# EDUCATIONAL QUALITY AND ASSESSMENT PROGRAMME [EQAP]

Pacific Community
Communauté du Pacifique

# SOUTH PACIFIC FORM SEVEN CERTIFICATE [SPFSC]

# MATHEMATICS with CALCULUS SYLLABUS

# GENERAL INFORMATION

The Office of the Educational Quality and Assessment Programme (EQAP) is located at:

3 Luke Street, Nabua
Suva
Fiji

Tel: (679) 3315600
Email: eqap@spc.int
Web: www.spc.int / www.eqap.org.fj

Any correspondence regarding this syllabus should be addressed to:

The Director EQAP
Pacific Community (SPC)
Private Mail Bag
Suva
Fiji

January 2004: 1st Edition
January 2011: 2nd Edition
April 2017: 3rd Edition
February 2019: 4th Edition
January 2020: 5th Edition

# SOUTH PACIFIC FORM SEVEN CERTIFICATE
# MATHEMATICS with CALCULUS

## Contents

# MATHEMATICS with CALCULUS

## PREAMBLE AND RATIONALE

This syllabus defines the requirements for the South Pacific Form Seven Certificate Mathematics (with Calculus) qualification.

There are explanatory notes provided below the table of specific learning outcomes in some strands. The specific learning outcomes in these strands are to be read in conjunction with these notes. Students also require knowledge and understanding of outcomes from the national Year 12 or Form 6 qualification or its equivalent, which are related to the specific outcomes of SPFSC Mathematics Calculus.

This syllabus subsumes/replaces all previous EQAP Mathematics with Calculus syllabus. The content and outcomes of the subject are aligned to the New Zealand National Certificate of Educational Assessment (NCEA) Level 3 Mathematics (with Calculus) Achievement Standards as published by New Zealand Qualifications Authority (NZQA).

The course is designed for students who wish to undertake university studies in mathematics and other related fields.

## COURSE AIM

Students undertaking this course are expected to:

- demonstrate mathematical skills, concepts and understanding of the mathematical processes, required for Measurement, Calculus, Trigonometry and Algebra at a level that is equivalent to that required for any Form 7 qualification or its equivalents including the NZ Universities Entrance, NCEA Level 3, University of the South Pacific (USP) Foundation , etc.
- apply these skills, concepts, and understanding to familiar and unfamiliar problems arising in real and simulated situations.
- demonstrate the ability to select and use appropriate mathematical techniques for problem solving.
- demonstrate the ability to reason logically and systematically.
- demonstrate the ability to communicate mathematical ideas.

## PREREQUISITES

Students taking this course are expected to have successfully completed the national Year 12 Senior Secondary Certificate Mathematics course or its equivalent.

## GENERAL OBJECTIVES

In a range of meaningful contexts, students will be engaged in thinking mathematically. They will solve problems and model situations that require them to:

1. apply algebraic techniques to real and complex numbers.
2. use and manipulate trigonometric functions and expressions.
3. demonstrate knowledge of basic and advanced concepts and techniques of differentiation and integration.

## CONTENT COMPONENTS

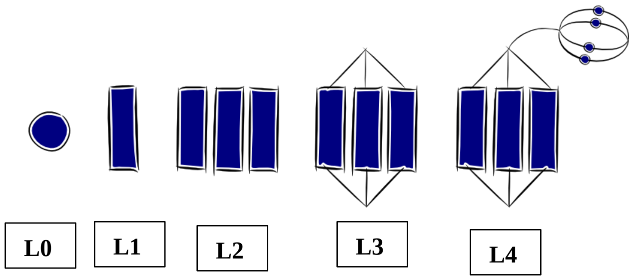
The content of the SPFSC Mathematics with Calculus course is organised under four strands and a number of sub-strands under each strand. These are outlined below:

| Strand Number | Strand Title | Sub strand number | Sub-strand title | External / Internal Assessment |
|---|---|---|---|---|
| 1.0 | **Algebra** | 1.1 | Algebra basic skills | External and Internal |
| | | 1.2 | Polynomial and non-linear equation | External |
| | | 1.3 | Complex numbers | External |
| 2.0 | **Trigonometry** | 2.1 | Trigonometry basic skills | External and Internal |
| | | 2.2 | Trigonometric functions to solve problems | External and Internal |
| | | 2.3 | Trigonometric identities | External and Internal |
| 3.0 | **Differentiation** | 3.1 | Differentiation basic skills | External |
| | | 3.2 | Discontinuity and limits of functions | External |
| | | 3.3 | Application of differentiation | Internal |
| | | 3.4 | Differentiate by sketching to solve problems | External |
| 4.0 | **Integration** | 4.1 | Integration basic skills | External |
| | | 4.2 | Use of variety of integration technique | Internal |
| | | 4.3 | Volumes of solids of revolution | External |
| | | 4.4 | Form and interpret solutions | External |

## UNPACKING LEARNING OUTCOMES

In this syllabus, Learning Outcomes are stated at three levels of generality: Major Learning Outcomes (MLOs) are stated at the strand level, Key Learning Outcomes (KLOs) are stated at the sub-strand level, and Specific Learning Outcomes (SLOs) are unpacked from the Key Learning Outcomes. Each SLO is a combination of a cognitive skill and a specific content component. Each SLO is given a skill level, (1, 2, 3 or 4) between, and this skill level results from the categorisation of the cognitive skill that is embedded in the SLO using the SOLO taxonomy[1].

The SOLO taxonomy provides a simple, reliable and robust model for three levels of understanding – surface, deep, and conceptual (Biggs and Collis 1982).




[1] Structure of Observed Learning Outcomes by Biggs and Collis (1982)

At the **prestructural** level (L0) of understanding, the task is inappropriately attacked, and the student has missed the point or needs help to start. The next two levels, unistructural and multistructural are associated with bringing in information (surface understanding). At the **unistructural** level (L1), one aspect of the task is picked up, and student understanding is disconnected and limited. The jump to the multistructural level is quantitative. At the **multistructural** level (L2), several aspects of the task are known but their relationships to each other and the whole are missed. The progression to relational and extended abstract outcomes is qualitative. At the **relational** level (L3), the aspects are linked and integrated, and contribute to a deeper and more coherent understanding of the whole. At the **extended abstract** level (L4), the new understanding at the relational level is re-thought at another conceptual level, looked at in a new way, and used as the basis for prediction, generalisation, reflection, or creation of new understanding (adapted from Hook and Mills 2011). [http://pamhook.com/solo-taxonomy/.. ]

The progression from Level 1 to Level 4 is exemplified in the progression from *define* →*describe* → *explain* → *discuss* with each succeeding level indicating a *higher level of understanding*, as follows:

- **define** – to state a basic definition of a concept [Unistructural or L1]
- **describe** – to give the characteristics of, or give an account of, or provide annotated diagrams. [Multistructural or L2]
- **explain** – to provide a reason for a relationship – an event and its impact, a cause and an effect, as to *how* or *why* something occurs. [Relational or L3]
- **discuss** – this means *linking ideas* (descriptions, explanations) to make generalisations or predictions or evaluations. It may involve relating, comparing, analysing, and justifying.
- **solve/calculate/compute** – to carry out a series of algorithms to arrive at a solution [Multistructural (L2) or Relational (L3) or even Extended (L4) depending on the complexity of the algorithm]. If there are two 'loadings' in the calculations (a standard problem) then skill level would be L2, if three 'loadings' (a complex problem) then L3 and four loadings (a more complex problem) for L4.

# LEARNING OUTCOMES

## STRAND 1.0 ALGEBRA

| **Major Learning Outcome** |
|---|
| Students are able to think mathematically and statistically and will be able to solve problems and model situations that require them to apply algebraic techniques to real and complex numbers. |

### SUB-STRAND 1.1 Algebra Basic Skills

| **Key Learning Outcome** |
|---|
| Students are able to demonstrate basic algebra skills. |

| No. | Specific Learning Outcomes<br>Students are able to: | Skill Level | SLO Code |
|---|---|---|---|
| 1 | **simplify** linear equations eliminating fractional terms | 1 | **Cal1.1.1.1** |
| 2 | **solve t**wo linear equations simultaneously | 1 | **Cal1.1.1.2** |
| 3 | **solve** linear inequations $\frac{x-3}{4}+1<\frac{x+1}{^-3}$ | 1 | **Cal1.1.1.3** |
| 4 | **rearrange** a formula to obtain the correct subject | 1 | **Cal1.1.1.4** |
| 5 | **factorise** quadratic equations | 1 | **Cal1.1.1.5** |
| 6 | **solve** quadratic equations by factorisation | 1 | **Cal1.1.1.6** |
| 7 | **solve** quadratic equations using quadratic formula | 2 | **Cal1.1.2.1** |
| 8 | **Factorise** a cubic function using factor theorem | 2 | **Cal1.1.2.2** |
| 9 | **apply** laws of indices to simplify exponential expressions | 1 | **Cal1.1.1.7** |
| 10 | **solve** straightforward exponential equations | 1 | **Cal1.1.1.8** |
| 11 | **apply** laws of logarithms to simplify logarithmic expressions | 1 | **Cal1.1.1.9** |
| 12 | **solve** straightforward logarithmic equations | 1 | **Cal1.1.1.10** |
| 13 | **express** a single algebraic fraction as a sum of its partial fractions where denominators of fractions are linear and or repeated roots | 2 | **Cal1.1.2.3** |
| 14 | **express** a single algebraic fraction as a sum of its partial fractions where denominators of fractions include a non-linear function or an irreducible quadratic function | 3 | **Cal1.1.3.1** |
| 15 | **divide** a polynomial by (x + a) . | 1 | **Cal1.1.1.11** |
| 16 | **find** the remainder to a function for x = a where (x - a) is not a factor. | 1 | **Cal1.1.1.12** |
| 17 | **expand** and simplify expressions of the form $(x + y)^x$ for n = 3 or 4 using the Binomial theorem. | 1 | **Cal1.1.1.13** |
| 18 | **find** specific terms in expansions where n is greater than 4 using the Binomial theorem. | 2 | **Cal1.1.2.4** |
| 19 | **find** the coefficients or constant term, or the term “independent of x” in expansions where n is greater than 4, using the Binomial theorem. | 3 | **Cal1.1.3.2** |
| 20 | **complete** the square of reducible quadratics of the form $ax^2 + bx + c$. | 1 | **Cal1.1.1.14** |
| 21 | **complete** the square of quadratics of the form $ax^2 + bx + c$. | 2 | **Cal1.1.2.5** |
| 22 | **solve** straightforward surd equations and check solutions. | 2 | **Cal1.1.2.6** |
| 23 | **prove** a given mathematical statement is true by using the method of mathematical induction, whereby the variable “n” in the statement provided is not a power. | 2 | **Cal1.1.2.7** |
| 24 | **prove** a given mathematical statement is true by using the method of mathematical induction, whereby the variable “n” in the statement to be proved is a power (the laws of indices is applied. | 3 | **Cal1.1.3.3** |
| 25 | **simplify** sums, differences, and products of surds | 1 | **Cal1.1.1.15** |
| 26 | **simplify** quotients of surds (including rationalizing). | 2 | **Cal1.1.2.8** |
| 27 | **use** and manipulate simple surds | 1 | **Cal1.1.1.16** |
| 28 | **use** and manipulate surds and other irrational numbers. | 2 | **Cal1.1.2.9** |

