivariate analysis; Time series analysis; and making inferences to compare two populations. Again, this is to help develop more data literate students.
- A focus on using methods to model situations and to solve problems rather than looking at sub-skills in isolation. This is in alignment with international trends.
- The removal of the Calculus topic to allow time for a more in-depth study of other topics. It was considered that those students who needed Calculus as a future pathway would need to enter the Mathematics in Calculus course anyway.
- The removal of the Sequences and Series topic was to allow time for more in-depth study of other topics. It was considered that much of the content was a repetition of what was covered in the national Year 12 syllabus and provided no future pathways for student enrolled in this course.
- The removal of binomial expansions through the use of the Binomial Theorem. It was considered that this aspect was no longer relevant in the context of this course and that a focus on permutations and combinations in the context of Probability was more suitable.
- The removal of approximating probability distributions and the introduction of combining probability distributions instead. With the changes in technology, approximating distributions has less relevance, whereas the ability to deal with more complex probability situations is now seen as being of importance.

- The inclusion of clear learning outcomes at different levels of generality (Major Learning Outcomes, Key Learning Outcomes, Specific Learning Outcomes)

The course content was then reorganised under four specific learning objectives and weightings were allocated to each objective based on teaching and learning time (including time needed for both formative and summative assessment). This, along with the change of emphasis towards understanding and using the statistical investigation process in its completeness, has led to a change in the internal and external assessment weightings.

COURSE AIM

The aim for this course is for students to realise the importance of Statistics and Modelling as a means for understanding what is happening in the real world.

PREREQUISITES

This aligned syllabus defines the requirements for the South Pacific Form Seven Certificate Mathematics (with Statistics) in terms of unpacked learning outcomes, external examination and internal assessment requirements.

Students also require knowledge and understanding of outcomes from the national Year 12 Mathematics syllabus, which are related to the specific outcomes of this syllabus.

The course is designed for students who may undertake further studies in a tertiary institution as well as for those students who will complete their formal education at the end of Year 13.

GENERAL OBJECTIVES

Students will be expected to:

1. develop knowledge and skills related to Probability in order to solve problems and to investigate situations involving elements of chance.
2. model situations using graphical methods in order to solve problems
3. carry out statistical investigations and understand statistical processes.
4. use numeric and algebraic methods to solve problems

CONTENT COMPONENTS

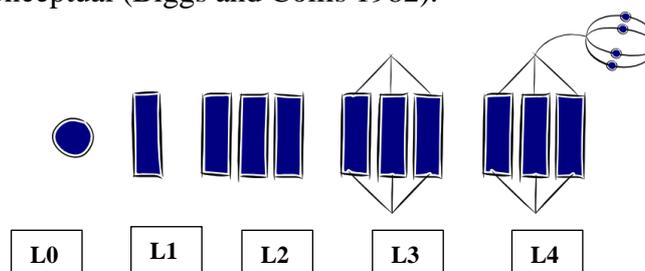
The content of the SPFSC Mathematics with Statistics course is organised under four strands and a number of sub-strands under each strand. These are outlined below:

STRAND NUMBER	STRAND TITLE	SUB STRAND NUMBER	SUB-STRAND TITLE
1.	Probability	1.1	Probability
		1.2	Discrete Probability Distribution
		1.3	Probability Distribution
2.	Modelling using Graphical Methods	2.1	Piecewise Function
		2.2	Power & Exponential Functions
		2.3	Linear Programming
3.	Statistical Investigations	3.1	Bivariate Data
		3.2	Time Series
		3.3	Comparing Two Populations
		3.4	Statistical Processes
4.	Numerical and Algebraic Methods	4.1	Linear Equations
		4.2	Non-Linear Equations

UNPACKING LEARNING OUTCOMES

In this syllabus, Learning Outcomes are stated at three levels of generality: Major Learning Outcomes (MLOs) are stated at the strand level, Key Learning Outcomes (KLOs) are stated at the sub-strand level, and Specific Learning Outcomes (SLOs) are unpacked from the Key Learning Outcomes. Each SLO is generally a combination of a cognitive skill and a specific content component. Each SLO is given a skill level, level 1 – 4, and this skill level results from the categorisation of the cognitive skill that is embedded in the SLO using the SOLO taxonomy¹.

The SOLO taxonomy provides a simple, reliable and robust model for three levels of understanding – surface, deep and conceptual (Biggs and Collis 1982).



At the **prestructural** level (L0) of understanding, the task is inappropriately attacked, and the student has missed the point or needs help to start. The next two levels, unistructural and multistructural are associated with bringing in information (surface understanding). At the **unistructural** level (L1), one aspect of the task is picked up, and student understanding is disconnected and limited. The jump to the multistructural level is quantitative. At the **multistructural** level (L2), several aspects of the task are known but their relationships to each other and the whole are missed. The progression to relational and extended abstract outcomes is qualitative. At the **relational** level (L3), the aspects are linked and integrated, and contribute to a deeper and more coherent understanding of the whole. At the **extended abstract** level (L4), the new understanding at the relational level is re-thought at another conceptual level, looked at in a new way, and used as the basis for prediction, generalisation, reflection, or creation of new understanding (adapted from Hook and Mills 2011). [<http://pamhook.com/solo-taxonomy/>]

The progression from Level 1 to Level 4 is exemplified in the progression from *define* → *describe* → *explain* → *discuss* with each succeeding level indicating a *higher level of understanding*, as follows:

- **define** – to state a basic definition of a concept [Unistructural or L1]
- **describe** – to give the characteristics of, or give an account of, or provide annotated diagrams. [Multistructural or L2]
- **explain** – to provide a reason for a relationship – an event and its impact, a cause and an effect, as to *how* or *why* something occurs. [Relational or L3]
- **discuss** – this means *linking ideas* (descriptions, explanations) to make generalisations or predictions or evaluations. It may involve relating, comparing, analysing, and justifying.
- **solve/calculate/compute** – to carry out a series of algorithms to arrive at a solution [Multistructural (L2) or Relational (L3) or even Extended (L4) depending on the complexity of the algorithm]. If there are two ‘loadings’ in the calculations (a standard problem) then skill level would be L2, if three ‘loadings’ (a complex problem) then L3 and four loadings (a more complex problem) for L4.

¹ Structure of Observed Learning Outcomes by Biggs and Collis (1982)

LEARNING OUTCOMES

STRAND 1: PROBABILITY

Major Learning Outcome (sta1):

Students are able to develop knowledge and skills related to Probability in order to solve problems and to investigate situations involving elements of chance.

SUB STRAND 1.1: Probability

Key Learning Outcome (sta1.1):

Students are able to solve probability problems.

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
1.	define an event in statistics.	1	Sta1.1.1.1
2.	identify an example of an event.	1	Sta1.1.1.2
3.	define trial.	1	Sta1.1.1.3
4.	define outcome.	1	Sta1.1.1.4
5.	define sample space.	1	Sta1.1.1.5
6.	define mutually exclusive events.	1	Sta1.1.1.6
7.	identify an example of mutually exclusive events.	1	Sta1.1.1.7
8.	define independent events.	1	Sta1.1.1.8
9.	identify an example of independent events.	1	Sta1.1.1.9
10.	define complementary events.	1	Sta1.1.1.10
11.	identify an example of complementary events.	1	Sta1.1.1.11
12.	define conditional events.	1	Sta1.1.1.12
13.	identify an example of conditional events.	1	Sta1.1.1.13
14.	define inclusive events.	1	Sta1.1.1.14
15.	identify an example of inclusive events.	1	Sta1.1.1.15
16.	define combined events.	1	Sta1.1.1.16
17.	identify an example of combined events.	1	Sta1.1.1.17
18.	compute probability of an event.	2	Sta1.1.2.1
19.	solve standard mutually exclusive event problems.	2	Sta1.1.2.2
20.	solve standard independent event problems.	2	Sta1.1.2.3
21.	solve standard complementary event problems.	2	Sta1.1.2.4
22.	solve standard conditional event problems.	2	Sta1.1.2.5
23.	solve standard problems using tree diagrams techniques.	2	Sta1.1.2.6
24.	solve standard problems using Venn diagrams techniques.	2	Sta1.1.2.7
25.	solve standard problems using tables of counts and relative frequencies techniques.	2	Sta1.1.2.8
26.	solve standard problems using theoretical and experimental probability techniques.	2	Sta1.1.2.9
27.	solve standard problems using theoretical and experimental probability techniques on exclusive events.	2	Sta1.1.2.10
28.	solve standard problems using permutations techniques.	2	Sta1.1.2.11
29.	solve standard problems using combinations techniques.	2	Sta1.1.2.12

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
30.	solve standard problems using theoretical and experimental probability techniques on either/or events.	2	Sta1.1.2.13
31.	solve standard problems using theoretical and experimental probability techniques on at least one event.	2	Sta1.1.2.14
32.	solve standard probability of inclusive events (both conditions are true).	2	Sta1.1.2.15
33.	determine the probability of complements of events.	2	Sta1.1.2.16
34.	solve standard combined events problems.	2	Sta1.1.2.17
35.	solve complex mutually exclusive events problems.	3	Sta1.1.3.1
36.	solve complex independent events problems.	3	Sta1.1.3.2
37.	solve complex complementary events problems.	3	Sta1.1.3.3
38.	solve complex conditional events problems.	3	Sta1.1.3.4
39.	solve complex combined events problems.	3	Sta1.1.3.5
40.	solve complex problems using tree diagrams techniques.	3	Sta1.1.3.6
41.	solve complex problems using Venn diagrams techniques.	3	Sta1.1.3.7
42.	solve complex problems using tables of counts and relative frequencies techniques.	3	Sta1.1.3.8
43.	solve complex problems using theoretical and experimental probability techniques.	3	Sta1.1.3.9
44.	solve complex problems using permutations techniques.	3	Sta1.1.3.10
45.	solve complex problems using combinations techniques.	3	Sta1.1.3.11
46.	solve more complex mutually exclusive events problems.	4	Sta1.1.4.1
47.	solve more complex independent events problems.	4	Sta1.1.4.2
48.	Solve more complex complementary events problems.	4	Sta1.1.4.3
49.	solve more complex conditional event problems.	4	Sta1.1.4.4
50.	solve more complex combined event problems.	4	Sta1.1.4.5
51.	solve more complex problems using tree diagrams techniques.	4	Sta1.1.4.6
52.	solve more complex problems using Venn diagrams techniques.	4	Sta1.1.4.7
53.	solve more complex problems using tables of counts and relative frequencies techniques.	4	Sta1.1.4.8
54.	solve more complex problems using theoretical and experimental probability techniques.	4	Sta1.1.4.9
55.	solve more complex problems using permutations techniques.	4	Sta1.1.4.10
56.	solve more complex problems using combinations techniques.	4	Sta1.1.4.11

SUB STRAND 1.2: Discrete Probability Distribution

Key Learning Outcome (sta1.2):

Solve problems involving expected values and variance of random variables from a discrete probability distribution.

