

MARKER CODE



Student Personal Identification Number

South Pacific Form Seven Certificate

MATHEMATICS WITH CALCULUS

2015

QUESTION and ANSWER BOOKLET

Time allowed: Two and half hours

INSTRUCTIONS

Write your **Student Personal Identification Number (SPIN)** in the space provided on the top right hand corner of this page.

Write your answers in the spaces provided in this booklet.

Show all working. Unless otherwise stated, numerical answers correct to **three significant figures** will be adequate.

If you need more space for answers, ask the Supervisor for extra paper. Write your SPIN on all extra sheets used and clearly number the questions. Attach the extra sheets at the appropriate places in this booklet.

Major Learning Outcomes (Achievement Standards)	Skill Level			Weight /Time
	Basic	Proficient	Advanced	
CaIA: Apply algebraic techniques to Real and Complex numbers.	8 questions	3 questions	2 questions	20% 42 min
CaIB: Demonstrate knowledge of advanced concepts and techniques of differentiation.	11 questions	4 questions	2 questions	25% 54 min
CaIC: Demonstrate knowledge of advanced concepts and techniques of integration.	11 questions	4 questions	2 questions	25% 54 min
TOTAL	30 Learning outcomes	11 Learning outcomes	6 Learning outcomes	150 min

Check that this booklet contains pages 2-31 in the correct order and that none of these pages is blank.
A 4-page booklet (No. 108/2) containing mathematical formulae and tables is provided.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

SECTION A: Apply algebraic techniques to real and complex numbers

Assessor's use only

A1

(a) Find the value of x if $\sqrt{2x + 15} = x$.

Basic	Level
Excellent	
Weak	
NR	

(b) Solve for x in the equation $\log_2(x + 4) - \log_2(x - 1) = 3$.

Basic	Level
Excellent	
Weak	
NR	

A2

(a) Solve the equation $16^x = 2^{3x-11}$.

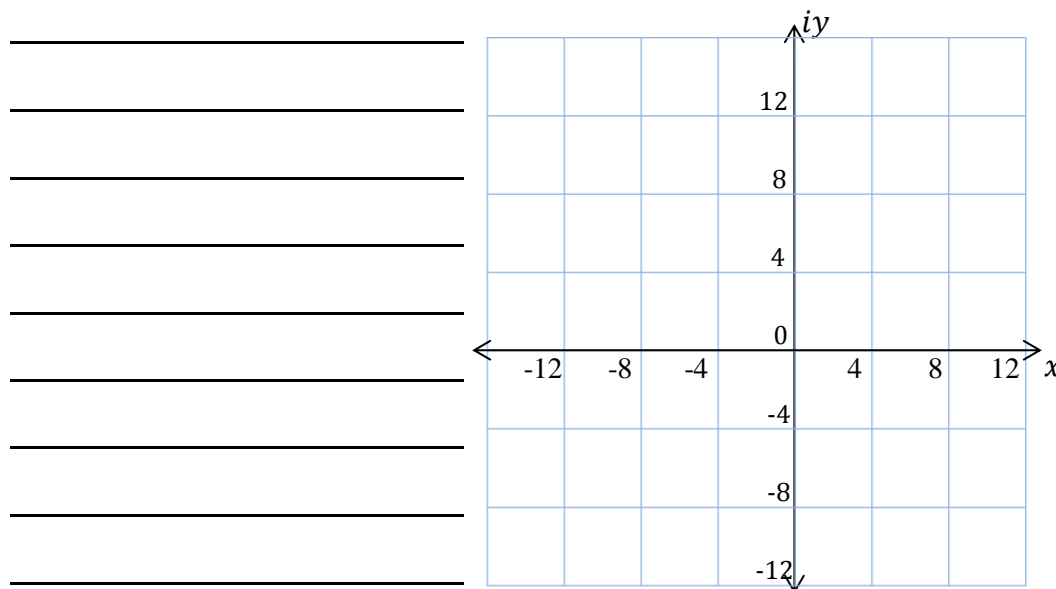
Basic	Level
Excellent	
Weak	
NR	

(b) Express $\frac{\sqrt{2}}{1+\sqrt{2}}$ in the form $a + b\sqrt{2}$.

Basic	Level
Excellent	
Weak	
NR	

A4

A complex number u in polar form is $u = 2cis\frac{\pi}{3}$. Write u^3 first in the polar form, and then draw the complex number u^3 in the Argand diagram below.