## SUB-STRAND 1.2 Polynomial and Non- Linear Equations

| **Key Learning Outcome** |
|---|
| Students are able to form and use polynomial and non-linear equations. |

| No. | Specific Learning Outcomes<br>Students are able to: | Skill Level | SLO Code |
|---|---|---|---|
| 1 | determine the variables in a contextual problem | 1 | Cal1.2.1.1 |
| 2 | form equations based on a contextual problem | 2 | Cal1.2.2.1 |
| 3 | **solve** problems that require translating word problems into simple mathematical expressions, involving two variables and two or three equations | 3 | Cal1.2.3.1 |
| 4 | **solve** problems that require translating word problems into complex mathematical expressions involving three variables and two or three equations | 4 | Cal1.2.4.1 |
| 5 | **analyse** the existence of solutions in the context of a simple situation involving word problems. | 3 | Cal1.2.3.2 |
| 6 | **analyse** the existence of solutions in the context of a complex situation involving word problems. | 4 | Cal1.2.4.2 |
| 7 | **solve** linear equations | 1 | Cal1.2.1.2 |
| 8 | **solve** simultaneous equations, where one equation is linear and the other a quadratic or cubic function | 3 | Cal1.2.3.3 |
| 9 | **solve** exponential equations that include expressing negative powers with positive exponents | 1 | Cal1.2.1.3 |
| 10 | **solve** exponential equations that include applying the laws of indices | 1 | Cal1.2.1.4 |
| 11 | **solve** exponential equations That can be expressed with the same base but different powers e.g $2^{2x} = 4^{x+1}$ | 2 | Cal1.2.2.2 |
| 12 | **solve** logarithmic equations, involving single logs, and addition and subtraction of logs | 2 | Cal1.2.2.3 |
| 13 | **solve** logarithmic equations involving the application of log rules of bringing down power as a coefficient. | 3 | Cal1.2.3.4 |
| 14 | **solve** rational equations | 3 | Cal1.2.3.5 |
| 15 | **solve** surd equations, including simplifying surds, expanding surds and writing surds in simplest forms | 2 | Cal1.2.2.4 |
| 16 | **solve** surd equations, including rationalizing the denominator | 3 | Cal1.2.3.6 |
| 17 | **solve** hyperbolic equations | 2 | Cal1.2.2.6 |
| 18 | **use** Remainder and Factor theorems involving straight forward substitution method and long division | 2 | Cal1.2.2.5 |
| 19 | **use** Remainder and Factor theorems to completely factorise a polynomial equation of degree 3 | 3 | Cal1.2.3.7 |
| 20 | **use** Remainder and Factor theorems to find unknowns in a polynomial equation | 4 | Cal1.2.4.3 |
| 21 | **Solve** a simultaneous system involving a hyperbolic function and a linear function | 4 | Cal1.2.4.4 |

## SUB-STRAND 1.3 Complex numbers

| **Key Learning Outcome** |
|---|
| Students are able to use and manipulate complex numbers. |

| No. | Specific Learning Outcomes<br>Students are able to: | Skill Level | SLO Code |
|---|---|---|---|
| 1 | **simplify** sums, differences, and products of complex numbers expressed in rectangular form. | 1 | Cal1.3.1.1 |
| 2 | **simplify** quotients of complex numbers expressed in rectangular form. | 2 | Cal1.3.2.1 |
| 3 | **convert** between rectangular ($a + ib$) and polar ($rcis\ \theta$) forms. | 1 | Cal1.3.1.2 |
| 4 | **use** Argand diagrams to represent complex number in the forms $a + ib$ , $rcis\ \theta$<br>The Argand diagram is either represented as an intercept on the x-axis or y-axis. | 1 | Cal1.3.1.3 |

| | | | |
|---|---|---|---|
| | It has only ONE representation. It has ONLY a Real component or ONLY an imaginary, i.e. $Z = 4i \quad or \quad Z = 2$ | | |
| **5** | **use** Argand diagrams to represent complex number in the forms $a + ib$ , *rcis* θ<br>The Argand diagram is plotted as a point on any of the four quadrants.<br>Example: It has BOTH a Real and Imaginary component, i.e. $Z = -2 + 3i$ | **2** | **Cal1.3.2.2** |
| **6** | **Interpret, manipulate and use** graphical representations of complex numbers, using polar and rectangular form on an Argand diagram. These problems include a point on the complex plane can be written in rectangular or polar form. | **2** | **Cal1.3.2.3** |
| **7** | **Interpret, manipulate and use** graphical representations of complex numbers, using polar and rectangular form on an Argand diagram. These problem include regions that are shaded on a line ($a \leq Re/Im(Z) \leq b$) that can be linked back to its rectangular or polar form. | **3** | **Cal1.3.3.1** |
| **8** | **interpret manipulate and use** graphical representations of complex numbers, using polar and rectangular form on an Argand diagram. (These problems include a region involving a circle that can be linked back to its rectangular/polar form. E.g. $-2 \leq \lvert z \rvert < 1$ ) | **4** | **Cal1.3.4.1** |
| **9** | **find** roots over the complex number system for polynomial equations with real coefficients, including the special case of the nth roots of *a* that come from solving equations of the form $z^n = a$, making links with their graphs.<br>(The equation of the complex number has only a Real component or Imaginary component, but not both. Example: Z = 2 or Z = -i ) | **2** | **Cal1.3.2.4** |
| **10** | **find** roots over the complex number system for polynomial equations with real coefficients, including the special case of the nth roots of *a* that come from solving equations of the form $z^n = a$, making links with their graphs.<br>(The equation of the complex number has both Real and Imaginary components and both components are whole numbers. Example: Z = -3 + 2i ) | **3** | **Cal1.3.3.2** |
| **11** | **find** roots over the complex number system for polynomial equations with real coefficients, including the special case of the nth roots of *a* that come from solving equations of the form $z^n = a$, making links with their graphs.<br>(The equation of the complex number has both Real and Imaginary components. One of the components must include a rational/surd. Example: $Z = \sqrt{2} + 2i$ ) | **3** | **Cal1.3.3.3** |
| **12** | **find** roots of equations of the form $z^n = a + ib$, $z^n = r\ cis\ \theta$ where n is a positive integer (includes the use of de Moivre's theorem to solve equations and Argand diagrams to represent relationships between solutions)<br>(This includes any roots from 9 – 11 above that are to be sketched on an Argand diagram.) | **4** | **Cal1.3.4.2** |

***Explanatory Notes***

The outcomes listed above require that students

- have an understanding of the need and relevance of different number systems leading to the development of ***i*** and complex numbers
- carry out mathematical operations on expressions incorporating ***i***, eg. powers of ***i***
- convert between rectangular and polar form
- carry out operations on *rcisθ*
- use de Moivre's theorem to solve equations of the form $z^{n} = a$ and display these solutions

## STRAND 2.0 TRIGNONOMETRY

**Major Learning Outcome**
Students are able to think mathematically by solving problems and model situations that require them to use and manipulate trigonometric functions and expressions.

### SUB-STRAND 2.1 Trigonometry Basic Skills

**Key Learning Outcome**
Students are able to use the basic trigonometry skills to solve problems

| No. | Specific Learning Outcomes<br>Students are able to: | Skill Level | SLO Code |
|---|---|---|---|
| 1 | **evaluate** reciprocal and/or inverse relationships | 1 | Cal2.1.1.1 |
| 2 | **find** exact values of trig expressions using special triangles | 1 | Cal2.1.1.2 |
| 3 | **Sketch** and label a basic trig function, e.g. y = A sin Bx, or y = A cos Bx | 1 | Cal2.1.1.3 |
| 4 | **Solve** a simple trig equation (of the form A sin x = c or A cos x = c or A tan x = c) | 1 | Cal2.1.1.4 |
| 5 | **solve** problems that involve manipulating trig expressions using trigonometric forms of Pythagoras theorem | 2 | Cal2.1.2.1 |
| 6 | **solve** problems that involve a simple manipulation of trig expressions using compound angle formula | 2 | Cal2.1.2.2 |
| 7 | **solve** problems that involve a simple manipulation of trig expressions using double angle formula | 2 | Cal2.1.2.3 |
| 8 | **solve** problems that involve a complex manipulation of trig expressions using compound angle formula | 3 | Cal2.1.3.1 |
| 9 | **solve** problems that involve a complex manipulation of trig expressions using double angle formula | 3 | Cal2.1.3.2 |
| 10 | **solve** problems that involve a complex manipulation of trig expressions using sums and products | 3 | Cal2.1.3.3 |
| 11 | **display** the graphs of inverse and/or reciprocal trigonometric functions (sin x, cos x, tan x) with x in radians or degrees showing the main features of the graphs. Consideration of restrictions on the domain of a function so that its inverse is also a function is required. | 3 | Cal2.1.3.4 |

***Explanatory Notes:***

A simple manipulation of trig expressions involves a two-step procedure to arrive at a possible solution, where the equation is equal to 0, ½ or 1. E.g. Solve sin A or cos A = 0, or sin A or cos A = ½ and sin A or cos A = 1. The angle values should be recognisable by sight at Year 13 level.

A complex manipulation of trig expressions involves a three step procedure to arrive at a possible solution. The equation to be solved is equal to a value other than 0, ½ or 1 and thus need the manipulation of the inverse trig function first in order to arrive at possible angles.

### SUB-STRAND 2.2: Trigonometric functions to solve problems

**Key Learning Outcome**
Students are able to select and form a trig function to solve problems.

| No. | Specific Learning Outcomes<br>Students are able to: | Skill Level | SLO Code |
|---|---|---|---|
| 1 | **solve** straightforward problems with models involving trigonometric functions of the form; y = A sin B (x + C) + D where C or D may be zero | 3 | Cal2.2.3.1 |
| 2 | **solve** straightforward problems with models involving trigonometric functions of the form; y = A cos B (x + C) + D where C or D may be zero | 3 | Cal2.2.3.2 |
| 3 | **solve** straightforward problems with models involving trigonometric functions of the form; y = A tan B (x + C) + D where C or D may be zero | 3 | Cal2.2.3.3 |

| | | | |
|---|---|---|---|
| **4** | **form** an equation for a model and use the model to solve problems;<br>y = A sin B (x + C) + D | **4** | **Cal2.2.3.4** |
| **5** | **form** an equation for a model and use the model to solve problems<br>y = A cos B (x + C) + D | **4** | **Cal2.2.3.5** |

***Explanatory Notes***

The above outcomes refer to trigonometric functions where C or D may be zero. Solutions of the problems may require knowledge of amplitude, period and frequency.