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
1.	define random variable.	1	Sta1.2.1.1
2.	define discrete random variable.	1	Sta1.2.1.2

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
3.	define variance.	1	Sta1.2.1.3
4.	define standard deviation.	1	Sta1.2.1.4
5.	define expected value.	1	Sta1.2.1.5
6.	define probability distribution.	1	Sta1.2.1.6
7.	calculate the expected value of discrete random variables in a standard problem.	2	Sta1.2.2.1
8.	calculate the variance of discrete random variables in a standard problem.	2	Sta1.2.2.2
9.	calculate the standard deviation of discrete random variables in a standard problem.	2	Sta1.2.2.3
10.	calculate the expected value of linear functions of random variables in a standard problem.	2	Sta1.2.2.4
11.	calculate the variance of linear functions of random variables in a standard problem.	2	Sta1.2.2.5
12.	calculate the expected value of sums of independent random variables in a standard problem.	2	Sta1.2.2.6
13.	calculate the variance of sums of independent random variables in a standard problem.	2	Sta1.2.2.7
14.	calculate and interpret the expected value of discrete random variables in a complex problem.	3	Sta1.2.3.1
15.	calculate and interpret the variance of discrete random variables in a complex problem.	3	Sta1.2.3.2
16.	calculate and interpret the standard deviation of discrete random variables in a complex problem.	3	Sta1.2.3.3

Explanatory Notes

Note: The random variables could come from both given probability distributions and ones that can be developed from tree diagrams.

SUB STRAND 1.3: Probability Distribution

Key Learning Outcome (sta1.3):

Using appropriate probability distributions to model a given situation and solve problems.

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
1.	define parameters.	1	Sta1.3.1.1
2.	identify properties of the binomial distribution.	1	Sta1.3.1.2
3.	identify parameters of a binomial distribution.	1	Sta1.3.1.3
4.	compute probabilities associated with a discrete probability distribution.	1	Sta1.3.1.4
5.	identify properties of the Poisson distribution.	1	Sta1.3.1.5
6.	identify the parameter of a Poisson distribution.	1	Sta1.3.1.6
7.	state the properties of a normal distribution.	1	Sta1.3.1.7
8.	identify parameters of a normal distribution.	1	Sta1.3.1.8
9.	compute parameters $[n, p]$ of a binomial distribution.	2	Sta1.3.2.1
10.	solve problems involving number of events using binomial distribution.	2	Sta1.3.2.2
11.	solve problems involving probabilities using binomial distribution.	2	Sta1.3.2.3

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
12.	solve problems using binomial distribution.	2	Sta1.3.2.4
13.	calculate parameter $[\lambda]$ for a Poisson distribution.	2	Sta1.3.2.5
14.	compute probabilities associated with a Poisson distribution.	2	Sta1.3.2.6
15.	calculate parameters $[n,\mu,\sigma]$ for a normal distribution.	2	Sta1.3.2.7
16.	compute probabilities associated with standard normal distribution.	2	Sta1.3.2.8
17.	compute probabilities associated with normal distribution.	2	Sta1.3.2.9
18.	calculate probabilities using formulae and tables.	2	Sta1.3.2.10
19.	solve problems using binomial distribution.	3	Sta1.3.3.1
20.	solve problems using Poisson distribution.	3	Sta1.3.3.2
21.	solve problems using normal distribution.	3	Sta1.3.3.3
22.	solve inverse normal type problems.	3	Sta1.3.3.4
23.	solve problems involving sum and difference of independent normal distribution random variables.	3	Sta1.3.3.5
24.	model problems using binomial distribution.	4	Sta1.3.4.1
25.	model problems using Poisson distribution.	4	Sta1.3.4.2
26.	model problems using normal (including standard normal) distribution.	4	Sta1.3.4.3
27.	solve problems involving combined distributions (e.g. a two-step problem requiring first a normal distribution calculation and then a binomial distribution calculation).	4	Sta1.3.4.4

STRAND 2: MODELLING USING GRAPHICAL METHODS

Major Learning Outcome (sta2):

Students are able to model situations using graphical methods in order to solve problems.

SUB STRAND 2.1: Piecewise Function

Key Learning Outcome (sta2.1):

Understanding simple piecewise functions in order to model data.

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
1.	state features of a linear function.	1	Sta2.1.1.1
2.	state features of a quadratic function.	1	Sta2.1.1.2
3.	define continuous function.	1	Sta2.1.1.3
4.	state properties of a continuous function.	1	Sta2.1.1.4
5.	define discontinuous function.	1	Sta2.1.1.5
6.	state properties of a discontinuous function.	1	Sta2.1.1.6
7.	determine equations from graphs of piecewise functions.	2	Sta2.1.2.1
8.	interpret piecewise functions which have components that are linear.	2	Sta2.1.2.2
9.	interpret piecewise functions which have components that are quadratic.	2	Sta2.1.2.3
10.	interpret piecewise functions which have components that are linear and quadratic.	2	Sta2.1.2.4
11.	determine values from graphs of a discontinuous function.	2	Sta2.1.2.5
12.	determine values from graphs of piecewise functions.	2	Sta2.1.2.6
13.	sketch simple piecewise functions (involving linear functions).	3	Sta2.1.3.1
14.	determine equations from graphs of simple piecewise functions.	3	Sta2.1.3.2
15.	sketch complex piecewise functions (involving linear and quadratic functions).	4	Sta2.1.4.1
16.	determine equations from graphs of complex piecewise functions.	4	Sta2.1.4.2

SUB STRAND 2.2: Power & Exponential Functions

Key Learning Outcome (sta2.2):

Model real data using power and exponential functions in order to solve problems.

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
1.	state the general form of a power function.	1	Sta2.2.1.1
2.	state the general forms of exponential functions.	1	Sta2.2.1.2
3.	identify parts of a power function.	1	Sta2.2.1.3
4.	identify parts of an exponential function.	1	Sta2.2.1.4
5.	draw graphs of power functions.	1	Sta2.2.1.5
6.	draw graphs of exponential functions.	1	Sta2.2.1.6
7.	solve simple power functions.	2	Sta2.2.2.1
8.	solve simple exponential functions.	2	sta2.2.2.2
9.	use log to calculate values of unknowns.	2	Sta2.2.2.3
10.	use natural log to calculate values of unknowns.	2	sta2.2.2.4
11.	show that variables are related by a power law.	3	sta2.2.3.1
12.	show that variables are related by an exponential law.	3	sta2.2.3.2
13.	draw graphs of hyperbolic functions.	3	sta2.2.3.3
14.	use log-log technique to transform variables to relate power functions of the form $y=ax^n$ (where n is a positive integer) to model simple real data.	3	sta2.2.3.4
15.	use semi-log technique to transform variables to relate exponential (index) functions of the form $y=am^x$ (where m can be any base) or $y=Ae^{kx}$ to model real data.	3	sta2.2.3.5
16.	solve simple problems using the models.	3	sta2.2.3.6
17.	solve problems by considering limitations of the model visually.	3	sta2.2.3.7
18.	use log-log technique to transform variables to relate power functions of the form $y= ax^n$ (where n is a positive integer) to model complex real data.	4	sta2.2.4.1
19.	use semi-log technique to transform variables to relate exponential (index) functions of the form $y= am^x$ (where m can be any base) or $y=Ae^{kx}$ to model complex real data.	4	sta2.2.4.2
20.	solve complex problems using the models.	4	sta2.2.4.3

SUB STRAND 2.3: Linear Programming

Key Learning Outcome (sta2.3):

Model situations using linear programming techniques to obtain optimal solution.

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
1.	state features of inequations.	1	sta2.3.1.1
2.	identify regions that are represented by inequations.	1	sta2.3.1.2
3.	define objective function.	1	sta2.3.1.3
4.	define constraints.	1	sta2.3.1.4
5.	define vertex.	1	sta2.3.1.5
6.	identify the objective function (function that is to be maximised or minimised) from a problem.	1	sta2.3.1.6
7.	Formulate objective function (function that is to be maximised or minimised) from a problem.	2	sta2.3.2.1
8.	formulate the constraints (the inequations that show the restrictions that must be considered) from a problem (NB: in some cases the constraints may be given).	2	sta2.3.2.2
9.	compute the point where the objective function is maximised or minimised to solve the problem, when the points are obvious or stated	2	sta2.3.2.3
10.	graph the constraints to obtain the feasible region and shade the region (shading in or shading out).	3	sta2.3.3.1
11.	compute the point where the objective function is maximised or minimised to solve the problem, when three corner points of the feasible region are to be determined first.	3	sta2.3.3.2
12.	construct the constraints (the inequations that show the restrictions that must be considered) from a complex word problem	4	sta2.3.4.1
13.	graph the constraints in a complex problem to obtain the feasible region and shade the region (shading in or shading out).	4	sta2.3.4.2
14.	compute the point where the objective function is maximised or minimised to solve the complex problem, where more than three corner points of the feasible region are to be determined first.	4	sta2.3.4.3

STRAND 3: STATISTICAL INVESTIGATIONS

Major Learning Outcome (sta3):

Students are able to carry out statistical investigations and understand statistical processes.

SUB STRAND 3.1: Bivariate Data

Key Learning Outcome (sta3.1):

Using statistical processes to undertake an investigation of bivariate data.