Basic	Level
Excellent	
Weak	
NR	

Basic	Level
Excellent	
Weak	
NR	

A5

The two roots of the quadratic equation $x^2 + mx + n = 0$ are -6 and 5 . Find the values of the constants m and n .

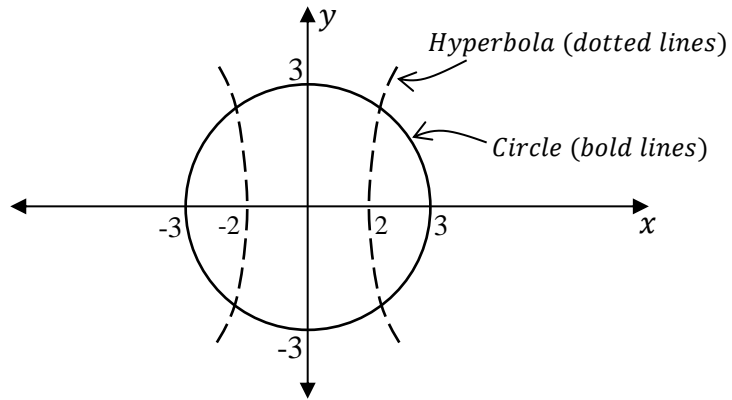
Basic	Level
Excellent	
Weak	
NR	

A6

Find the x-coordinates of the points of intersection of the hyperbola and the circle, whose equations are:

$$3x^2 - 4y^2 = 12 \quad \textit{hyperbola}$$

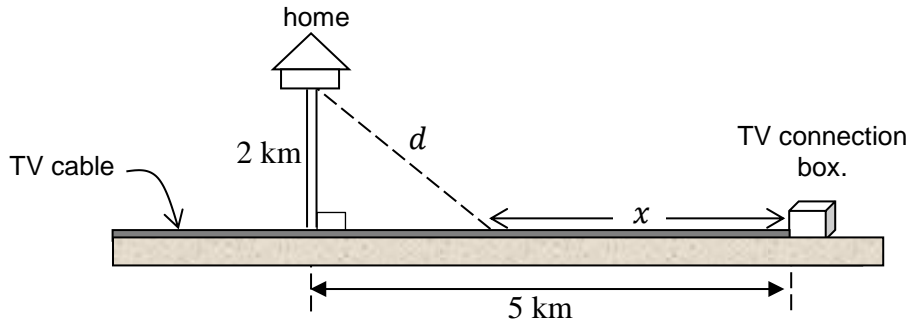
$$x^2 + y^2 = 3^2 \quad \textit{circle}$$



Proficient	Level
Excellent	
Moderate	
Weak	
NR	

A7

The cost of connecting a home to the nearest TV connection box depends on the length x of the TV cable (costing \$80/km) along the road, and the length d of the TV cable (costing \$100/km) off the road, where x and d are in kilometres. The home is 2 km from the road and 5 km along the road to the TV connection box.



Derive the expression for the total cost C as a function of x only.

Proficient	Level
Excellent	
Moderate	
Weak	
NR	

A9

Find the roots of the equation $(z - 1)^4 = 16i$.

Advanced	Level
Exceed	
Excellent	
Moderate	
Low	
Weak	
NR	

SECTION B Demonstrate knowledge of advanced concepts and techniques of differentiation.
Answer ALL Questions

Assessor's use only

B1

Find $\frac{dy}{dx}$ for each of the following. You do not need to simplify your answer.

(a) $y = 4(\sin 3x)^2$

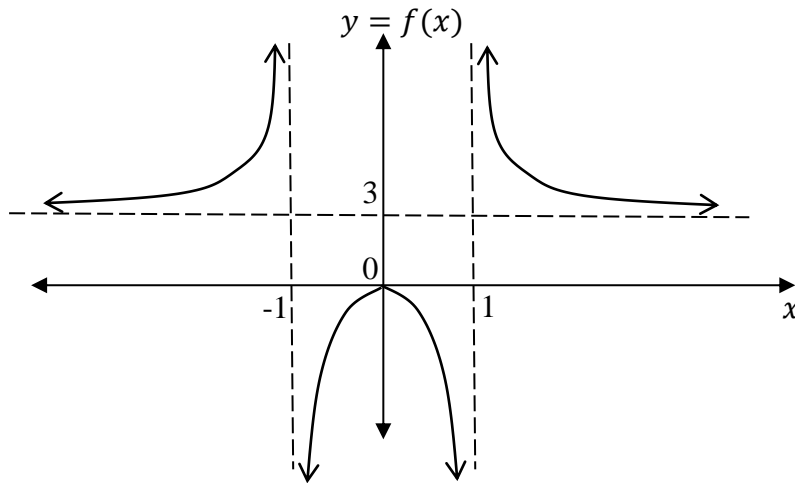
Basic	Level
Excellent	
Weak	
NR	

(b) $y = e^{2x} \ln x$

Basic	Level
Excellent	
Weak	
NR	

B2

(a)