### SUB-STRAND 2.3: Prove Trigonometric identities

**Key Learning Outcome**

Students are able to prove trigonometric identities using various formulae

| No. | Specific Learning Outcomes<br>Students are able to: | Skill Level | SLO Code |
|---|---|---|---|
| **1** | **prove** trigonometry identities (involving **direct substitution** of reciprocal relationships $cot\theta, cosec\theta\ and\ \sec\theta$), by solving equations to provide a general solution or a solution within a specified domain. | **1** | **Cal2.3.1.1** |
| **2** | **prove** trigonometry identities (involving reciprocal relationships but involves **algebraic working**), by manipulating equations to provide a general solution or a solution within a specified domain. | **2** | **Cal2.3.2.1** |
| **3** | **prove** trigonometry identities (involving Pythagorean identities), by manipulating equations to provide a general solution or a solution within a specified domain. | **2** | **Cal2.3.2.2** |
| **4** | **prove** trigonometry identities (involving compound angle formulae), by manipulating equations to provide a general solution or a solution within a specified domain. | **3** | **Cal2.3.3.1** |
| **5** | **prove** trigonometry identities (involving double angle formulae), by manipulating equations to provide a general solution or a solution within a specified domain. | **3** | **Cal2.3.3.2** |
| **6** | **prove** trigonometry identities (involving sum and product formulae), by manipulating equations to provide a general solution or a solution within a specified domain. | **3** | **Cal2.3.3.3** |

## STRAND 3.0 DIFFERENTIATION

| **Major Learning Outcome** |
|---|
| Students are able to solve problems and model situations that require them to demonstrate knowledge of advanced concepts and techniques of differentiation. |

### SUB-STRAND 3.1 Differentiation Basic Skills

| **Key Learning Outcome** |
|---|
| Students are able to use basic differentiation skills to solve simple problems |

| **No.** | **Specific Learning Outcomes** **Students are able to:** | **Skill Level** | **SLO Code** |
|---|---|---|---|
| **1** | **identify** features of a piecewise function of f(a) | **1** | **Cal3.1.1.1** |
| **2** | **identify** features of a piecewise function if the function is discontinuous | **1** | **Cal3.1.1.2** |
| **3** | **identify** features of a piecewise function if the limit exits | **1** | **Cal3.1.1.3** |
| **4** | **identify** features of a piecewise function if the function is differentiable | **1** | **Cal3.1.1.4** |
| **5** | find limits of piecewise functions | **1** | **Cal3.1.1.5** |
| **6** | **use** the first principles to differentiate a function (only for polynomials of degree ≤ 3) using $\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$ | **3** | **Cal3.1.3.1** |
| **7** | **differentiate** sums of functions | **2** | **Cal3.1.2.1** |
| **8** | **differentiate** quotients, where f and g are both singular functions | **2** | **Cal3.1.2.2** |
| **9** | **differentiate** composite functions (chain rule), where f and g are both singular functions | **2** | **Cal3.1.2.3** |
| **10** | **differentiate** parametric functions | **2** | **Cal3.1.2.4** |
| **11** | **finding** the second derivatives of a given function | **2** | **Cal3.1.2.5** |
| **12** | **solve** problems by finding the maxima or minima with proof for polynomial and rational functions | **3** | **Cal3.1.3.2** |
| **13** | **differentiate** to find the points of inflection | **3** | **Cal3.1.3.3** |

***Explanatory Notes:***

Singular functions include sin ax, cos ax, tan ax, $ax^n$, $e^{ax}$, etc. For level 2 SLOs, exact simplification is presumed to be a possessed skill in year 13.

### SUB-STRAND 3.2 Discontinuity and limits of functions

| **Key Learning Outcome** |
|---|
| Students are able to identify discontinuity and limits of functions |

| **No.** | **Specific Learning Outcomes** **Students are able to:** | **Skill Level** | **SLO Code** |
|---|---|---|---|
| **1** | **link** features of graphs with the limiting behaviour of functions for either a cubic or quadratic function | **3** | **Cal3.2.3.1** |
| **2** | **link** features of graphs with the limiting behaviour of functions for either a hyperbolic or polynomial Function | **4** | **Cal3.2.4.1** |
| **3** | **use** limiting features of functions to sketch piecewise and rational graphs | **3** | **Cal3.2.3.2** |
| **4** | **use** limiting features of functions to sketch complex graphs | **4** | **Cal3.2.4.2** |
| **5** | **find** limits algebraically, and numerically by considering behaviour of a function as x approaches a specific value from above and below | **2** | **Cal3.2.2.1** |
| **6** | **find** limits algebraically and numerically by considering behaviour as x approaches +∞ or -∞ | **2** | **Cal3.2.2.2** |
| **7** | **demonstrate** understanding of continuity at a point (the limit as x tends to *a* of f(x) = f(a)). | **4** | **Cal3.2.4.3** |
| **8** | **identify** discontinuities algebraically | **2** | **Cal3.2.2.3** |
| **9** | **link** informally concepts of continuity and differentiability | **4** | **Cal3.2.4.4** |

## SUB-STRAND 3.3 Application of differentiation technique

| **Key Learning Outcome** |
|---|
| Students are able to apply a variety of differentiation technique to functions and relations. |

| No. | Specific Learning Outcomes<br>Students are able to: | Skill Level | SLO Code |
|---|---|---|---|
| 1 | **use** implicit differentiation or parametric differentiation to differentiate simple functions (functions include: polynomials, and trigonometric functions ) | 3 | Cal3.3.3.1 |
| 2 | **use** implicit differentiation or parametric differentiation to differentiate complex functions (functions include: $Ae^{px}$, ln(ax +b), power functions such as $Ap^{x}$ ) | 4 | Cal3.3.4.1 |
| 3 | **use** a range of differentiation techniques to find an equation of a tangent or normal | 3 | Cal3.3.3.2 |
| 4 | **use** a range of differentiation techniques to solve optimization of simple functions (maxima, minima or point of inflection) linked to only **one** differentiation, $f(x) \rightarrow f'(x)$ . | 2 | Cal3.3.2.1 |
| 5 | **use** a range of differentiation techniques to solve optimization problems (maxima, minima or point of inflection) that are linked to **two** differentiations, $f(x) \rightarrow f'(x) \rightarrow f''(x)$ | 3 | Cal3.3.3.3 |
| 6 | **use** a range of differentiation techniques to solve optimization problems (maxima, minima or point of inflection) that involve many overlapping equations or equations to be differentiated are not obviously stated. | 4 | Cal3.3.4.2 |
| 7 | **use** differentiation to find the rates of change involving simple and straight forward equations | 2 | Cal3.3.2.2 |
| 8 | **use** differentiation to find the rates of change, using the Chain rule. | 3 | Cal3.3.3.4 |
| 9 | **use** differentiation to find the rates of change, that uses the Chain rule with two variables involved. The relationship of both variables is given in the problem. | 4 | Cal3.3.4.3 |
| 10 | **use differentiation to solve simple** problems in kinematics, e.g. in relation to projectile motions etc. where time and distances are to be found, or one level of differentiation e.g. from d(t) → v(t), and substitution of time may not be necessary. | 2 | Cal3.3.2.3 |
| 11 | **use differentiation to solve** problems in kinematics, e.g. in relation to projectile motions etc. where descriptions of motion are involved, as in $d(t) \rightarrow v(t)\ or\ v(t) \rightarrow a(t)$ and values of time are substituted | 3 | Cal3.3.3.5 |
| 12 | **apply** differentiation to problems in kinematics in which "2 derivatives" or 2nd differentiation applies, e.g. $d(t) \rightarrow find\ a(t)$, and values of t are to be substituted | 4 | Cal3.3.4.4 |

## SUB-STRAND 3.4 Differentiate to solve problems by sketching graphs

| **Key Learning Outcome** |
|---|
| Students are able to use differentiation to solve problems involving sketching graphs of polynomials and derivatives. |

| No. | Specific Learning Outcomes<br>Students are able to: | Skill Level | SLO Code |
|---|---|---|---|
| 1 | **sketch** graphs of polynomials of degree ≥3 by using differentiation to identify features such as turning points, points of inflection and region of concavity | 3 | Cal3.4.3.1 |
| 2 | **sketch** the graph of a derived function from a given graph | 3 | Cal3.4.3.2 |
| 3 | **identify** features of given graphs (selection from limits, differentiability, discontinuity, gradients, concavity, turning points, points of inflection) | 2 | Cal3.4.2.1 |
| 4 | **sketch** graphs/derivatives to demonstrate knowledge of the above features. | 3 | Cal3.4.3.3 |

# STRAND 4.0: INTEGRATION

| **Major Learning Outcome** |
|---|
| Students are able to solve problems and model situations that require them to demonstrate knowledge of advanced concepts and techniques of integration. |

## SUB-STRAND 4.1: Integration Basic Skills

| **Key Learning Outcome** |
|---|
| Students are able to use basic integration skills to solve simple problems. |

| No. | Specific Learning Outcomes<br>Students are able to: | Skill Level | SLO Code |
|---|---|---|---|
| 1 | **integrate** functions of $ax^n$ , where n is a Real number | 1 | **Cal4.1.1.1** |
| 2 | **integrate** polynomial functions | 1 | **Cal4.1.1.2** |
| 3 | **integrate** exponential functions of the form $Ae^{f(x)}$, where f(x) = ax + b | 1 | **Cal4.1.1.3** |
| 4 | **integrate** trigonometric functions, including products | 2 | **Cal4.1.2.1** |
| 5 | **integrate** rational functions of the type $y = \frac{ax+b}{cx+d}$ | 3 | **Cal4.1.3.1** |
| 6 | **find** definite integrals in a simple integration | 2 | **Cal4.1.2.2** |
| 7 | **use** integration to **solve** 'straightforward' problems involving areas, where there is a singular function and the limits of integration are obvious in the problem) | 2 | **Cal4.1.2.3** |
| 8 | **use definite** integration to **solve** problems involving areas (the limits of integration are not obvious and may have to obtained from a sketch of the function/s) | 3 | **Cal4.1.3.2** |
| 9 | **use** anti-differentiation to form and solve differential equations of first and second order with variables easily separated | 3 | **Cal4.1.3.3** |