Specific Learning Outcomes (SLO): Students are able to	Skill level	SLO code
1. define bivariate data.	1	sta3.1.1.1
2. State/identify the Pearson's correlation coefficient, r , formula.	1	sta3.1.1.2
3. state the properties of the Pearson's correlation coefficient, r .	1	sta3.1.1.3
4. state features of a scatter plot.	1	sta3.1.1.4
5. identify variables of interest from a given data set.	1	sta3.1.1.5
6. state a purpose for a statistical investigation.	1	sta3.1.1.6
7. Identify the appropriate sampling method to be used	1	Sta3.1.1.7
8. identify a sampling method appropriate to the purpose of the investigation		sta3.1.1.8
9. Identify the variables of interest for the investigation	1	Sta3.1.1.9
10 Identify the sources of variation in the investigation	1	Sta3.1.1.10
11 describe the methods of managing variations for best results	2	Sta3.1.2.1
12 describe the sampling method appropriate for the purpose	2	sta3.1.2.2
13 describe the data collection procedure	2	Sta3.1.2.3
14 identify trends of a relationship.	1	sta3.1.1.11
15 identify the strength of a relationship.	1	sta3.1.1.12
16 make statements based on simple data shown on graphs.	2	sta3.1.2.4
17 formulate relationship questions to be investigated	2	sta3.1.2.5
18 describe variables of interest from a given data set.	2	sta3.1.2.6
19 collect data for the investigation.	2	sta3.1.2.7
20 display collected data in an appropriate format.	2	sta3.1.2.8
21 describe the trend of relationship in an investigation.	2	sta3.1.2.9
22 describe the strength of relationship in an investigation.	2	sta3.1.2.10
23 compile logically, statistical (bivariate) data, results and conclusion.	2	sta3.1.2.11
24 display collected data as a scatter plot and draw the line of best fit.	3	sta3.1.3.1
25 make concluding statements based on the line of best fit.	3	sta3.1.3.2
26 calculate the Pearson's correlation coefficient, r of a given data set and relate to the data set.	3	sta3.1.3.3
27 make concluding statements based on a given value of the Pearson's correlation coefficient, r .	2	sta3.1.2.12
28 calculate the equation of line of best fit and relate to the data set;	3	sta3.1.3.4
29 discuss trends and strength of relationship with supporting evidence.	4	sta3.1.4.2
30 draw conclusions based on the investigative question and supported with evidence.	4	sta3.1.4.3

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
31	evaluate the strengths and limitations of the investigation design and/or process	4	Sta3.1.4.4

SUB STRAND 3.2: Time Series

Key Learning Outcome (sta3.2):

Using statistical processes to undertake an investigation of time series data.

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
1.	define time series.	1	sta3.2.1.1
2.	define secular trend.	1	sta3.2.1.2
3.	define periodic or seasonal movements.	1	sta3.2.1.3
4.	define erratic or residual variation.	1	sta3.2.1.4
5.	define cyclic variation.	1	sta3.2.1.5
6.	define moving average.	1	sta3.2.1.6
7.	define the order of a moving average.	1	sta3.2.1.7
8.	define moving mean.	1	sta3.2.1.8
9.	define moving median.	1	sta3.2.1.9
10.	state a purpose for a time series investigation.	1	sta3.2.1.10
11.	identify features of a time series distribution.	1	sta3.2.1.11
12.	select variables of interest from a given time series data set.	1	sta3.2.1.12
13.	write a relationship question on time series to be investigated.	1	sta3.2.1.13
14.	state the trend of a relationship.	1	sta3.2.1.14
15.	state the strength of a relationship.	1	sta3.2.1.15
16.	describe variables of interest from a given time series data set.	2	sta3.2.2.1
17.	describe features of the time series (seasonal and long term trends, cyclic trends, outliers, any unusual features).	2	sta3.2.2.2
18.	compute the trend value.	2	sta3.2.2.3
19.	describe the trend in a time series investigation.	2	sta3.2.2.4
20.	compute the smoothed data using moving means.	2	sta3.2.2.5
21.	compute smoothed data using moving medians.	2	sta3.2.2.6
22.	analyse the data - smoothing data (moving medians or moving means and, where appropriate centred moving means);	3	sta3.2.3.1
23.	plot raw and smoothed data on a time series graph; fitting a trend line (linear) based on the smoothed data (either using technology or using visual methods);	3	sta3.2.3.2
24.	formulate the equation of the trend line and relate to or calculate average seasonal effects	3	sta3.2.3.3

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
25.	predict the moving average.	3	sta3.2.3.4
26.	use the trend line and seasonality to make a prediction.	3	sta3.2.3.5
27.	analyse the behaviour of smoothing data (moving medians or moving means and, where appropriate centred moving means).	3	sta3.2.3.6
28.	plot raw and smoothed data on a time series graph.	3	sta3.2.3.7
29.	fit a trend line (linear) based on the smoothed data using visual or technology method.	3	sta3.2.3.8
30.	calculate seasonal effects	3	Sta3.2.3.9
31.	explain the appropriateness of the prediction with respect to (i) the possibility of other models for the trend (e.g. non-linear, piecewise trends), or (ii) limitations of the prediction.	3	sta3.2.3.10
32.	explain the effect of seasonally adjusted values.	3	sta3.2.3.11
33.	develop a conclusion by discussing the features of the time series (seasonal and long term trends, cyclic trends, outliers, any unusual features);	4	sta3.2.4.1
34.	discuss trends and strength of a time series relationship with supporting evidence.	4	sta3.2.4.2
35.	draw conclusion and/or make a prediction based on discussion of findings	4	sta3.2.4.3
36.	discuss the limitations of the prediction	4	Sta3.2.4.4

SUB STRAND 3.3: Comparing Two Populations

Key Learning Outcome (sta3.3):

Using statistical processes to undertake an investigation of numerical data in order to make a comparison between two populations.

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
1.	define sample or populations to be studied.	1	sta3.3.1.1
2.	Identify appropriate sampling method.	1	sta3.3.1.2
3.	Identify outliers in the sample/population.	1	sta3.3.1.4
4.	identify the variable of interest of an investigation (planning).	1	sta3.3.1.5
5.	identify the populations to be investigated (planning).	1	sta3.3.1.6
6.	identify sources of variation.	1	sta3.3.1.7
7.	identify methods to manage variations.	1	sta3.3.1.8
8.	state the different methods of sampling.	1	sta3.3.1.9
9.	state a comparison question to be investigated.	1	sta3.3.1.10
10	describe the sampling method used in the investigation.	2	sta3.3.2.1
11	plan/describe methods of collecting and recording data.	2	sta3.3.2.2
12	plan/describe the investigation by identifying the comparison question, variables of interest; the two populations; sources of variation and establishing methods to manage them.	2	sta3.3.2.3

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
13	describe sources of variation and establishing methods to manage them (e.g. considering outside variables in data collection methods)	2	sta3.3.2.4
14	collect data to be investigated, and tabulate appropriately.	2	sta3.3.2.5
15	describe sources of variation on the collected data.	2	sta3.3.2.7
16	analyse the data by summarizing data with statistical measures;	3	sta3.3.3.1
17	draw and label appropriate graphical displays	3	sta3.3.3.2
18	reflect on the data patterns and form a hypothesis	3	sta3.3.3.3
19	establish methods to manage variations (e.g. considering outside variables in data collection methods).	3	sta3.3.3.4
20	make inferences using confidence interval (difference of two means or proportions).	3	sta3.3.3.5
21	discuss limitations of the investigation with supporting evidence.	4	sta3.3.4.1
22	draw conclusions based on the results of the investigation and support with evidence.	4	sta3.3.4.2

SUB STRAND 3.4: Statistical Processes

Key Learning Outcome (sta3.4):

Answer questions related to statistical processes.

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
1.	define margin of error.	1	sta3.4.1.1
2.	define sample size.	1	sta3.4.1.2
3.	define point estimate.	1	sta3.4.1.3
4.	define sample statistics.	1	sta3.4.1.4
5.	define population parameters.	1	sta3.4.1.5
6.	define statistical inference.	1	sta3.4.1.6
7.	define confidence level.	1	sta3.4.1.7
8.	state the central limit theorem.	1	sta3.4.1.8
9.	compute the margin of error.	2	sta3.4.2.1
10.	compute the sample size (n) using the margin of error.	2	sta3.4.2.2
11.	compute the point estimate of the population mean (sample mean).	2	sta3.4.2.3
12.	compute the point estimate of the population proportion (sample proportion).	2	sta3.4.2.4
13.	compute the standard error in a given confidence interval.	2	sta3.4.2.5
14.	express the confidence interval in a statement.	3	sta3.4.3.1
15.	explain how the margin of error changes when certain parameters of the population/sample or confidence interval are altered.	3	sta3.4.3.2
16.	apply the central limit theorem to solve problems related to confidence interval.	3	sta3.4.3.3
17.	apply point estimates calculations to solve problems related to confidence interval.	3	sta3.4.3.4
18.	calculate the 90% confidence interval for a population mean.	3	sta3.4.3.5
19.	calculate the 95% confidence interval for a population mean.	3	sta3.4.3.6

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
20.	calculate the 99% confidence interval for a population mean.	3	sta3.4.3.7
21.	calculate the 90% confidence interval for a population proportion.	3	sta3.4.3.8
22.	calculate the 95% confidence interval for a population proportion.	3	sta3.4.3.9
23.	calculate the 99% confidence interval for a population proportion.	3	sta3.4.3.10
24.	calculate the 90% confidence interval for the difference in means.	3	sta3.4.3.11
25.	calculate the 95% confidence interval for the difference in means.	3	sta3.4.3.12
26.	calculate the 99% confidence interval for the difference in means.	3	sta3.4.3.13
27.	calculate the 90% confidence interval for the difference in proportions.	3	sta3.4.3.14
28.	calculate the 95% confidence interval for the difference in proportions.	3	sta3.4.3.15
29.	calculate the 99% confidence interval for the difference in proportions.	3	sta3.4.3.16
30.	compare and contrast standard error and margin of error.	3	sta3.4.3.17
31.	explain the impact of the size of the confidence interval on the margin of error.	3	sta3.4.3.18
32.	solve problems related to confidence interval.	4	sta3.4.4.1
33.	predict the effect of the margin of error.	4	sta3.4.4.2
34.	interpret confidence interval for population means.	4	sta3.4.4.3
35.	interpret confidence interval for population proportions.	4	sta3.4.4.4
36.	evaluate the validity of claims using confidence interval.	4	sta3.4.4.5

Explanatory Notes

Assessment will involve a selection from:

- performing or commenting on aspects of the processes involved in outcomes 3.1, 3.2, 3.3
- understanding the central limit theorem
- performing calculations related to confidence intervals (mean, proportion, difference of mean, difference of proportion) at the 90%, 95% & 99% level of confidence and interpreting these intervals
- understanding point estimates and margin of error
- calculating the sample size (n) using the margin of error

All relevant formula that students will require for calculations indicated in the table of SLOs above are to be provided in the formula sheet as well as part of items in tests and examinations, where necessary.

STRAND 4: NUMERICAL AND ALGEBRAIC METHODS

Major Learning Outcome (sta4):

Students are able to use numeric and algebraic methods to solve problems.

SUB STRAND 4.1: Linear Equations

Key Learning Outcome (sta4.1):

Solve systems of 3 x 3 linear equations; Interpret solutions to linear systems of equations; Solve problems that can be modelled by systems of linear equations.