Using the above sketch of the function $f(x)$, find

i. $\lim_{x \rightarrow +\infty} f(x) = \underline{\hspace{2cm}}$

ii. $\lim_{x \rightarrow +1^+} f(x) = \underline{\hspace{2cm}}$

Basic	Level
Excellent	
Weak	
NR	

(b) The distance of a moving particle from a point P is given by $x = 3t - 4t^3$. At what time is the velocity zero?

Basic	Level
Excellent	
Weak	
NR	

B7

Use implicit differentiation to find $\frac{dy}{dx}$ for the curve $y^2 + xy - 2x^2 = 2$.

Proficient	Level
Excellent	
Moderate	
Weak	
NR	

B8

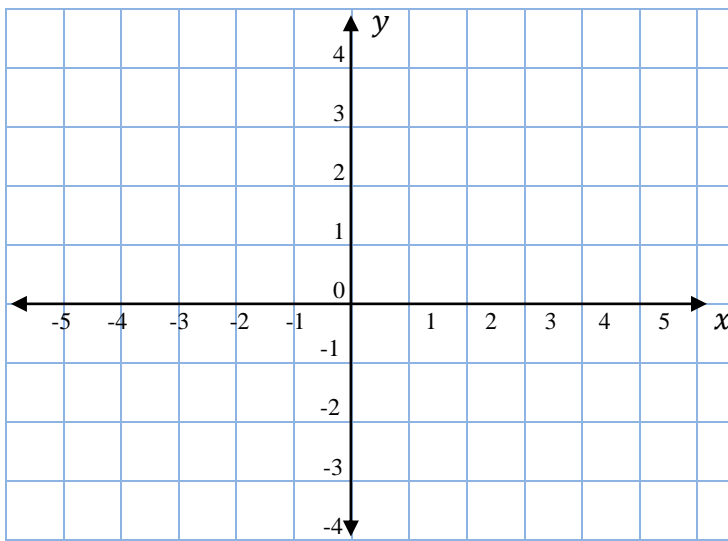
A piecewise function is given as $f(x) = \begin{cases} 1 - nx^2 & \text{if } x \leq 1 \\ nx & \text{if } x > 1 \end{cases}$

Find the value of n which makes $f(x)$ continuous at $x = 1$.

Proficient	Level
Excellent	
Moderate	
Weak	
NR	

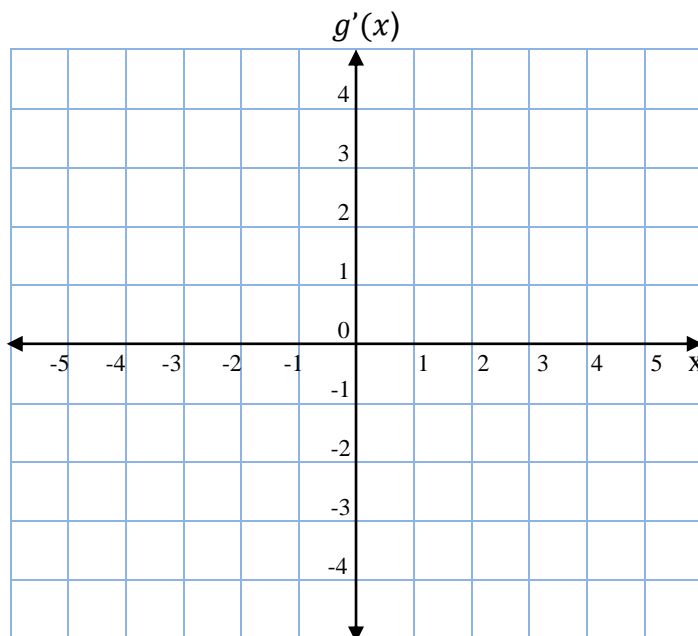
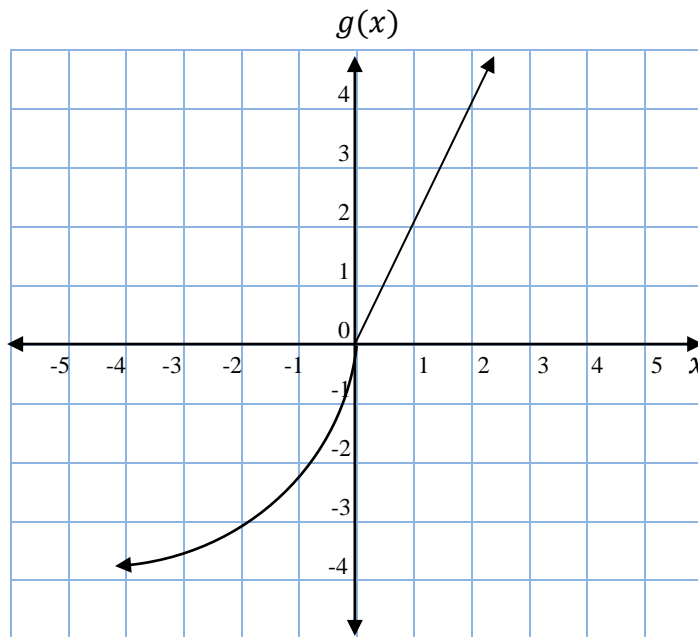
B9