## SUB-STRAND 4.2: Use of variety of integration techniques

| **Key Learning Outcome** |
|---|
| Students are able to apply a variety of integration and anti-differentiation techniques to functions and relations, using both analytical and numerical methods. |

| No. | Specific Learning Outcomes<br>Students are able to: | Skill Level | SLO Code |
|---|---|---|---|
| 1 | use and apply numerical methods to find areas under the curves functions include polynomials, $x^n, n \in R$, $e^x$ , sin(x), cos(x), f '(g(x)).g' (x), $\frac{f'(x)}{f(x)}$ | 3 | **Cal4.2.3.1** |
| 2 | use and apply numerical methods to find areas *between* the curves including polynomials, $x^n, n \in R$, $e^x$ , sin(x), cos(x), f '(g(x)).g' (x), $\frac{f'(x)}{f(x)}$ | 4 | **Cal4.2.4.1** |
| 3 | find area for odd, even and periodic functions | 3 | **Cal4.2.3.2** |
| 4 | **apply** anti-differentiation to problems in kinematics. Any question that only involves substitution of "time", but no integration, i.e. substituting "*t*" into the formula. Initial (t=0) or t = a | 2 | **Cal4.2.1.1** |
| 5 | **apply** anti-differentiation to solve word problems in kinematics, involving the application of one level of integration, e.g. $a(t) \to v(t) \quad or \quad v(t) \to d(t)$ | 3 | **Cal4.2.3.3** |
| 6 | **apply** anti-differentiation to solve word problems in kinematics, involving the application of two levels of integration e.g. $a(t) \to find\ d(t)$ | 4 | **Cal4.2.4.2** |
| 7 | find area of kinematic equations | 3 | **Cal4.2.3.4** |

***Explanatory Notes:***

1. integration by substitution is restricted only to those involving simple algebraic substitutions
2. integration using partial fraction is to be used as an alternative method of integration for students to use where appropriate, but are not tested directly in this syllabus

## SUB-STRAND 4.3: Integration: Volumes of solids of revolution

| **Key Learning Outcome** |
|---|
| Students are able to use integration to find the volumes of solids of revolution. |

| **No.** | **Specific Learning Outcomes**<br>**Students are able to:** | **Skill Level** | **SLO Code** |
|---|---|---|---|
| **1** | **use** of integration to **find volumes** of revolution of simple function (such as $y = ax^n$) and around an axis parallel to the x or y axis. | **3** | **Cal4.3.3.1** |
| **2** | **use** of integration to **find volumes** of revolution of complex function (such as exponential or trig function) and around an axis parallel to the x or y axis, and limits of integration are not obvious in the problem. | **4** | **Cal4.3.4.1** |
| **3** | **use** of integration to **find volumes** of revolution of area between two functions (linear, trigonometric, quadratic, exponential, logarithmic, hyperbola), and in which the limits of integration are obvious in the problem. | **4** | **Cal4.3.4.2** |

***Explanatory notes***

1. Simple functions do not involve sums, products, or combinations of the standard functions other than expressions such as $ax^2 + bx + c$ or $A - e^{ax}$ .
2. Complex functions are those that do involve sums, products and combinations of the standard functions.
3. For areas between two function (as in SLO3 above, L4), the limits of integration are obvious in the problem.

## SUB-STRAND 4.4: Form and interpret solutions of differential equations

| **Key Learning Outcome** |
|---|
| Students are able to form differential equations and interpret the solutions |

| **No.** | **Specific Learning Outcomes**<br>**Students are able to:** | **Skill Level** | **SLO Code** |
|---|---|---|---|
| **1** | **use** anti-differentiation to form and solve simple differential equations with the rate of change directly or inversely proportional to the variable of interest (functions include: polynomials, $e^x$, ln(x), trig functions, $x^n, n\in R$) | 3 | **Cal4.4.3.1** |
| **2** | **use** anti-differentiation to form and solve complex differential equations with the rate of change directly or inversely proportional to the variable of interest (functions include: polynomials, $e^x$, ln(x), trig functions, $x^n, n\in R$) | 4 | **Cal4.4.4.1** |
| 3 | **use** anti-differentiation to form and solve differential equations of first order only with variables easily separated (functions include: polynomials, $e^x$, ln(x), trig functions, $x^n, n\in R$) | 2 | **Cal4.4.2.1** |
| **4** | **use** anti-differentiation to form and solve simple differential equations of first and second order with variables easily separated (functions include: polynomials, $e^x$, ln(x), trig functions, $x^n, n\in R$) | 3 | **Cal4.4.3.2** |
| **5** | **use** anti-differentiation to form and solve complex differential equations of first and second order with variables easily separated (functions include: polynomials, $e^x$, ln(x), trig functions, $x^n, n\in R$) | 4 | **Cal4.4.4.2** |
| **6** | **form** differential equations of the type with the rate of change directly proportional to the variable of interest (functions include: polynomials, $e^x$, ln(x), trig functions, $x^n, n\in R$) | 2 | **Cal4.4.2.2** |
| **7** | **solve** differential equations of the first order type with variables easily separated | 3 | **Cal4.4.3.3** |

### *Explanatory notes*

Student applies boundary or initial conditions to solutions of differential equations. Student are expected to distinguish between families of solutions and exact solutions using given boundary or initial conditions and interprets these solutions.

**Possible context elaborations**

$\frac{dy}{dx} = ysinx$

$\frac{d^2y}{dx^2} = polynomial$

$\frac{dy}{dx} = e^x$

$\frac{dy}{dx} = \frac{1}{x}$

Situations which students will be expected to form a model for and solve include:

Growth and decay $\frac{dy}{dx} = ky$

Newton's law of cooling $\frac{dT}{dt} = k(T - T_0)$

Kinematics

### *Explanatory Notes*

Once solutions to differential equations are available students are required to:

- make predictions about a situation based on the solution
- make further calculations with particular solution of differential equation to check accuracy of model
- comment on limitations of their model in relation to uncontrolled factors, etc.

## ASSESSMENT

Assessment in Mathematics Calculus course is in two parts:

1. External Assessment (EA) is worth 60.
2. Internal Assessment (IA) is worth 40.

The principal, or his/her nominee, will certify a proposed IA programme for the course. This is to be submitted to EQAP early in the year.

The Principal, or his nominee, will also certify that the internal assessment requirements have been fulfilled.

## Suggested Teaching Time and Weightings

| | Outcomes | External / internal | Approximate weighting | Suggested Time |
|---|---|---|---|---|
| 1 | **Algebra:** Students will be able to apply algebraic techniques to real and complex numbers. | *External*<br>*Internal* | 20<br>10 | *8 weeks* |
| 2 | **Trigonometry:** Students will be able to use trigonometric functions as well as apply its relationship to solve problems.<br>*Demonstrate basic skills and use trigonometric functions to solve simple problems. Prove trigonometric identities* | *External*<br>*Internal* | 10<br>10 | *5 weeks* |
| 3 | **Differentiation:** Students will be able to demonstrate knowledge of advanced concepts and techniques of differentiation.<br>*Demonstrate basic skills of differentiation and solve problems by sketching.* | *External*<br>*Internal* | 15<br>10 | *8 weeks* |
| 4 | **Integration:** Students will be able to demonstrate knowledge of advanced concepts and techniques of integration.<br>*Demonstrate basic skills and use a variety of integration technique to solve problems* | *External*<br>*Internal* | 15<br>10 | *7 weeks* |
| | **Total** | | **100** | **28 weeks** |

## Assessment Blueprint

| Strand | Assessment Type | SKILL LEVEL/ SCORE | | | | Weight |
|---|---|---|---|---|---|---|
| | | Level 1 SS = 1 | Level 2 SS = 2 | Level 3 SS = 3 | Level 4 SS = 4 | |
| 1. Algebra | EA | | | | | 20 |
| | IA | | 2 | 2 | | 10 |
| 2. Trigonometry | EA | | | | | 10 |
| | IA | | 2 | 2 | | 10 |
| 3. Differentiation | EA | | | | | 15 |
| | IA | | 2 | 2 | | 10 |
| 4. Integration | EA | | | | | 15 |
| | IA | | | 2 | 1 | 10 |
| **Total number of items** | | **20** | **15** | **10** | **5** | **100** |
| **Total skill score** | | **20** | **30** | **30** | **20** | |

## External Assessment

The syllabus contains the Major Learning Outcome (MLO), the Key Learning Outcome (MLO) and the Specific Learning Outcomes.. Examination questions, which require specific mathematical knowledge, will be based on these specific outcomes.

The external assessment will contribute 60% of the final grade. The table below gives an approximate weighting for each major learning outcome in the examination. This is based on the the distribution of learning outcomes for each strand. The four strands in the syllabus are reflected in the examination paper, and students will be given **3 hours** (plus 10 minutes of reading time) for the examination.

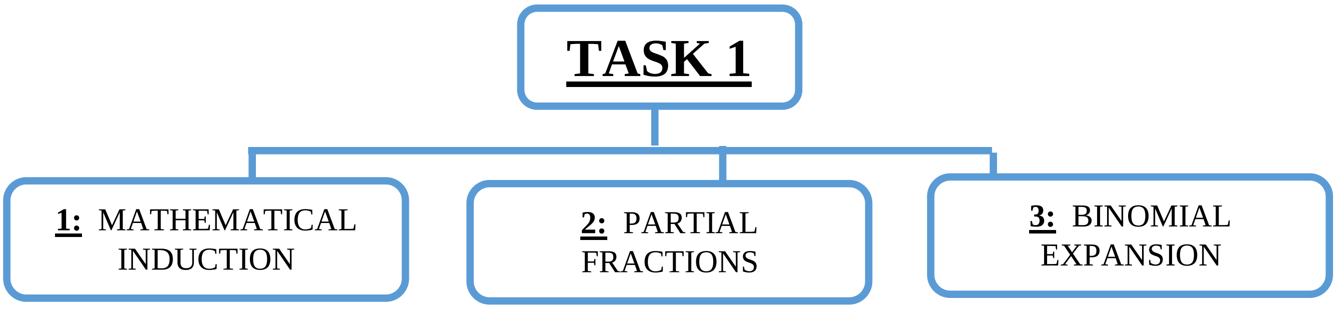
| Strand | | Outcomes<br>*In a range of meaningful contexts, students are able to solve problems and model situations that require them to:* | Percentage per section | Suggested Exam Time |
|---|---|---|---|---|
| 1 | Algebra: | Apply algebraic techniques to real and complex numbers. | 20% | 60 mins |
| 2 | Trigonometry | Use trigonometric functions as well as apply its relationships to solve problems. | 10% | 30 mins |
| 3 | Differentiation: | Demonstrate knowledge of advanced concepts and techniques of differentiation. | 15% | 45 mins |
| 4 | Integration: | Demonstrate knowledge of advanced concepts and techniques of integration. | 15% | 45 mins |

## Internal Assessment

The four IA tasks are Common Assessment Tasks (CATs), and they assess the specific learning outcomes (SLOs) indicated. The task instructions for each task are provided below.

| TOPICS | Task No. | Assessment Type | SOLO LEVELS | | | | Weighting |
|---|---|---|---|---|---|---|---|
| | | | Level 1 | Level 2 | Level 3 | Level 4 | |
| ALGEBRA | **TASK 1** | INDIVIDUAL ASSIGNMENT | | 2<br>4 | 2<br>6 | | 10 |
| TRIGNOMETRY | **TASK 2** | GROUP TASK | | 2<br>4 | 2<br>6 | | 10 |
| DIFFERENTIATION | **TASK 3** | PRACTICAL ACTIVITY | | 2<br>4 | 2<br>6 | | 10 |
| INTEGRATION | **TASK 4** | GROUP TASK | | | 2<br>6 | 1<br>4 | 10 |

*IA TASK 1 Instructions*




The concepts of binomial expansion, mathematical induction and partial fractions are often given a lot of emphasis by teachers and students. This emphasis reflects a similar level of emphasis given to these in the Year 13 modules in the USP Foundation programme. The inclusion of a task on these three concepts gives recognition to this emphasis. The inclusion of an IA task on Strand 1 also gives recognition to the large number of SLOs within this strand.