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
1.	state the different types of solutions in solving linear equations.	1	sta4.1.1.1
2.	write linear systems of equations for a given basic situation.	2	sta4.1.2.1
3.	solve simple linear systems of equations for a given situation by elimination and substitution method.	2	sta4.1.2.2
4.	solve simple linear systems of equations for a given situation by graphing method.	2	sta4.1.2.3
5.	describe the nature of solutions of a linear systems of equations.	2	sta4.1.2.4
6.	determine the condition for infinitely many solutions to a given set of equations.	2	sta4.1.2.5
7.	solve linear systems of equations (use of matrices is accepted).	3	sta4.1.3.1
8.	write linear systems of equations for a given complex situation, involving three equations in three variables.	3	sta4.1.3.2
9.	solve application word problems by elimination and substitution method.	3	sta4.1.3.3
10.	solve application word problems by graphing method.	3	sta4.1.3.4
11.	interpret the nature of solutions of linear system of equation with description.	3	sta4.1.3.5
12.	solve 3 x 3 linear system of equation based on real life situations (use of matrices is accepted).	4	sta4.1.4.1

Explanatory Notes

Assessment will focus on using simultaneous equations to solve problems in context:

- solving 3x3 linear systems of equations (use of matrices is accepted).
- forming linear systems of equations for a given situation and solving them in context.
- determining and interpreting the nature of solutions, including
- Unique or dependent solutions
- No solution
- Geometric interpretations

SUB STRAND 4.2: Non-Linear Equations

Key Learning Outcome (sta4.2):

Use bisection method or Newton-Raphson method to solve non-linear equations to a given precision.

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
1.	state the advantage or disadvantage of using the Newton-Raphson method.	1	sta4.2.1.1
2.	state the advantage or disadvantage of using the Bisection method.	1	sta4.2.1.3
3.	determine the presence of roots of a hyperbolic function between two x values.	2	sta4.2.2.1
4.	determine the nature of the roots after using the Newton-Raphson to improve the approximation to a given precision.	2	sta4.2.2.2
5.	differentiate power functions in order to use the Newton-Raphson Method.	3	sta4.2.3.1
6.	use the Newton-Raphson method to solve the equation $f(x)$.	3	sta4.2.3.2
7.	use the Bisection method to solve the equation $f(x)$.	3	sta4.2.3.3
8.	solve a non-linear equation using the Newton-Raphson method with a given starting value.	4	sta4.2.4.1
9.	solve a non-linear equation using the Bisection Method with a given starting interval.	4	sta4.2.4.2
10.	use a suitable method to find an approximate solution to a non-linear equation.	4	sta4.2.4.3

Explanatory Notes

Assessment will focus on:

- use of the Newton-Raphson or the Bisection Method (or both) to improve the approximation to a given precision.*
- differentiate simple power functions in order to use the Newton-Raphson Method (this involves derivatives of polynomials only. If needed, derivatives of other functions will be given)*

NB: Students will be expected to be familiar with both the Newton-Raphson and the bisection methods.

The starting values will be provided.

ASSESSMENT

Assessment in this Mathematics with Statistics course is in two parts - external assessment and internal assessment. The respective weightings are

1. **External Assessment (EA) : 60%**
2. **Internal Assessment (IA) : 40%**

The principal, or his/her nominee, will certify that the syllabus requirements have been fulfilled.

Suggested Teaching Time and Weightings

STRAND	EXTERNAL/ INTERNAL	SUGGESTED TIME ALLOCATION	APPROXIMATE WEIGHTING
1. Strand 1 : Probability	<i>External</i>	<i>8 weeks</i>	<i>20 %</i>
2. Strand 2 : Modelling Using Graphical Methods	<i>External</i>	<i>5 weeks</i>	<i>17 %</i>
3. Strand 3 : Statistical Investigations	<i>External & Internal</i>	<i>12 weeks (4 weeks external / 8 weeks internal)</i>	<i>50 % (10% external / 40% internal)</i>
4. Strand 4 : Numerical and Algebraic Methods	<i>External</i>	<i>3 weeks</i>	<i>13 %</i>
Total		<i>28 weeks</i>	<i>100%</i>

Assessment Blueprint

Strand	Assessment Type	SKILL LEVEL/ SCORE				Weight
		1	2	3	4	
1. Probability	EA					20
2. Modelling using graphical methods	EA					17
3. Statistical investigations	EA					10
	IA	3	5	5	3	40
4. Numerical and algebraic methods	EA					13
Total number of items		20	15	10	5	100
Total skill score		20	30	30	20	

External Examination

This will be a **three-hour** written examination and will be out of **70%**.

The external written examination will assess learning outcomes from all four strands/sections in the following proportions:

Strand / Section		Major Learning Outcomes	Total score per section	Suggested Exam Time
1	Probability	Develop knowledge and skills related to Probability in order to solve problems and to investigate situations involving elements of chance.	20	60 mins
2	Modelling using graphical methods.	Model situations using graphical methods in order to solve problems.	17	50 mins
3	Statistical investigations	Understand statistical processes.	10	30 mins
4	Numerical and algebraic methods	Use numeric and algebraic methods to solve problems	13	40 mins

Learning outcomes that have been assessed using internal assessment will not be assessed in the external assessment.

Students would be expected to know, understand and apply any mathematical skill related to the outcomes. Examination questions, which require specific mathematical knowledge, will be based on the outcomes. However, students can be assessed the same learning outcomes using an unfamiliar context to assess students' application of the mathematical concepts in the real world context.

Emphasis or the number of questions in the examination will reflect the teaching time and weightings allocated to each topic.

A standard list of formulae will be incorporated in the examination paper. Additional formulae may be provided where appropriate. A copy of the formulae list is attached as Appendix 1 in this syllabus.

The use of calculators and computers is an integral part of mathematics. Students will be required to supply their own calculators for use in the examination. The use of calculators during the examination must conform to the EQAP Rules and Procedures governing electronic calculators.

Internal Assessment

There are three internal assessment tasks and these include:

Task 1: Statistical Investigations- Bivariate Project (15%)

Task 2: Statistical Investigations- Time Series Project (10%)

Task 3: Statistical Investigations- Comparison Project (15%)

These assessments will complement the written examination. They emphasize those specific outcomes in the syllabus for which assessment by an external examination is less appropriate, e.g., carrying out statistical investigations.

To meet the internally assessed component requirement, students will carry out three tasks and then submit three pieces of written or word processed work, reporting on the statistical investigations undertaken. The three investigations to be carried out are briefly described below.

The use of computer technology in the investigations is strongly encouraged, including the use of appropriate statistical packages, or simple graphing packages, where available.

At the beginning of each year, each school presenting students for the South Pacific Form Seven Certificate Mathematics (with Statistics) assessment must complete an Internal Assessment programme proposal and forward to EQAP by the date set down by the Director. The form must clearly indicate the time (start and completion date) each investigation is intended to be undertaken. The Internal Assessment Programme Proposal template is provided as Appendix 3. Teachers are advised to fill in all the required details based on information provided in the syllabus as well as those from their schools and submit the proposal to EQAP by the stipulated due date. Failure of a school to submit an acceptable IA programme proposal will result in possible disqualification of the IA program for that school.

Students should be given ample notification about an Internal Assessment task and ample time to complete the task. It is recommended that students be provided with the scoring rubrics for each task so that they are fully informed of the criteria against which their task responses will be assessed. A sample of candidate responses to all internal assessment tasks undertaken must be available for verification on request until 30 November of the year of the examination.

The moderation of Internal Assessment will be done in accordance with EQAP policy as specified from time to time.

The assessment/scoring schedules for the three IA tasks are included in the next section.

IA Task 1 - Statistical Investigation using Bivariate data

Instruction to Students:

The investigation will involve:

1. Posing a relationship question to be investigated and verify this with the teacher
2. Planning the investigation: based on the relationship questions agreed with the teacher, students are to
 - ✓ identify the population
 - ✓ identify the variables of interest
 - ✓ identifying the sources of variation and establishing methods to manage them
 - ✓ choosing the sampling method
 - ✓ plan methods of collecting and recording data
3. gathering the data
 - ✓ collecting and recording data
4. analysing the data
 - ✓ plotting data on a scatter plot
 - ✓ fitting a line of best fit (either using technology or using the method of two medians)
 - ✓ determining the equation of the line of best fit and relate this to the data set.
 - ✓ calculating the Pearson's correlation coefficient, r (NB: if students do not have access to computer technology, teachers may need to consider restricting the amount of data that is to be analysed)
5. developing a conclusion
 - ✓ interpreting the trend
 - ✓ interpreting the strength of the relationship (correlation)
 - ✓ writing a conclusion to answer the investigative question
 - ✓ discuss causality
 - ✓ discussing limitations
6. writing a report on the above.
7. referring to the scoring rubric when writing the report.

IA Task 2 – Statistical Investigation 2 – Time Series Project

The investigation will involve:

1. choosing variables of interest from an existing data set
2. stating a purpose for the investigation
3. analysing the data
 - ✓ smoothing data (moving medians or moving means and, where appropriate centred moving means)
 - ✓ plotting raw and smoothed data on a time series graph
 - ✓ fitting a trend line (linear) based on the smoothed data (either using technology or using visual methods)
 - ✓ formulate the equation of the trend line.
 - ✓ calculate average seasonal effects
- developing a conclusion
 - ✓ discuss the features of the time series (seasonal and long term trends, cyclic trends, outliers, any unusual features)
 - ✓ use the trend line and seasonality to make a prediction
 - ✓ discuss the appropriateness of the prediction with respect to:
 - the possibility of other models for the trend (e.g. non-linear, piecewise trends)
 - limitations of the prediction
 - ✓ consider and comment on seasonally adjusted values
- writing a report on the above.
- referring to the scoring rubric when writing the report.

IA Task 3 – Statistical Investigation 3 – Comparison Project

The investigation will involve:

- posing a comparison question to be investigated
- planning the investigation
 - ✓ identifying the variable of interest
 - ✓ identifying the two populations
 - ✓ identifying sources of variation and establishing methods to manage them (e.g. Considering outside variables in data collection methods)
 - ✓ choosing a sampling method
 - ✓ planning methods of collecting and recording data
- gathering the data
 - ✓ collecting and recording data
 - ✓ managing & sorting data
 - ✓ cleaning data (removal of outliers)
- analysing the data
 - ✓ summarising data with statistical measures
 - ✓ drawing appropriate displays
 - ✓ reflecting on the distribution
 - ✓ formulating a hypothesis
- developing a conclusion
 - ✓ interpreting data
 - ✓ making inferences using confidence interval (difference of two means or proportions).
 - ✓ writing a conclusion to answer the investigative question
 - ✓ discussing limitations
- writing a report on the above.
- referring to the scoring rubric when writing the report.