**TASK DESCRIPTION**

This is an INDIVIDUAL ASSIGMENT. Students can discuss the content amongst themselves in class, or in their groups – but each student is to submit his or her own answer paper to the teacher by the due date. The idea is to have collaboration and interactive discussions, whereby the teacher moves around as a facilitator to see that constructive discussions are carried out, thus providing opportunities for students to learn from each other as well as from their teacher. The idea is centred on PEER TUTORING, whereby students themselves come up with ways to answer the questions in the Assignment, rather than relying on the teacher for their source. Teachers must refrain from providing direct answers to students, and also to see that students do not end up plagiarizing or copying each other's work.

-It is recommended that one full week of 5 - 6 lessons are dedicated to this task, and that the teacher collects student worksheets at the end of the week.

The task document itself with the scoring rubric will be provided from EQAP at the appropriate point in time, so teachers will be responsible for the implementation within the specifications provided above and for the scoring. However, teachers are advised to prepare worksheets based on the learning outcomes for this task for students to work on to provide them with the experience and skills that they will need for the actual task, as well as for the final examination.

The learning outcomes for Task 1 are taken from Strand 1, sub-strand 1.1

*IA TASK 2 Instructions*

**TASK 2**

**1:** PROVING TRIG IDENTITIES

**2:** TRIG BASIC SKILLS

**Task 2 is worth 10%. The task is based on Strand 2: Trigonometry**
*The key learning outcome that is assessed in this task is the student's ability to demonstrate basic trig skills as well as proving trigonometric identities.*

The Specific Learning Outcomes for Task 2 are taken from Strand 2, sub-strands 2.1 and 2.3. This task, together with the scoring rubric, will be provided from EQAP at the appropriate time in the year. Teachers will be responsible for the implementation within the specification provided below, as well as for the scoring of student responses. The scoring rubric will also be provided from EQAP.

Proving identities is a high level mathematical skill that involves manipulation of multiple possibilities, so it augurs well to be attempted as an assignment or task, rather than be attempted as items in a written timed test.

**Task Specifications:**

1. The task is a group task, and the teacher is to allocate students into their respective **groups of two or three**. A group of 4 may be necessary in some situations but it will end up being a large group, so teachers are cautioned about organising students into groups of 4s.
2. The SLOs that are to be targeted in this task are taken from Strand 2. Teachers are hereby advised to create worksheets for students that contain items that suit those learning outcomes, to give students much needed practice in manipulation of trig identities and other basic trig skills so that they gain the skills necessary for the completion of this task as well as similar items that may appear in the final examination.
3. The actual task document as well as the scoring rubric will be sent to schools from EQAP at the appropriate point in the year. **.** This is very important for teachers to note.
4. There will be **ONE common version of Task 2** and teachers are to give this to each group. Students are to be encouraged to work collaboratively in solving the task problems; however, each student is to submit individual papers to the teacher for scoring.
5. Students are to be given time in-class to work on their task. At the end of each class time, the teacher is to collect the students' work and return the same to them to work on during the next class time. In this way, the efforts of students is more meaningful and fairer. Teachers are to make independent decisions about allocation of scores for students who are absent during lessons in which groups are working on their tasks.
6. It is expected that students spend about 5 - 6 lessons on this task.
7. Teachers are to refer to the scoring rubric provided from EQAP and follow this closely when scoring students' responses to Task 1.

*IA TASK 3 Instructions*

**Task 3 is worth 10%. The task is based on Strands 3.**

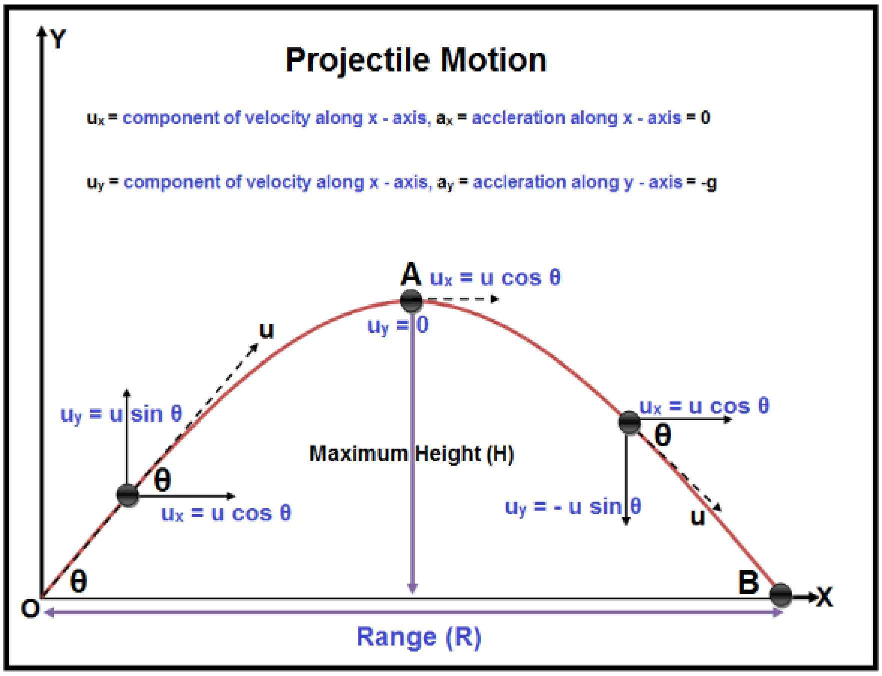
# TASK 3

**DIFFERENTIATION**

**TASK 3A:** Differentiation 10%
**PRACTICAL AND THEORETICAL**

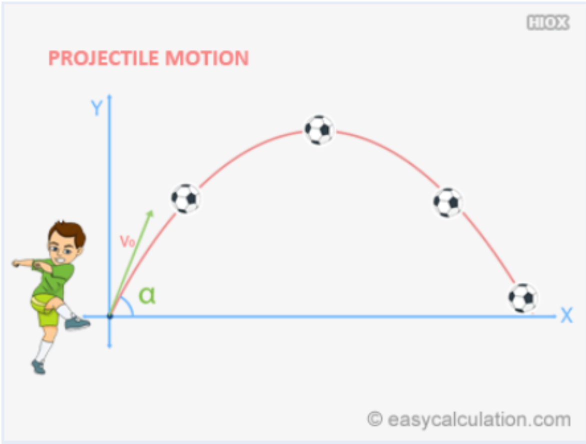
This task is designed to see how students can use differentiation and integration in real life application problems in the area of kinematics.

**Task 3.**
The focus of this task is the real-life applications in the kinematic area of projectile motions, using differentiation of basic trig functions. A demonstration is provided in the chart below. .




The idea behind this task, is to **transition away from having a pen-to-paper test**, and apply the concept using a real-life situation, that could be <u>contextualised to any setting</u> the student is exposed to.
For instance, applying projectile motion to:

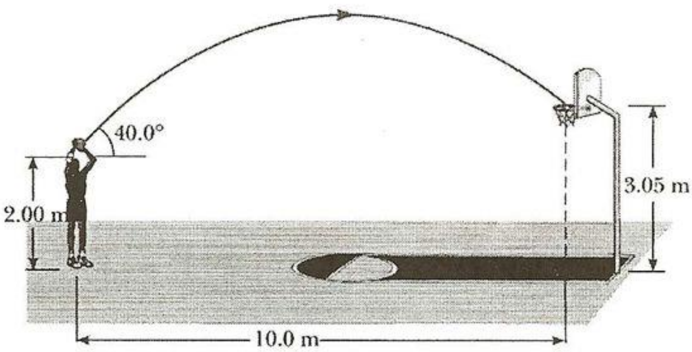
- the movement of a kicked “Soccer” ball.




- Throwing a spear in fishing or pig hunting.

- Catapulting a stone (or bird) using a sling, as in the “Angry Birds” application on smart phones.

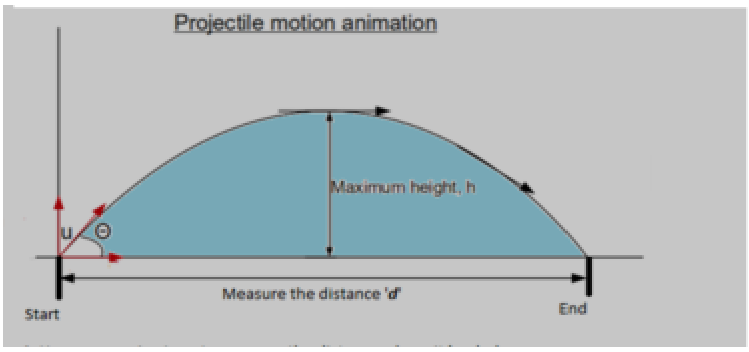

- Sinking a basketball or netball in a hoop.



The key idea is to **contextualise the task activity**, so students can relate to a real life application of mathematics in the form of integration and/or differentiation related to projectile motions.

Task 3 is a school based task with a Common Assessment Frame. The common frame is that the chosen activity is to deal with projectile motions but the actual choice of the type of projectile motion (e.g. soccer or basketball or throwing a spear or catapulting a stone) from which measurements are to be taken is left to teachers and students to decide. Detailed task instructions and the scoring rubric will be provided from EQAP at the appropriate point in the year, and will have the following features:

1. Task 3 is based on learning outcomes in Strands 3 (Differentiation), sub-strand 3.3.
2. There are **two** parts to Task 3.
    a. In the first part, students have to make some measurements, in a practical setting, and use the data collected to carry out some basic calculations.
    b. For the second part, students will use their knowledge of the content to derive their answers. This second part will be provided from EQAP at the appropriate time in the year.
3. For the first part, students are to choose the application area i.e. soccer or basketball or throwing a spear or catapulting a stone or something else. They are to be given **2 or 3** lessons to complete the practical part of throwing or kicking or catapulting an object, from which they are to measure the following (some approximations will be necessary):




| Measurements to be taken | | |
|---|---|---|
| 1 | Angle $\boldsymbol{\theta}$ | |
| 2 | Distance ***d*** | |
| 3 | Time of flight ***t*** | |

4. The required calculations are outlined below:
    a. Find the relative velocity V, where $V = \frac{d}{t\ x\ \cos\theta}$
    b. Find the velocity of the object, $u$ where $u = V sin\theta$
    c. Determine the formula for displacement $D$ where $D = ut + \frac{1}{2}at^2$
5. The rest of the problems to be solved for Task 3 will be provided from EQAP at the appropriate time in the year.
6. Other details for the task are to be included in the IA Program that the teacher is to send to EQAP for approval;
7. The exact day and week of the administration of Task 3 is suggested in the 'Calendar', and 'IA Due Dates' documents from EQAP.

**TASK 4**

**Task 4 is worth 10%. The task is based on Strands 4.**

**TASK 4**

**INTEGRATION**

**TASK 4:** Integration 10%
**THEORETICAL**

The application of integration is an essential component of mathematical applications in real life situations. Finding areas enclosed between boundaries and volumes of revolution are such important applications. Task 4 provides an opportunity for students to work collaboratively in order to solve application problems on areas and volumes together. Teachers are encouraged to provide students with worksheets containing problems that are based on learning outcomes from sub-strands 4.1 and 4.3, upon which they can collaborate, learn together and practice essential skills that relate to the process of finding areas and volumes of revolution.

1. Task 4 is based on the topic of Integration and the learning outcomes to be assessed in this task are taken from Strand 4, sub-strands 4.1 and 4.3
2. Task 4 will be administered as a controlled assignment, and the task paper together with the scoring rubric will be provided by EQAP at the appropriate time in the year. The date for administration of this CAT will be indicated in the IA Due Dates document that will be provided from EQAP at the beginning of the year.
3. Teachers are expected to ensure that the task is administered properly and scored according to the scoring rubric that will be provided. Proper administration includes regular monitoring by teachers to ensure that there is no plagiarism or cheating.
4. Students are to work collaboratively in groups of 2s or 3s, and they are to be encouraged to discuss the various methods of solving each problem; however, each student is to submit his or her own answer paper.
5. Teachers are encouraged to provide students with worksheets containing problems that assess the learning outcomes in sub-strands 4.1 and 4.3 and facilitate their learning to provide them with the experience and skills necessary for them to engage with Task 4, as well as similar items that will appear in the final examination.
6. Students are to be provided with 2 - 3 lessons to complete this task.

**INTERNAL ASSESSMENT (IA) PROGRAM APPROVAL**

At the beginning of each year, each school presenting students for the South Pacific Form Seven Certificate Mathematics (with Calculus) assessment must complete an Internal Assessment Program Proposal and forward to EQAP by the date set down by the Director. The proposal must make clear a number of things. These include The time (start and completion date) for each task. In the case of the CATs, the start and end date is the same as it is a test. Refer to the IA Due Dates document sent from EQAP to ensure that the start and end dates that is proposed for the school fall within or very close to the dates indicated in the IA Due Dates document. Also, the details for each task that teachers and students are to make decisions about, etc.

Since the syllabus is a new revision for 2020, and all IA tasks are new, all schools are expected to submit a full IA Program proposal. A copy of the IA Program proposal template is provided as Appendix 2 and all teachers are urged to complete the details for the IA proposal for each subject, in compliance with the requirements stipulated in the template. Completed IA program proposals are to be submitted to EQAP for approval by the stipulated due dates.

The IA Program and copies of all assessment tasks and assessment schedules used, as well as a sample of candidate responses to all internal assessment work undertaken, must be available for verification during the IA verification visit.

The moderation of Internal Assessment will be done in accordance with EQAP policy as specified from time to time.

It is recommended that at the start of the year students are given copies of the learning outcomes and the task descriptions or the IA Programme.

# APPENDICES

## Appendix 1: Sample TASK Scoring Rubrics

### Task 1: ALGEBRA (10%)

| # | SLO Code | Skill Score | Evidence of correct response | Student Response Level | | | |
|---|---|---|---|---|---|---|---|
| | | | | 4 | 3 | 2 | 1 |
| 1 | **Cal1.1.2.7** | 2 | | | | | |
| 2 | **Cal1.1.3.3** | 3 | | | | | |
| 3 | **Cal1.1.3.1** | 3 | | | | | |
| 4 | **Cal1.1.2.3** | 2 | | | | | |
| 5 | **Cal1.1.2.4** | 2 | | | | | |
| 6 | **Cal1.1.3.2** | 3 | | | | | |

### Task 2: TRIGONOMETRY (10%)

| # | SLO Code | Skill Score | Evidence of correct response | Student Response Level | | | |
|---|---|---|---|---|---|---|---|
| | | | | 4 | 3 | 2 | 1 |
| 1 | **Cal2.1.2.1** | 2 | | | | | |
| 2 | **Cal2.1.2.3** | 2 | | | | | |
| 3 | **Cal2.1.3.2** | 3 | | | | | |
| 4 | **Cal2.3.2.1** | 2 | | | | | |
| 5 | **Cal2.3.3.3** | 3 | | | | | |
| 6 | **Cal2.3.3.1** | 3 | | | | | |

### Task 3: DIFFERENTIATION (10%)

| # | SLO Code | Skill Score | Evidence of correct response | Student Response Level | | | |
|---|---|---|---|---|---|---|---|
| | | | | 4 | 3 | 2 | 1 |
| 1 | **Cal3.3.3.5** | 3 | | | | | |
| 2 | **Cal3.3.2.3** | 2 | | | | | |
| 3 | **Cal3.3.3.1** | 3 | | | | | |
| 4 | **Cal3.1.2.4** | 2 | | | | | |
| 5 | **Cal3.3.3.4** | 3 | | | | | |

Task 4: INTEGRATION (10%)

| | SLO code | | Evidence | 4 | 3 | 2 | 1 |
|---|---|---|---|---|---|---|---|
| 1 | **Cal4.1.3.1** | 3 | | | | | |
| 2 | **Cal4.1.2.3** | 2 | | | | | |
| 3 | **Cal4.1.3.2** | 3 | | | | | |
| 4 | **Cal4.3.3.1** | 3 | | | | | |
| 5 | **Cal4.3.4.1** | 4 | | | | | |
| 6 | **Cal4.3.4.2** | 4 | | | | | |

## Appendix 2: IA Program Proposal Template

### FULL IA PROGRAM 2020

---

**Page 1:** **COVER PAGE**

| |
|---|
| **xxxxxx SEC SCHOOL**<br>**SPFSC 2020**<br>**Xxxxx (subject): FULL IA PROGRAM**<br><br>Name: |

**Page 2:** **INSERT IA SUMMARY FORM HERE**

(To be completed, signed by both the teacher and the School Principal of his/her Nominee/school stamped/scan/insert)

**Pages 3-6:**

**1 Task title: Task 1:**___________________________

The title should be brief and include a reference to the particular syllabus topic or skill which is being assessed by the task.

Example: *"Research Topic – Investigation of a Social Issue."*

**2 Learning Outcomes: List the Specific Learning Outcomes (SLOs) to be assessed by the task**

These are found in the syllabus and need to be identified before the tasks are constructed.

Example: *Describe a feature of ….*

*(Copy and paste directly from the aligned Syllabus: it must show strand, sub strand and SLOs)*

**3. Assessment/Task:**

**Describe the task as a form of assessment to measure student achievements of the above learning outcomes at different stages of the lesson/task implementation.**

*(Think of what are the best types of assessment for the above LOs so that your students can demonstrate they have achieved the learning outcomes. Also include how you will pre-assess their knowledge at the beginning of the lesson and how you will continuously assess them throughout the strand/topic to monitor their learning progress. The summative assessments are the final IA tasks.)*

*e.g.* Diagnostic: *(can be oral questions/short tests/ surveys/questionnaires to find out what students already know before the lesson)*

*Formative: 1. This is the formative use of the summative assessment such as the drafts submitted, self-assessment, peer assessment, teacher assessment of the drafts and specific feedback provided to improve the task. 2. For CATs – this can be similar items prepared by teachers using the SLOs and given to students for practice. After scoring, the feedback needs to be given to improve learning. If majority students not doing well then re-teach using another strategy, assess and monitor learning.*

*Summative: (these are the final IA tasks or the CATs to measure how much the students have learnt/achieved after the learning period)*

**4 Resources: List materials required for completing the task (for learning & demonstrating the achievement for the SLOs.**

This must specify any material items such as books, documents, maps, stimulus material, equipment required by the task, including use of technology.

**5 Guidelines for the teacher on advance preparation requirements**

a) **time required** by the student for task completion (monitoring progress)
b) recommended dates/date range for task completion
c) organization of room and hardware to facilitate task completion (learning assessment).

(After the task has been completed and scored, teachers will need an IA score capture sheet to record the performance of all students in the class.)

**6 Guidelines for the teacher on task completion and task control**

This must specify:

- the role of the teacher during the period of task completion
- instructions that are to be given by the teacher to the students
- actions that are required of the teacher during task completion

**7 Preparation by the students beforehand**

If students are required to prepare in advance of the task date, preparatory notes must indicate the requirements. For example, students may need to collect support materials for a task that is supervised in a classroom.

**8 Task outline for the student**

This outline is a brief description of the task that the student is to complete. It is a general description without specific detail.

Example: *Your task is to focus on an important social issue. After investigating that issue, you need to process information collected and suggest possible courses of action that authorities could take.*

**9 Task detail for the student**

This must provide a detailed description of the task in the sequence that the student would be expected to follow during task completion. This must clearly state:

- what the student is expected to do
- what the student is expected to record and present for assessment.

**10. Feedback & Support**

Allocate time for:

i. Student's self-assessment and correction
ii. Peer assessment, feedback, and time for improvement
iii. Teacher assessment, feedback, and time for time improvement

(NB: State how this will be carried out)

**11. Final submission & scoring**

State when the final task is due and how it will be assessed. State how the school (HOD/SPFSC Coordinator) will monitor the scoring of the tasks.