Scoring Rubrics

IA Task 1 Scoring Rubric

Statistical Investigation using BIVARIATE data (15%)

ITEM #	SLO CODE	SKILL LEVEL	EVIDENCE	STUDENT RESPONSE LEVEL			
				Level 1	Level 2	Level 3	Level 4
1a	sta3.1.1.6	1	Purpose stated	Purpose of investigation clearly stated.			
1b	Sta3.1.1.5	1	Variables of interest are identified	One variable identified	Two variables are identified		
1c	Sta3.1.2.2	2	Sampling procedure is described	Procedure is partly correct	Procedure is fully correct.		
1d	sta3.1.2.7	2	Data collected	Data collected but not clearly recorded or cleaned.	Data collected, clearly recorded and cleaned where appropriate.		
1e	sta3.1.2.8	2	Collected data displayed in an appropriate form	Data collected but not clearly recorded in an appropriate form	Data collected and recorded appropriately		
1f	sta3.1.3.1	3	Scatter plot clear, and appropriate line of best fit drawn	Scatter plot partially correct, no line of best fit or incorrect.	Any one correctly provided; scatter plot, appropriate line of best fit	Scatter plot, and appropriate line of best fit are correct and labelled	
1g	Sta3.1.3.4	3	Equation of the line of the best fit is determined and related to the data set	Equation of line of best fit is calculated but not fully correct	Equation of line of best fit is correctly determined.	Equation of line of best fit is correctly determined and related appropriately to the data set.	
1h	Sta3.1.3.3	3	Pearson's correlation coefficient is calculated	Calculation has one appropriate step (e.g. correct formula) but the rest are not.	Calculation is complete but partly incorrect	Calculation is correct contributing to correct final value	

ii	sta3.1.4.3	4	Clear conclusion, showing interpretation of strength in context of relationship with evidence.	Strength correctly identified	Clear interpretation of strength in context of relationship	Clear conclusion showing interpretation of strength in context of relationship	Clear conclusion, showing interpretation of strength in context of relationship with evidence.
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IA Task 2 Scoring Rubric

TIME SERIES (10%)

QUESTION #	SLO CODE	SKILL LEVEL	EVIDENCE	STUDENT RESPONSE LEVEL			
				Level 1	Level 2	Level 3	Level 4
2a	sta3.2.1.10	1	Purpose stated.	Purpose of an investigation is clearly stated.			
2b	Sta3.2.1.13	1	Relationship question is stated	Relationship question is appropriate to the selected data set			
2c	sta3.2.2.1	2	Variables identified and clearly described.	Variables identified only.	Variables identified and clearly described		
2d	Sta3.2.2.5	2	Smoothed data are calculated	Smoothed data are presented but with some errors	Smoothed data calculated correctly		
2e	sta3.2.3.1	3	Table, graphs of raw and smoothed data	Data presented (as a table or graph) or calculation is partially correct.	Data presented in a table along with smoothed values and graph but with some errors	Table, graphs of raw and smoothed data all correct	
2f	Sta3.2.3.3	3	Equation of trend line is formulated and related to seasonal effects	Equation of trend line is given, but may have error	Equation of trend line is fully correct	Equation of trend line is fully correct and related to seasonal effects	
2g	sta3.2.2.4	2	Trend clearly described.	Trend identified.	Trend described with two relevant points or more		
2h	Sta3.2.3.9	3	Seasonal effects are calculated	One value is presented, but working may not be fully correct	Two or more effects are calculated but working may not be fully correct	Two or more effects are calculated with fully correct working	

2i	sta3.2.4.3	4	Clear conclusion with appropriate discussions on features, prediction and seasonally adjusted values of the time series.	A feature or correct prediction is identified.	Features described and correct prediction is made.	All features described, a correct prediction is made and discussed the appropriateness of the prediction.	Clear conclusion with appropriate discussions on features, prediction and seasonally adjusted values of the time series.
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IA Task 3 Scoring Rubric
COMPARISON PROJECT (15%)

ITEM #	SLO CODE	SKILL LEVEL	EVIDENCE	STUDENT RESPONSE LEVEL			
				Level 1	Level 2	Level 3	Level 4
3a	sta3.3.1.10	1	Comparison question clearly stated	Question clearly stated			
3b	Sta3.3.1.6	1	The two populations under study are named	Populations are named correctly			
3c	sta3.3.2.2	2	Methods of collecting and recording data clearly stated	Either one clearly stated (methods of collecting or recording).	Both methods of collecting and recording data clearly stated.		
3d	sta3.3.2.1	2	Sampling method clearly described	Sampling method identified.	Sampling method clearly described, using two or more appropriate ideas.		
3e	sta3.3.2.5	2	Data collected, clearly recorded and cleaned where appropriate.	Data collected but not clearly recorded or cleaned.	Data collected, clearly recorded and cleaned.		
3f	Sta3.3.3.1	3	Summary of data with appropriate measures of central tendency and spread;	An appropriate measure of central tendency or an appropriate measure of spread is provided.	Two or more measures are provided correctly;	Two or more measures are provided and related appropriately to the data set and the investigation purpose.	

ITEM #	SLO CODE	SKILL LEVEL	EVIDENCE	STUDENT RESPONSE LEVEL			
				Level 1	Level 2	Level 3	Level 4
3g	sta3.3.3.2	3	Appropriate display/s provided and labelled	Presented data through an appropriate display, but very limited.	An appropriate display is provided and labelled but with some minor mistakes or omissions.	Appropriate display is provided and fully and correctly labelled.	
3h	sta3.3.4.2	4	Clear conclusion that answers the investigative question, making a clear and justified inference back in the population.	A concluding statement is given but it is one idea only so rather limited.	A concluding statement or two are provided containing two or more unrelated ideas.	Concluding statements are provided that had appropriate ideas that are related well.	Clear conclusion that answers the investigative question, making a clear and justified inference back in the population.
3i	Sta3.3.4.1	4	Discussion of limitations in their data study, and discussing any possible further investigation that the data may have highlighted.	State one limitation	States two or more limitations or describe a limitation, using two or more ideas.	Explains why the limitation influences the results.	The limitation is discussed in relation to possible further investigation that the data may have highlighted.

APPENDICES

Appendix 1: Formulas and Tables

MATHEMATICS WITH STATISTICS — USEFUL FORMULAE AND TABLES

Straight Line

$$\text{Gradient } m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Equation } y - y_1 = m(x - x_1)$$

Quadratic Equations

$$\text{If } ax^2 + bx + c = 0$$

$$\text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Bisection Method

$$x_2 = \frac{x_0 + x_1}{2}$$

Newton-Raphson Method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Differentiation

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
x^n	nx^{n-1}
$\ln x$	$\frac{1}{x}$
e^{ax}	ae^{ax}

Differential Equation

$$\text{If } \frac{dy}{dx} = ky \text{ then } y = Ae^{kx}$$

Arithmetic Sequences and Series

$$t_n = a + (n-1)d$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

Geometric Sequences and Series

$$t_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} \quad \text{for } r \neq 1$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Exponential Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \text{ (for all } x \text{)}$$

Logarithms

$$\ln x = \log_e x$$

$$\text{If } y = b^x \text{ then } \log_b y = x$$

$$\text{If } y = e^x \text{ then } x = \ln y = \log_e y$$

$$\log_b x + \log_b y = \log_b xy$$

$$\log_b x - \log_b y = \log_b \frac{x}{y}$$

$$\log_b x^n = n \log_b x$$

Binomial Theorem

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + \binom{n}{n}b^n$$

$$\text{where } \binom{n}{r} = {}^nC_r = \frac{n!}{(n-r)!r!} \quad \text{Note: } {}^nC_r = \frac{n!}{(n-r)!}$$

Some values of $\binom{n}{r}$ are given in the table below.

Binomial Coefficients

$r \backslash n$	0	1	2	3	4	5	6	7	8	9	10
0	1										
1	1	1									
2	1	2	1								
3	1	3	3	1							
4	1	4	6	4	1						
5	1	5	10	10	5	1					
6	1	6	15	20	15	6	1				
7	1	7	21	35	35	21	7	1			
8	1	8	28	56	70	56	28	8	1		
9	1	9	36	84	126	126	84	36	9	1	
10	1	10	45	120	210	252	210	120	45	10	1
11	1	11	55	165	330	462	462	330	165	55	11
12	1	12	66	220	495	792	924	792	495	220	66
13	1	13	78	286	715	1287	1716	1716	1287	715	286
14	1	14	91	364	1001	2002	3003	3432	3003	2002	1001
15	1	15	105	455	1365	3003	5005	6435	6435	5005	3003

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Expectation Algebra

$$E(aX + b) = aE(X) + b$$

$$\text{Var}(aX + b) = a^2 \text{Var}(X)$$

$$E(aX + bY) = aE(X) + bE(Y)$$

$$\text{Var}(aX + bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y),$$

(if X, Y are independent)

Mean and Variance of Data

$$\bar{x} = \frac{\sum f_i x_i}{n} \quad s^2 = \frac{\sum f_i x_i^2 - \frac{(\sum f_i x_i)^2}{n}}{n}$$

Mean and Variance of a Random Variable

$$\mu = E(X) = \sum x_i P(X = x_i) = E(X^2) - [E(X)]^2$$

Distribution of Sample Statistics

Statistic	Mean	Standard Deviation
Sample Mean	$E(\bar{X}) = \mu$	$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$ (std. error of the mean)
Sample Proportion	$E(P) = \pi$	$\sigma_P = \sqrt{\frac{\pi(1-\pi)}{n}}$ (std. error of the proportion)
Difference of Means (of two independent samples)	$E(\bar{X}_1 - \bar{X}_2) = \mu_1 - \mu_2$	$\sigma_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

Confidence Intervals

Mean $\bar{X} - z \cdot \sigma_{\bar{X}} < \mu < \bar{X} + z \cdot \sigma_{\bar{X}}$

Proportion $P - z \cdot \sigma_P < \pi < P + z \cdot \sigma_P$

Difference of two means $(\bar{X}_1 - \bar{X}_2) - z \cdot \sigma_{\bar{X}_1 - \bar{X}_2} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + z \cdot \sigma_{\bar{X}_1 - \bar{X}_2}$

Standard Normal Distribution



$$Z = \frac{X - \mu}{\sigma}$$

Each entry gives the probability that the standardised normal random variable Z lies between 0 and z.