**12 Scoring Rubric**

Copy and paste directly from the aligned Syllabus the relevant scoring rubrics

**13 Assessment score capture sheet for the task**

This will be provided by EQAP

**(Repeat 1-13 for other tasks)**

# Appendix 3: List of useful formulae and tables

## MATHEMATICS WITH CALCULUS – USEFUL FORMULAE AND TABLES

### ALGEBRA

**Quadratic Formula**

If $ax^2 + bx + c = 0$

then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

**Arithmetic Sequences and Series**

$t_n = a + (n-1)d$

$S_n = \frac{n}{2}[2a + (n-1)d]$

**Binomial Theorem**

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + \binom{n}{n}b^n$$

where $\binom{n}{r} = {}^nC_r = \frac{n!}{(n-r)!r!}$

Some values of $\binom{n}{r}$ are given in the table below.

**Logarithms**

$y = \log_b x \Leftrightarrow x = b^y$

$\log_b(xy) = \log_b x + \log_b y$

$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

$\log_b(x^n) = n \log_b x$

$\log_b x = \frac{\log_a x}{\log_a b}$

**Geometric Sequences and Series**

$t_n = ar^{n-1}$

$S_n = \frac{a(1-r^n)}{1-r} \quad r \neq 1$

$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$

**Binomial Coefficients**

| n \ r | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | | | | | | | | | | |
| 1 | 1 | 1 | | | | | | | | | |
| 2 | 1 | 2 | 1 | | | | | | | | |
| 3 | 1 | 3 | 3 | 1 | | | | | | | |
| 4 | 1 | 4 | 6 | 4 | 1 | | | | | | |
| 5 | 1 | 5 | 10 | 10 | 5 | 1 | | | | | |
| 6 | 1 | 6 | 15 | 20 | 15 | 6 | 1 | | | | |
| 7 | 1 | 7 | 21 | 35 | 35 | 21 | 7 | 1 | | | |
| 8 | 1 | 8 | 28 | 56 | 70 | 56 | 28 | 8 | 1 | | |
| 9 | 1 | 9 | 36 | 84 | 126 | 126 | 84 | 36 | 9 | 1 | |
| 10 | 1 | 10 | 45 | 120 | 210 | 252 | 210 | 120 | 45 | 10 | 1 |
| 11 | 1 | 11 | 55 | 165 | 330 | 462 | 462 | 330 | 165 | 55 | 11 |
| 12 | 1 | 12 | 66 | 220 | 495 | 792 | 924 | 792 | 495 | 220 | 66 |
| 13 | 1 | 13 | 78 | 286 | 715 | 1287 | 1716 | 1716 | 1287 | 715 | 286 |
| 14 | 1 | 14 | 91 | 364 | 1001 | 2002 | 3003 | 3432 | 3003 | 2002 | 1001 |
| 15 | 1 | 15 | 105 | 455 | 1365 | 3003 | 5005 | 6435 | 6435 | 5005 | 3003 |

**Complex Numbers**

$z = x + iy = r\,\text{cis}\theta = r(\cos\theta + i\sin\theta)$

$\overline{z} = x - iy = r\,\text{cis}(-\theta) = r(\cos\theta - i\sin\theta)$

$r = |z| = \sqrt{z\overline{z}} = \sqrt{x^2 + y^2}$

$\theta = \arg(z)$ where $\cos\theta = \frac{x}{r}$ , $\sin\theta = \frac{y}{r}$

$(r\,\text{cis}\theta)^n = r^n\,\text{cis}(n\theta)$ for integer $n$ (De Moivre's Theorem)

## CALCULUS

### Differentiation

| $y$ | $\frac{dy}{dx}$ |
|---|---|
| $c$ | $0$ |
| $x^n$ | $nx^{n-1}$ |
| $\ln x$ | $\frac{1}{x}$ |
| $e^{ax}$ | $ae^{ax}$ |
| $\sin x$ | $\cos x$ |
| $\cos x$ | $-\sin x$ |
| $\tan x$ | $\sec^2 x$ |
| $\sec x$ | $\sec x \tan x$ |
| $\operatorname{cosec} x$ | $-\operatorname{cosec} x \cot x$ |
| $\cot x$ | $-\operatorname{cosec}^2 x$ |

### Integration

| $y$ | $\int y\,dx$ |
|---|---|
| $k$ | $kx+c$ |
| $x^n$ | $\frac{1}{n+1}x^{n+1}+c$ |
| $\frac{1}{x}$ | $\ln\lvert x\rvert+c$ |
| $\frac{f'(x)}{f(x)}$ | $\ln\lvert f(x)\rvert+c$ |

### Product Rule

$(f.g)' = f.g' + g.f'$

or if $y = uv$ then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

### Quotient Rule

$\left(\frac{f}{g}\right)' = \frac{g.f' - f.g'}{g^2}$

or if $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

### Volume of Revolution

$y = f(x)$ between $x = a$ and $x = b$

rotated about the $x$ - axis.

Volume $= \int_a^b \pi y^2 dx$

### Composite Function or Chain Rule

$\left(f(g)\right)' = f'(g).g'$

or if $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$

## COORDINATE GEOMETRY

### Straight Line

Gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$

Equation $y - y_1 = m(x - x_1)$

Distance $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Midpoint $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

### Circle

$x^2 + y^2 = r^2$ or $(r\cos\theta, r\sin\theta)$

has centre $(0,0)$ and radius $r$

$(x-a)^2 + (y-b)^2 = r^2$

has centre $(a,b)$ and radius $r$

### Parabola

$y^2 = 4ax$ or $(at^2, 2at)$

### Ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ or $(a\cos\theta, b\sin\theta)$

### Hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ or $(a\sec\theta, b\tan\theta)$

asymptotes $y = \pm\frac{b}{a}x$

## MEASUREMENT

### Triangle

area $= \frac{1}{2}ab\sin C$

### Trapezium

area $= \frac{1}{2}(a+b)h$

### Circle

circumference $= 2\pi r$

area $= \pi r^2$

### Sector

area $= \frac{1}{2}r^2\theta$

arc length $= r\theta$

### Cylinder

volume $= \pi r^2 h$

curved surface area $= 2\pi rh$

### Cone

volume $= \frac{1}{3}\pi r^2 h$

curved surface area $= \pi rl$ where $l$ = slant height

### Sphere

volume $= \frac{4}{3}\pi r^3$

surface area $= 4\pi r^2$

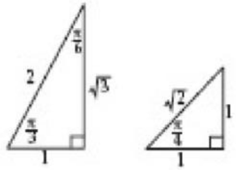
## TRIGONOMETRY

$\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

$\sec\theta = \dfrac{1}{\cos\theta}$

$\cot\theta = \dfrac{1}{\tan\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$




### Sine Rule

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

### Cosine Rule

$c^2 = a^2 + b^2 - 2ab\cos C$

### Identities

$\cos^2\theta + \sin^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$

### Compound Angles

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

### Double Angles

$\sin 2A = 2\sin A\cos A$

$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A\\ &= 2\cos^2 A - 1\\ &= 1 - 2\sin^2 A\end{aligned}$$

### Products

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$2\cos A\sin B = \sin(A+B) - \sin(A-B)$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$2\sin A\sin B = \cos(A-B) - \cos(A+B)$

### Sums

$\sin C + \sin D = 2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

$\sin C - \sin D = 2\cos\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

$\cos C + \cos D = 2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

$\cos C - \cos D = -2\sin\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

### General Solutions

If $\sin\theta = \sin\alpha$ then $\theta = n\pi + (-1)^n\alpha$

If $\cos\theta = \cos\alpha$ then $\theta = 2n\pi \pm \alpha$

If $\tan\theta = \tan\alpha$ then $\theta = n\pi + \alpha$

where $n$ is any integer

## NUMERICAL METHODS

### Trapezium Rule

$\int_a^b f(x)\,dx \approx \frac{1}{2}h\left[y_0 + y_n + 2(y_1 + y_2 + \dots + y_{n-1})\right]$

where $h = \dfrac{b-a}{n}$ and $y_r = f(x_r)$

### Simpson's Rule

$\int_a^b f(x)\,dx \approx \frac{1}{3}h\left[y_0 + y_n + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2})\right]$

where $h = \dfrac{b-a}{n}$, $y_r = f(x_r)$ and $n$ is even.

**MAC – IA**

## Appendix 4: IA Program Summary Form

# SOUTH PACIFIC FORM SEVEN CERTIFICATE

### Internal Assessment Summary Form

# MATHEMATICS WITH CALCULUS

Country: ____________________ School: ______________________________________

| Task | Task Description/Focus | Start Date | End Date | Weighting |
|---|---|---|---|---|
| **1. Algebra** | | | | **10%** |
| **2. Trigonometry** | | | | **10%** |
| **3. Differentiation** | | | | **10%** |
| **4. Integration** | | | | **10%** |
| | **Total** | | | **40%** |

***Note***:
1. Be specific about dates, not just Week 3 Term 1, etc.
2. Assessment Schedules/Scoring Rubrics for the tasks will be provided by EQAP. Teachers must use these when scoring students' work.
3. All IA Score Capture Sheets will be provided by EQAP to schools.

***Teacher's Name and Signature***: _____________________________________ *Date: ………….*

***Principal's Name and Signature***: ___________________________________ *Date: ………….*

**A full IA program is to be submitted together with this IA Summary Form.**

# Appendix 5: Electronic calculators in examinations

Written or printed materials (including information and routines stored in the programmable memory of calculators) are prohibited.

Due to rapid changes in calculator technology, the Board will regularly review its policy on the use of calculators in examinations. However, every effort is made to ensure that schools are given adequate notice of policy changes.

The following policy aims to compromise between rewarding the appropriate use of technology while giving consideration to associated equity issues. The Board has a responsibility to ensure fairness and equity to all candidates.

Examination setters are aware of calculator technology, and take calculator capability into account in the design of examination questions and marking schedules.

## General Policy

The EQAP *Assessment and Certification Rules and Procedures for Secondary Schools* allow candidates to use a calculator in any of its examinations provided that the calculator is silent, hand-held, non-printing and contains its own power source. However, calculators may not be used to pass information to other candidates, bring information into the examination, or as a dictionary/translator.

EQAP encourages examiners to set papers that examine understanding of concepts in such a way that the use of sophisticated calculators is not a significant advantage. Examination questions may require details of working steps to be shown to ensure that candidates understand the key concepts being assessed.

EQAP's policy on calculators in examinations allows the legitimate use of most types of calculator, including graphical and programmable calculators. The intention of the policy is to support the directions of curriculum development and encourage the appropriate use of calculator technology. The policy does not allow the use of calculators to contravene other examination rules and procedures.

The exception to the above paragraph is the use of any calculator that has symbolic algebraic manipulation capability. These will continue to be prohibited in all of the Board's examinations as they may offer candidates who use them a significant advantage over other candidates.

The following models have currently been identified as having this capability:

- Texas Instruments T189
- Texas Instruments T192
- Texas Instruments T192 Plus
- Casio CFX 9970G
- Casio Algebra FX 2.0
- Hewlett Packard HP48G
- Hewlett Packard HP48GX
- Hewlett Packard HP49G

EQAP may from time to time publish more detailed rules for the use of calculators, or further add to the list of prohibited calculators.

## Information for Students

The *Instructions to Candidates* booklet, which is issued to all candidates prior to the examination period, summarises the above rules and procedures and also states:

## Calculators

Candidates are recommended to take a calculator into the examination room for subjects where they have used a calculator during the year.  For subjects where a scientific calculator has been used during the year, this should be taken to the examination.

Candidates bringing more sophisticated calculators into an examination room may be subjected to additional scrutiny by supervisors.

Any possible misuse of calculators during an examination will be handled through the Board’s standard procedures for investigating possible misconduct in examinations.

# ADVISORY SECTION

## Suggested Texts

This is a list of only some Mathematics texts that are available and have been used for teaching the course for University Bursary Mathematics with Calculus. It is important teachers use this as only a guide and check current book lists available through publishers and book retailers.

**A. Suggested Text**

1. Delta Mathematics - Barton, D.; Johnson, W. & Laird S

**B. Supplementary Texts**

1. Year 13 Study Guide, *Mathematics with Calculus* - Sidebotham, T. ESA
2. Longman Write-on, *Notes – Calculus* _ Barton, D
3. Bursary Calculus - Questions from the last 8 bursary papers with suggested answers.

   Really Useful Resources
   Box 19-939
   Woolston
   Christchurch

4. Study Pass reference notes, Year 13 Calculus - *info@studypass.co.nz*

## Websites

5. Calculus Website

   i). *www.bbc.uk/education/asguru/maths/intro.shtml is part of www.bbc.co.uk*

   *ii) www.ies.co.jp/math/java/calc/index/html is part of www.ies.co.jp/math/indexeng.html*

   *iii) www.ies.co.jp/math/java/comp/index.html is part of www.ies.co.jp/math/*

   *iv) www.ies.co.jp/math/java/conics/index.html is part of www.ies.co.jp/math/*

   v) www.unc.edu/~rowlett/units.html is part of *www.unc.edu/*

   vi) www.mathforum.org/pow/

   vii)www.ies.co.jp/math/java/misc/index.html is part of *www.ies.co.jp/math/indexeng.html*

   viii) www.btinternet.com/~rfbarrow/

**The End**

No. 108/2

EDUCATIONAL QUALITY
AND
ASSESSMENT PROGRAMME

# SOUTH PACIFIC FORM SEVEN CERTIFICATE

# MATHEMATICS WITH CALCULUS

FORMULAE AND TABLES BOOKLET

**YOU MAY KEEP THIS FORMULAE AND TABLES BOOKLET AT THE END OF THE EXAMINATION.**

# MATHEMATICS WITH CALCULUS – USEFUL FORMULAE AND TABLES

## ALGEBRA

### Quadratic Formula

If $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

### Arithmetic Sequences and Series

$t_n = a + (n-1)d$

$S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$

### Binomial Theorem

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + \binom{n}{n}b^n$$

where $\dbinom{n}{r} = {}^nC_r = \dfrac{n!}{(n-r)!\,r!}$

Some values of $\dbinom{n}{r}$ are given in the table below.

### Logarithms

$y = \log_b x \Leftrightarrow x = b^y$

$\log_b(xy) = \log_b x + \log_b y$

$\log_b\left(\dfrac{x}{y}\right) = \log_b x - \log_b y$

$\log_b\left(x^n\right) = n\log_b x$

$\log_b x = \dfrac{\log_a x}{\log_a b}$

### Geometric Sequences and Series

$t_n = ar^{n-1}$

$S_n = \dfrac{a\left(1-r^n\right)}{1-r}$ $\quad r \neq 1$

$S_\infty = \dfrac{a}{1-r}$ $\quad$ for $|r| < 1$

### Binomial Coefficients

| n \ r | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | | | | | | | | | | |
| 1 | 1 | 1 | | | | | | | | | |
| 2 | 1 | 2 | 1 | | | | | | | | |
| 3 | 1 | 3 | 3 | 1 | | | | | | | |
| 4 | 1 | 4 | 6 | 4 | 1 | | | | | | |
| 5 | 1 | 5 | 10 | 10 | 5 | 1 | | | | | |
| 6 | 1 | 6 | 15 | 20 | 15 | 6 | 1 | | | | |
| 7 | 1 | 7 | 21 | 35 | 35 | 21 | 7 | 1 | | | |
| 8 | 1 | 8 | 28 | 56 | 70 | 56 | 28 | 8 | 1 | | |
| 9 | 1 | 9 | 36 | 84 | 126 | 126 | 84 | 36 | 9 | 1 | |
| 10 | 1 | 10 | 45 | 120 | 210 | 252 | 210 | 120 | 45 | 10 | 1 |
| 11 | 1 | 11 | 55 | 165 | 330 | 462 | 462 | 330 | 165 | 55 | 11 |
| 12 | 1 | 12 | 66 | 220 | 495 | 792 | 924 | 792 | 495 | 220 | 66 |
| 13 | 1 | 13 | 78 | 286 | 715 | 1287 | 1716 | 1716 | 1287 | 715 | 286 |
| 14 | 1 | 14 | 91 | 364 | 1001 | 2002 | 3003 | 3432 | 3003 | 2002 | 1001 |
| 15 | 1 | 15 | 105 | 455 | 1365 | 3003 | 5005 | 6435 | 6435 | 5005 | 3003 |

### Complex Numbers

$z = x + iy = r\,\mathrm{cis}\,\theta = r(\cos\theta + i\sin\theta)$

$\bar{z} = x - iy = r\,\mathrm{cis}(-\theta) = r(\cos\theta - i\sin\theta)$

$r = |z| = \sqrt{z\bar{z}} = \sqrt{x^2 + y^2}$

$\theta = \arg(z)$ where $\cos\theta = \dfrac{x}{r}$ , $\sin\theta = \dfrac{y}{r}$

$(r\,\mathrm{cis}\,\theta)^n = r^n\,\mathrm{cis}(n\theta)$ for integer $n$ (De Moivre's Theorem)

## CALCULUS

### Differentiation

| $y$ | $\frac{dy}{dx}$ |
|---|---|
| $c$ | $0$ |
| $x^n$ | $nx^{n-1}$ |
| $\ln x$ | $\frac{1}{x}$ |
| $e^{ax}$ | $ae^{ax}$ |
| $\sin x$ | $\cos x$ |
| $\cos x$ | $-\sin x$ |
| $\tan x$ | $\sec^2 x$ |
| $\sec x$ | $\sec x \tan x$ |
| $\operatorname{cosec} x$ | $-\operatorname{cosec} x \cot x$ |
| $\cot x$ | $-\operatorname{cosec}^2 x$ |

### Integration

| $y$ | $\int y\,dx$ |
|---|---|
| $k$ | $kx + c$ |
| $x^n$ | $\frac{1}{n+1}x^{n+1} + c$ |
| $\frac{1}{x}$ | $\ln\lvert x\rvert + c$ |
| $\frac{f'(x)}{f(x)}$ | $\ln\lvert f(x)\rvert + c$ |

### Product Rule

$(f.g)' = f.g' + g.f'$

or if $y = uv$ then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

### Quotient Rule

$\left(\frac{f}{g}\right)' = \frac{g.f' - f.g'}{g^2}$

or if $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

### Volume of Revolution

$y = f(x)$ between $x = a$ and $x = b$ rotated about the $x$ - axis.

Volume $= \int_a^b \pi y^2\,dx$

### Composite Function or Chain Rule

$(f(g))' = f'(g).g'$

or if $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$

## COORDINATE GEOMETRY

### Straight Line

Gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$

Equation $y - y_1 = m(x - x_1)$

Distance $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Midpoint $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

### Circle

$x^2 + y^2 = r^2$ or $(r\cos\theta, r\sin\theta)$

has centre $(0,0)$ and radius $r$

$(x-a)^2 + (y-b)^2 = r^2$

has centre $(a,b)$ and radius $r$

### Parabola

$y^2 = 4ax$ or $(at^2, 2at)$

### Ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ or $(a\cos\theta, b\sin\theta)$

### Hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ or $(a\sec\theta, b\tan\theta)$

asymptotes $y = \pm\frac{b}{a}x$

## MEASUREMENT

### Triangle

area $= \frac{1}{2}ab\sin C$

### Trapezium

area $= \frac{1}{2}(a+b)h$

### Circle

circumference $= 2\pi r$

area $= \pi r^2$

### Sector

area $= \frac{1}{2}r^2\theta$

arc length $= r\theta$

### Cylinder

volume $= \pi r^2 h$

curved surface area $= 2\pi rh$

### Cone

volume $= \frac{1}{3}\pi r^2 h$

curved surface area $= \pi rl$ where $l =$ slant height

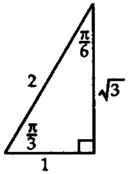
### Sphere

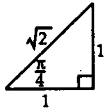
volume $= \frac{4}{3}\pi r^3$

surface area $= 4\pi r^2$

## TRIGONOMETRY

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$
$$\sec\theta = \frac{1}{\cos\theta}$$
$$\cot\theta = \frac{1}{\tan\theta}$$
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$




### Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

### Cosine Rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

### Identities

$$\cos^2\theta + \sin^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$$

### Compound Angles

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

### Double Angles

$$\sin 2A = 2\sin A\cos A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$
$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A\\ &= 2\cos^2 A - 1\\ &= 1 - 2\sin^2 A\end{aligned}$$

### Products

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$
$$2\cos A\sin B = \sin(A+B) - \sin(A-B)$$
$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$
$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

### Sums

$$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$$
$$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$$
$$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$$
$$\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$$

### General Solutions

If $\sin\theta = \sin\alpha$ then $\theta = n\pi + (-1)^n\alpha$

If $\cos\theta = \cos\alpha$ then $\theta = 2n\pi \pm \alpha$

If $\tan\theta = \tan\alpha$ then $\theta = n\pi + \alpha$

where $n$ is any integer

## NUMERICAL METHODS

### Trapezium Rule

$$\int_a^b f(x)\,dx \approx \frac{1}{2}h\left[y_0 + y_n + 2\left(y_1 + y_2 + \ldots + y_{n-1}\right)\right]$$

where $h = \dfrac{b-a}{n}$ and $y_r = f(x_r)$

### Simpson's Rule

$$\int_a^b f(x)\,dx \approx \frac{1}{3}h\left[y_0 + y_n + 4\left(y_1 + y_3 + \ldots + y_{n-1}\right) + 2\left(y_2 + y_4 + \ldots + y_{n-2}\right)\right]$$

where $h = \dfrac{b-a}{n}$, $y_r = f(x_r)$ and $n$ is even.