Differences

z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359	4	8	12	16	20	24	28	32	36
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754	4	8	12	16	20	24	28	32	36
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141	4	8	12	15	19	22	27	31	35
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517	4	8	11	15	19	22	26	30	34
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879	4	7	11	14	18	22	25	29	32
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224	3	7	10	14	17	21	24	27	31
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549	3	6	10	13	16	19	23	26	29
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852	3	6	9	12	15	18	21	24	27
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133	3	6	8	11	14	17	19	22	25
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389	3	5	8	10	13	15	18	20	23
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621	2	5	7	9	12	14	16	18	21
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830	2	4	6	8	10	12	14	16	19
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015	2	4	5	7	9	11	13	15	16
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177	2	3	5	6	8	10	11	13	14
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319	1	3	4	6	7	8	10	11	13
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441	1	2	4	5	6	7	8	10	11
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545	1	2	3	4	5	6	7	8	9
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633	1	2	3	4	5	6	7	8	8
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706	1	1	2	3	4	5	6	6	6
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767	1	1	2	2	3	4	4	5	5
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817	0	1	1	2	2	3	3	4	4
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857	0	1	1	2	2	2	3	3	4
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890	0	1	1	2	2	2	3	3	3
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916	0	1	1	1	2	2	2	2	2
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936	0	1	1	1	1	1	1	1	2
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952	0	0	0	1	1	1	1	1	1
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964	0	0	0	1	1	1	1	1	1
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974	0	0	0	1	1	1	1	1	1
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981	0	0	0	0	0	0	0	0	1
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986	0	0	0	0	0	0	0	0	1
3.0	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4989	.4990	.4990	0	0	0	0	0	0	0	0	0
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993	0	0	0	0	0	0	0	0	0
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4994	.4995	.4995	0	0	0	0	0	0	0	0	0
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997	0	0	0	0	0	0	0	0	0
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998	.4998	0	0	0	0	0	0	0	0	0
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	0	0	0	0	0	0	0	0	0
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	0	0	0	0	0	0	0	0	0
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	0	0	0	0	0	0	0	0	0
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	0	0	0	0	0	0	0	0	0
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	0	0	0	0	0	0	0	0	0

Binomial Distribution

$$P(X = x) = \binom{n}{x} \pi^x (1 - \pi)^{n-x}$$

$$\left(\mu = n\pi, \quad \sigma = \sqrt{n\pi(1 - \pi)} \right)$$

Each entry gives the probability that a binomial random variable X , with the parameters n and π , has the value x .

n	x	0.05	0.1	0.15	$1/6$	0.2	0.25	$\pi/3$	$1/3$	0.35	0.4	0.45	0.5
4	0	0.8145	0.6561	0.5220	0.4823	0.4096	0.3164	0.2401	0.1975	0.1785	0.1296	0.0915	0.0625
	1	0.1715	0.2916	0.3685	0.3858	0.4096	0.4219	0.4116	0.3951	0.3845	0.3456	0.2995	0.2500
	2	0.0135	0.0486	0.0975	0.1157	0.1536	0.2109	0.2646	0.3105	0.3683	0.4375	0.5075	0.3750
	3	0.0005	0.0036	0.0115	0.0154	0.0256	0.0469	0.0756	0.0988	0.1115	0.1536	0.2005	0.2500
	4	0.0001	0.0005	0.0008	0.0016	0.0039	0.0081	0.0123	0.0140	0.0156	0.0256	0.0410	0.0625
5	0	0.7738	0.5905	0.4437	0.4019	0.3277	0.2373	0.1681	0.1317	0.1160	0.0778	0.0503	0.0313
	1	0.2036	0.3261	0.3915	0.4019	0.4096	0.3955	0.3602	0.3124	0.2592	0.2069	0.1513	0.1000
	2	0.0114	0.0729	0.1382	0.1608	0.2045	0.2637	0.3087	0.3416	0.3566	0.3369	0.3125	0.2875
	3	0.0011	0.0081	0.0244	0.0372	0.0512	0.0679	0.1323	0.1616	0.1811	0.2004	0.2157	0.2125
	4	0.0005	0.0022	0.0032	0.0064	0.0146	0.0284	0.0412	0.0488	0.0568	0.1128	0.1563	0.1563
	5	0.0001	0.0001	0.0003	0.0010	0.0024	0.0041	0.0053	0.0102	0.0102	0.0185	0.0313	0.0313
6	0	0.7351	0.5314	0.3771	0.3349	0.2621	0.1780	0.1176	0.0878	0.0754	0.0467	0.0277	0.0156
	1	0.2321	0.3543	0.3993	0.4019	0.3932	0.3560	0.3025	0.2634	0.2437	0.1866	0.1359	0.0938
	2	0.0305	0.0984	0.1762	0.2099	0.2458	0.2966	0.3241	0.3292	0.3280	0.3110	0.2780	0.2344
	3	0.0021	0.0146	0.0415	0.0536	0.0819	0.1318	0.1852	0.2195	0.2355	0.2765	0.3032	0.3125
	4	0.0001	0.0012	0.0055	0.0080	0.0154	0.0330	0.0595	0.0823	0.0951	0.1382	0.1861	0.2344
	5	0.0001	0.0004	0.0006	0.0015	0.0044	0.0102	0.0165	0.0205	0.0369	0.0609	0.0938	0.0938
	6	0.0001	0.0001	0.0002	0.0001	0.0002	0.0007	0.0014	0.0018	0.0018	0.0083	0.0156	0.0156
7	0	0.6983	0.4783	0.3206	0.2791	0.2097	0.1335	0.0824	0.0585	0.0490	0.0280	0.0152	0.0078
	1	0.2573	0.3720	0.3960	0.3907	0.3670	0.3115	0.2471	0.2048	0.1848	0.1306	0.0872	0.0547
	2	0.0406	0.1240	0.2097	0.2534	0.3115	0.3177	0.3073	0.2985	0.2813	0.2140	0.1641	0.1244
	3	0.0036	0.0250	0.0617	0.0781	0.1147	0.1730	0.2269	0.2561	0.2679	0.2903	0.2918	0.2734
	4	0.0002	0.0026	0.0109	0.0156	0.0287	0.0577	0.0972	0.1280	0.1442	0.1935	0.2388	0.2734
	5	0.0002	0.0012	0.0019	0.0043	0.0115	0.0250	0.0384	0.0466	0.0774	0.1172	0.1641	0.1641
	6	0.0001	0.0001	0.0001	0.0004	0.0013	0.0036	0.0064	0.0084	0.0172	0.0330	0.0547	0.0547
	7	0.0001	0.0001	0.0001	0.0001	0.0002	0.0005	0.0006	0.0006	0.0016	0.0037	0.0078	0.0078
8	0	0.6634	0.4305	0.2725	0.2326	0.1678	0.1001	0.0576	0.0390	0.0319	0.0168	0.0084	0.0039
	1	0.2793	0.3826	0.3847	0.3721	0.3355	0.2670	0.1977	0.1561	0.1373	0.0896	0.0564	0.0313
	2	0.0515	0.1488	0.2376	0.2645	0.2929	0.3115	0.2965	0.2751	0.2587	0.2098	0.1548	0.1122
	3	0.0054	0.0333	0.0839	0.1042	0.1468	0.2176	0.2541	0.2731	0.2786	0.2568	0.2188	0.1734
	4	0.0004	0.0046	0.0185	0.0260	0.0459	0.0865	0.1361	0.1707	0.1875	0.2522	0.2627	0.2734
	5	0.0004	0.0004	0.0002	0.0004	0.0009	0.0023	0.0047	0.0083	0.0088	0.1239	0.1719	0.2188
	6	0.0001	0.0002	0.0004	0.0011	0.0038	0.0072	0.0100	0.0171	0.0217	0.0413	0.0703	0.1094
	7	0.0001	0.0001	0.0004	0.0001	0.0004	0.0001	0.0004	0.0001	0.0002	0.0007	0.0164	0.0313
	8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0007	0.0117	0.0313
9	0	0.6302	0.3874	0.2316	0.1938	0.1342	0.0751	0.0404	0.0260	0.0207	0.0101	0.0046	0.0020
	1	0.2985	0.3874	0.3679	0.3489	0.3020	0.2253	0.1536	0.1171	0.1004	0.0605	0.0339	0.0176
	2	0.0629	0.1722	0.2597	0.2791	0.3020	0.3003	0.2668	0.2341	0.2162	0.1612	0.1110	0.0703
	3	0.0077	0.0446	0.1069	0.1302	0.1762	0.2336	0.2668	0.2731	0.2716	0.2508	0.2119	0.1641
	4	0.0006	0.0074	0.0283	0.0391	0.0661	0.1168	0.1715	0.2048	0.2194	0.2508	0.2600	0.2461
	5	0.0008	0.0050	0.0078	0.0165	0.0389	0.0735	0.1024	0.1181	0.1672	0.2128	0.2461	0.2461
	6	0.0001	0.0006	0.0010	0.0028	0.0087	0.0210	0.0341	0.0424	0.0743	0.1160	0.1641	0.1641
	7	0.0001	0.0001	0.0003	0.0012	0.0039	0.0092	0.0171	0.0233	0.0398	0.0622	0.0407	0.0703
	8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0004	0.0001	0.0001	0.0003	0.0003	0.0003	0.0020
	9	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
10	0	0.5987	0.3487	0.1969	0.1615	0.1074	0.0563	0.0282	0.0173	0.0135	0.0060	0.0025	0.0010
	1	0.3151	0.3874	0.3474	0.3230	0.2684	0.1877	0.1211	0.0867	0.0725	0.0403	0.0207	0.0098
	2	0.0746	0.1937	0.2759	0.2907	0.3020	0.2816	0.2335	0.1951	0.1757	0.1209	0.0763	0.0439
	3	0.0105	0.0514	0.1298	0.1550	0.2013	0.2503	0.2668	0.2601	0.2522	0.2150	0.1665	0.1172
	4	0.0010	0.0112	0.0401	0.0543	0.0881	0.1460	0.2001	0.2276	0.2377	0.2508	0.2384	0.2051
	5	0.0001	0.0015	0.0085	0.0130	0.0264	0.0584	0.1029	0.1366	0.1536	0.2007	0.2540	0.2461
	6	0.0001	0.0001	0.0012	0.0022	0.0055	0.0162	0.0368	0.0569	0.0689	0.1115	0.1596	0.2051
	7	0.0001	0.0001	0.0002	0.0002	0.0008	0.0031	0.0090	0.0163	0.0212	0.0425	0.0746	0.1172
	8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0004	0.0001	0.0001	0.0001	0.0004	0.0029	0.0439
	9	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0042	0.0098
	10	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0010

(all other entries < 0.0001)

Poisson Distribution

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$\left(\mu = \lambda, \quad \sigma = \sqrt{\lambda} \right)$$

Each entry gives the probability that a Poisson random variable X , with parameter λ , has the value x .

x	λ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066	0.3679	0.3329
1	0.0905	0.1637	0.2222	0.2881	0.3533	0.4295	0.5057	0.5820	0.6583	0.7346	0.8109
2	0.0042	0.0104	0.0193	0.0307	0.0456	0.0649	0.0887	0.1170	0.1498	0.1867	0.2277
3	0.0002	0.0001	0.0003	0.0007	0.0016	0.0030	0.0050	0.0077	0.0111	0.0153	0.0203
4	0.0001	0.0001	0.0001	0.0002	0.0004	0.0007	0.0012	0.0020	0.0031	0.0045	0.0061
5	0.0001	0.0001	0.0001	0.0001	0.0002	0.0004	0.0007	0.0012	0.0020	0.0031	0.0045
6	0.0001	0.0001	0.0001	0.0001	0.0002	0.0004	0.0007	0.0012	0.0020	0.0031	0.0045
7	0.0001	0.0001	0.0001	0.0001	0.0002	0.0004	0.0007	0.0012	0.0020	0.0031	0.0045
8	0.0001	0.0001	0.0001	0.0001	0.0002	0.0004	0.0007	0.0012	0.0020	0.0031	0.0045
9	0.0001	0.0001	0.0001	0.0001	0.0002	0.0004	0.0007	0.0012	0.0020	0.0031	0.0045
10	0.0001	0.0001	0.0001	0.0001	0.0002	0.0004	0.0007	0.0012	0.0020	0.0031	0.0045
11	0.0001	0.0001	0.0001	0.0001	0.0002	0.0004	0.0007	0.0012	0.0020	0.0031	0.0045
12	0.0001	0.0001	0.0001	0.0001	0.0002	0.0004	0.0007	0.0012	0.0020	0.0031	0.0045
13	0.0001	0.0001	0.0001	0.0001	0.0002	0.0004	0.0007	0.0012	0.0020	0.0031	0.0045
14	0.0001	0.0001	0.0001	0.0001	0.0002	0.0004	0.0007	0.0012	0.0020	0.0031	0.0045
15	0.0001	0.0001	0.0001	0.0001	0.0002	0.0004	0.0007	0.0012	0.0020	0.0031	0.0045
16	0.0001	0.0001	0								

Appendix 2: Verb Taxonomy

BLOOMS TAXONOMY	SOLO TAXONOMY	SKILL LEVEL SCORE	SKILL LEVEL BAND	DESCRIPTORS
Knowledge	Unistructural <ul style="list-style-type: none"> students make simple and obvious connections between pieces of information 	1	1	Mastery of the basic knowledge and skills that are fundamental for proficient work.
	Multistructural <ul style="list-style-type: none"> a number of connections are made, but not the metaconnections between them 	2		
Comprehension	Relational <ul style="list-style-type: none"> students see the significance of how the various pieces of information relate to one another 	3	2	Solid academic performance for the given learning outcome and competency over challenging subject matter including subject-matter knowledge, application of such knowledge to real world situations.
Application				
Synthesis, Analysis, Evaluation	Extended Abstract <ul style="list-style-type: none"> at this level students can make connections beyond the scope of the problem or question, to generalise or transfer learning into a new situation 	4	3	Presumes mastery of both the Basic and Proficient levels and represents superior academic performance.

FULL IA PROGRAM

Page 1: COVER PAGE

<p style="text-align: center;">xxxxxx SEC SCHOOL SPFSC 2020 Xxxxx (subject): FULL IA PROGRAM</p> <p style="text-align: center;">Name:</p>

Page 2: INSERT IA SUMMARY FORM HERE

(To be completed, signed by both the teacher and the School Principal of his/her Nominee/school stamped/scan/insert)

Pages 3-6:

1 Task title: Task 1: _____

The title should be brief and include a reference to the particular syllabus topic or skill which is being assessed by the task.

Example: *“Research Topic – Investigation of a Social Issue.”*

2 Learning Outcomes: List the Specific Learning Outcomes (SLOs) to be assessed by the task

These are found in the syllabus and need to be identified before the tasks are constructed.

Example: *Describe a feature of*

(Copy and paste directly from the aligned Syllabus: it must show strand, sub strand and SLOs)

3. Assessment/Task:

Describe the task as a form of assessment to measure student achievements of the above learning outcomes at different stages of the lesson/task implementation.

(Think of what are the best types of assessment for the above LOs so that your students can demonstrate they have achieved the learning outcomes. Also include how you will pre-assess their knowledge at the beginning of the lesson and how you will continuously assess them throughout the strand/topic to monitor their learning progress. The summative assessments are the final IA tasks.)

e.g. Diagnostic: *(can be oral questions/short tests/ surveys/questionnaires to find out what students already know before the lesson)*

Formative: 1. This is the formative use of the summative assessment such as the drafts submitted, self-assessment, peer assessment, teacher assessment of the drafts and specific feedback provided to improve the task. 2. For CATs – this can be similar items prepared by teachers using the SLOs and given to students for practice. After scoring, the feedback needs to be given to improve learning. If majority students not doing well then re-teach using another strategy, assess and monitor learning.

Summative: (these are the final IA tasks or the CATs to measure how much the students have learnt/achieved after the learning period)

4 Resources: List materials required for completing the task (for learning & demonstrating the achievement for the SLOs.

This must specify any material items such as books, documents, maps, stimulus material, equipment required by the task, including use of technology.

5 Guidelines for the teacher on advance preparation requirements

- a) **time required** by the student for task completion (monitoring progress)
- b) recommended dates/date range for task completion
- c) organization of room and hardware to facilitate task completion (learning assessment).

(After the task has been completed and scored, teachers will need an IA score capture sheet to record the performance of all students in the class.)

6 Guidelines for the teacher on task completion and task control

This must specify:

- the role of the teacher during the period of task completion
- instructions that are to be given by the teacher to the students
- actions that are required of the teacher during task completion

7 Preparation by the students beforehand

If students are required to prepare in advance of the task date, preparatory notes must indicate the requirements. For example, students may need to collect support materials for a task that is supervised in a classroom.

8 Task outline for the student

This outline is a brief description of the task that the student is to complete. It is a general description without specific detail.

Example: *Your task is to focus on an important social issue. After investigating that issue, you need to process information collected and suggest possible courses of action that authorities could take.*

9 Task detail for the student

This must provide a detailed description of the task in the sequence that the student would be expected to follow during task completion. This must clearly state:

- what the student is expected to do
- what the student is expected to record and present for assessment.

10. Feedback & Support

Allocate time for:

- i. Student's self-assessment and correction
- ii. Peer assessment, feedback, and time for improvement
- iii. Teacher assessment, feedback, and time for time improvement

(NB: State how this will be carried out)

11. Final submission & scoring

State when the final task is due and how it will be assessed. State how the school (HOD/SPFSC Coordinator) will monitor the scoring of the tasks.

12. Scoring Rubric

Copy and paste directly from the aligned Syllabus the relevant scoring rubrics

13. Assessment score capture sheet for the task

This will be provided by EQAP

(Repeat 1-13 for other tasks)

SOUTH PACIFIC FORM SEVEN CERTIFICATE
Internal Assessment Summary Form

MATHEMATICS WITH STATISTICS

Country: _____ **School:** _____

Task	Task Description/Focus	Start Date	End Date	Weighting
1. Bivariate project				15%
2. Time Series project				10%
3. Comparison project				15%
	Total			40%

- Note:**
1. Be specific about the dates, not just Week 3, Term 1, etc
 2. Scoring rubrics/assessment schedules for the tasks are provided in Appendix 3, 4 and 5. Teachers must use these when assessing students' IA tasks.
 3. All IA Score Capture Sheets will be provided by EQAP to schools.

Teacher's Name and Signature: _____ **Date:** _____

Principal's Name and Signature: _____ **Date:** _____

A full IA program is to be submitted together with this IA Summary Form.

ADVISORY SECTION

Recommended Texts & Resources

This is a list of only some Mathematics texts that are available and have been used for teaching courses for Mathematics with Statistics. It is important teachers use this as only a guide and check current book lists available through publishers and book retailers.

A. Suggested Text

1. Sigma Mathematics - Barton, D. Longman, 1998 (2nd Edition)
2. Sigma Mathematics - Barton, D. Longman, 2006 (3rd Edition)

B. Supplementary Texts

1. Sigma mathematics Workbook - David Barton, 2007
2. Achieving in Statistics - W. Geldof, 2006
3. Statistics Workbook - Lakeland, R & Nugent, C, Nulake, 1998
4. Year 13 Study Guide, *Mathematics with Statistics* - Barrett, ESA
5. Longman write-on Notes – *Statistics* - Barton, D.
6. Bursary Statistics – Questions from the last 8 bursary papers with suggested answers.
Really Useful Resources
Box 19-939
Woolston
Christchurch
7. Schaum’s Outlines, Probability and Statistics, 2nd edition, Spiegel et al, McGraw Hill – a metal teacher resource.
8. Study Pass reference notes: Year 13 Statistics, info@studypass.co.nz
9. Time Series resources - Stats NZ - Yearbook
 - PCINFOS online. - Met service
 - weather stats - Business Indicators
 - share index (<http://www.world-exchanges.org/>)
 - exchange rates - The internet
10. Some Statistics Websites:
 - NZ Ministry of Education resources www.tki.org.nz
 - Census at schools data, analysis tools and resources www.censusatschool.org.nz
 - University of Auckland www.stat.auckland.ac.nz
 - Secretariat of the Pacific Community website www.spc.int
 - Tonga Statistics department www.spc.int/prism/country/to/stats/ - Statistics New Zealand www.stats.govt.nz

Sample Teaching Programme

It is important that teachers recognize that while this course is divided up into 4 specific objectives, the teaching programme should not necessarily have each objective taught as one big chunk.

Below is one example of a teaching programme that splits the objectives into smaller topics of learning. This is merely an example of what could be done, it is not intended that schools will be expected to follow this format. Schools need to be flexible to the needs of their school and community when planning the teaching order of this syllabus.

Teaching Programme Example – Form 7 Mathematics with Statistics

Week	Learning Outcome	Syllabus Reference	Assessment
Term I			
1 - 3	Solve probability problems.	1.1	
4 - 5	Solve problems involving expected values and variance of random variables from discrete probability distributions.	1.2	
6 - 8	Use statistical processes to undertake an investigation of bivariate data.	3.1	Internal assessment #1
9 - 10	Understanding simple piecewise functions in order to model data.	2.1	
11 - 13	Model real data using power and exponential functions in order to solve problems.	2.2	
Term II			
1 - 3	Using appropriate probability distributions to model a given situation and solve problems.	1.3	
4	Exam Revision		
5	Exam Time		Exam
6 - 8	Use statistical processes to undertake an investigation of time series data.	3.2	Internal assessment #2
9 - 10	Simultaneous equations.	4.1	
11 - 14	Use statistical processes to undertake an investigation of numerical data in order to make a comparison between two populations.	3.3	Internal assessment #3
Term III			
1 - 2	Model situations using linear programming techniques to obtain an optimum solution.	2.3	
3 - 4	Use the bisection or Newton-Raphson method to solve non-linear equations to a given precision.	4.2	
5 - 6	Answer questions related to statistical processes.	3.4	
7	Revision.		
8	Exam Time		Exam
9	Go over Exam.		
10 - 11	Revision/exam preparation.		

Issues & Recommendations for Assessment Governing Bodies

1. *Technology:*

At the present time the prevalent calculators being used by students are scientific ones. A growing trend in Asian markets is to produce low cost scientific calculators with extra chips that allow some algebraic manipulation. There is also potential for graphical calculators to become more prevalent as the prices of these drop. It is important that the examiner has an awareness of what calculator technology is being used by students and that question styles are adapted so that students who do not have the more advanced technology are not penalized. This is particularly of importance in the topics covered by learning objectives 2 and 4, where the use of a graphics calculator could potentially advantage a student both in simplifying a problem and in saving them a lot of time. While the temptation may be to just ban such technology, we cannot ignore its development and also the maintenance of an exclusion list becomes impractical as the market starts to become flooded with so many new brands. The onus on ensuring fairness must stand with the examiner structuring questions in a way that minimizes the advantages of one calculator over another.

It is also important that students are able to access suitable computer technology and statistical packages to deal with the analysis of data. To understand data students need to be able to deal with a suitable volume of data and a wide variety of variables. It is impractical for students to manage that volume of data by hand and suitable packages need to be provided. The use of such packages also will help keep students up to date with international trends in data analysis. Teachers will also need professional development in using such packages.

2. *Development of Statistical Thinking:*

Statistics and the development of statistical thinking in our students is becoming of increasing importance in our world. We are now in a data driven world. Technology has allowed us to deal with large amounts of data to get clearer pictures to assist societies in our economic and social development. We need students who understand the whole statistical process, not just the mechanics. They need to be able to gather, manage, analyse and interpret data, while considering variation. These understandings take time, so it is important that a greater emphasis on Statistics is given throughout all years of secondary education. This is a trend that is starting to occur internationally and it is important that the Pacific Islands region gives it some careful consideration.

3. *Professional Development*

There has been a new topic added to the scheme, analysis of bivariate data. Teachers will need resources and professional development in this area. There is also a changing trend internationally starting to come about on how Statistics is taught. In the past, Statistics syllabuss were based on a mathematical perspective on statistics. Currently the emphasis has changed to developing an understanding of how statistics are dealt with in the real world. There is less importance placed on the actual performance of calculations (which become trivialized with technology) and more importance placed on understanding what the measures calculated are actually telling us about the data and how it relates back to the population. With this change in thinking it is very important that teachers are provided support and professional development to help them understand the impact of this change on what has to happen in the teaching and learning of Statistics in the classroom.

The End

EDUCATIONAL QUALITY
AND
ASSESSMENT PROGRAMME



Pacific
Community
Communauté
du Pacifique



**SOUTH PACIFIC FORM SEVEN
CERTIFICATE**

MATHEMATICS WITH STATISTICS

FORMULAE AND TABLES BOOKLET

**YOU MAY KEEP THIS FORMULAE AND TABLES BOOKLET
AT THE END OF THE EXAMINATION.**

MATHEMATICS WITH STATISTICS — USEFUL FORMULAE AND TABLES

Straight Line

$$\text{Gradient } m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Equation } y - y_1 = m(x - x_1)$$

Quadratic Equations

$$\text{If } ax^2 + bx + c = 0$$

$$\text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Bisection Method

$$x_2 = \frac{x_0 + x_1}{2}$$

Newton-Raphson Method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Differentiation

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
x^n	nx^{n-1}
$\ln x$	$\frac{1}{x}$
e^{ax}	ae^{ax}

Differential Equation

$$\text{If } \frac{dy}{dx} = ky \text{ then } y = Ae^{kx}$$

Arithmetic Sequences and Series

$$t_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Geometric Sequences and Series

$$t_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} \quad \text{for } r \neq 1$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Exponential Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \text{ (for all } x)$$

Logarithms

$$\ln x = \log_e x$$

$$\text{If } y = b^x \text{ then } \log_b y = x$$

$$\text{If } y = e^x \text{ then } x = \ln y = \log_e y$$

$$\log_b x + \log_b y = \log_b xy$$

$$\log_b x - \log_b y = \log_b \frac{x}{y}$$

$$\log_b x^n = n \log_b x$$

Binomial Theorem

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + \binom{n}{n}b^n$$

$$\text{where } \binom{n}{r} = {}^nC_r = \frac{n!}{(n-r)!r!}$$

$$\text{Note: } {}^nP_r = \frac{n!}{(n-r)!}$$

Some values of $\binom{n}{r}$ are given in the table below.

Binomial Coefficients

$n \backslash r$	0	1	2	3	4	5	6	7	8	9	10
0	1										
1	1	1									
2	1	2	1								
3	1	3	3	1							
4	1	4	6	4	1						
5	1	5	10	10	5	1					
6	1	6	15	20	15	6	1				
7	1	7	21	35	35	21	7	1			
8	1	8	28	56	70	56	28	8	1		
9	1	9	36	84	126	126	84	36	9	1	
10	1	10	45	120	210	252	210	120	45	10	1
11	1	11	55	165	330	462	462	330	165	55	11
12	1	12	66	220	495	792	924	792	495	220	66
13	1	13	78	286	715	1287	1716	1716	1287	715	286
14	1	14	91	364	1001	2002	3003	3432	3003	2002	1001
15	1	15	105	455	1365	3003	5005	6435	6435	5005	3003

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Mean and Variance of Data

$$\bar{x} = \frac{\sum fx}{n} \quad s^2 = \frac{\sum fx^2 - \frac{(\sum fx)^2}{n}}{n}$$

Mean and Variance of a Random Variable

$$\mu = E(X) \quad \sigma^2 = \text{Var}(X)$$

$$= \sum x.P(X = x) \quad = E[X^2] - [E(X)]^2$$

Distribution of Sample Statistics

Statistic	Mean	Standard Deviation
Sample Mean	$E(\bar{X}) = \mu$	$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$ (std. error of the mean)
Sample Proportion	$E(P) = \pi$	$\sigma_P = \sqrt{\frac{\pi(1-\pi)}{n}}$ (std. error of the proportion)
Difference of Means (of two independent samples)	$E(\bar{X}_1 - \bar{X}_2) = \mu_1 - \mu_2$	$\sigma_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

Confidence Intervals

Mean $\bar{X} - z.\sigma_{\bar{X}} < \mu < \bar{X} + z.\sigma_{\bar{X}}$

Proportion $P - z.\sigma_P < \pi < P + z.\sigma_P$

Difference of two means $(\bar{X}_1 - \bar{X}_2) - z.\sigma_{\bar{X}_1 - \bar{X}_2} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + z.\sigma_{\bar{X}_1 - \bar{X}_2}$

Expectation Algebra

$$E[aX + b] = aE[X] + b$$

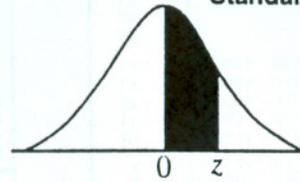
$$\text{Var}[aX + b] = a^2\text{Var}[X]$$

$$E[aX + bY] = aE[X] + bE[Y]$$

$$\text{Var}[aX + bY] = a^2\text{Var}[X] + b^2\text{Var}[Y],$$

(if X, Y are independent)

Standard Normal Distribution



$$\left(Z = \frac{X - \mu}{\sigma} \right)$$

Each entry gives the probability that the standardised normal random variable Z lies between 0 and z.

z	Differences																		
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359	4	8	12	16	20	24	28	32	36
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754	4	8	12	16	20	24	28	32	36
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141	4	8	12	15	19	22	27	31	35
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517	4	8	11	15	19	22	26	30	34
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879	4	7	11	14	18	22	25	29	32
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224	3	7	10	14	17	21	24	27	31
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549	3	6	10	13	16	19	23	26	29
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852	3	6	9	12	15	18	21	24	27
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133	3	6	8	11	14	17	19	22	25
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389	3	5	8	10	13	15	18	20	23
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621	2	5	7	9	12	14	16	18	21
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830	2	4	6	8	10	12	14	16	19
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015	2	4	5	7	9	11	13	15	16
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177	2	3	5	6	8	10	11	13	14
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319	1	3	4	6	7	8	10	11	13
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441	1	2	4	5	6	7	8	10	11
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545	1	2	3	4	5	6	7	8	9
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633	1	2	3	3	4	5	6	7	8
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706	1	1	2	3	4	4	5	6	6
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767	1	1	2	2	3	4	4	5	5
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817	0	1	1	2	2	3	3	4	4
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857	0	1	1	2	2	2	3	3	4
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890	0	1	1	1	2	2	2	3	3
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916	0	0	1	1	1	2	2	2	2
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936	0	0	1	1	1	1	1	2	2
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952	0	0	0	1	1	1	1	1	1
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964	0	0	0	0	1	1	1	1	1
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974	0	0	0	0	0	1	1	1	1
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981	0	0	0	0	0	0	0	0	1
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986	0	0	0	0	0	0	0	0	1
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990	0	0	0	0	0	0	0	0	0
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993	0	0	0	0	0	0	0	0	0
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995	0	0	0	0	0	0	0	0	0
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997	0	0	0	0	0	0	0	0	0
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998	.4998	0	0	0	0	0	0	0	0	0
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	0	0	0	0	0	0	0	0	0
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	0	0	0	0	0	0	0	0	0
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	0	0	0	0	0	0	0	0	0
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.5000	.5000	0	0	0	0	0	0	0	0	0
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	0	0	0	0	0	0	0	0	0