Find the coordinates of the turning points of $f(x) = 2x^2 - x^4$. Hence sketch below the graph of $f(x)$, showing also the x-intercepts in the sketch.



Proficient	Level
Excellent	
Moderate	
Weak	
NR	

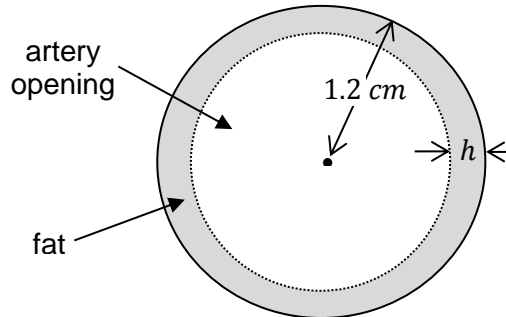
B10 The graph drawn below is that of a function $g(x)$. On the similar grid below this graph sketch the graph of the derived function $g'(x)$.



Proficient	Level
Excellent	
Moderate	
Weak	
NR	

B11

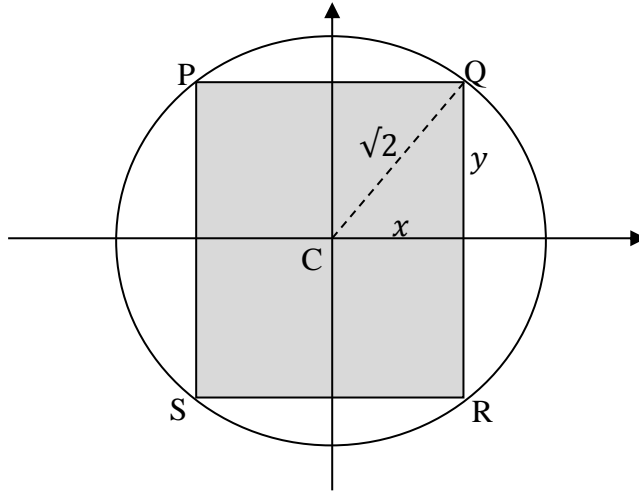
Fat, of uniform thickness h , is being deposited on the walls of an artery at a constant rate of 0.001 cm/month. The walls of the artery are assumed to be circular and of radius 1.2 cm.



At what rate is the cross-sectional area A of the artery opening changing when the fat deposit is 0.4 cm thick?

Advanced	Level
Exceed	
Excellent	
Moderate	
Low	
Weak	
NR	

B12 The rectangle PQRS has corners on the circumference of the circle with centre C and radius $r = \sqrt{2}$.



Calculate the value of x that will give the maximum area (shaded) of the rectangle PQRS.

Advanced	Level
Excellent	
Moderate	
Low	
Weak	
NR	

SECTION C Demonstrate knowledge of advanced concepts and techniques of integration.
Answer ALL Questions

Assessor's use only

C1 Find the following integrals. There is no need to simplify the answer.

(a) $\int \frac{4x + 3x^2}{x} dx$

(b) $\int \sin(2\theta + 1) d\theta$

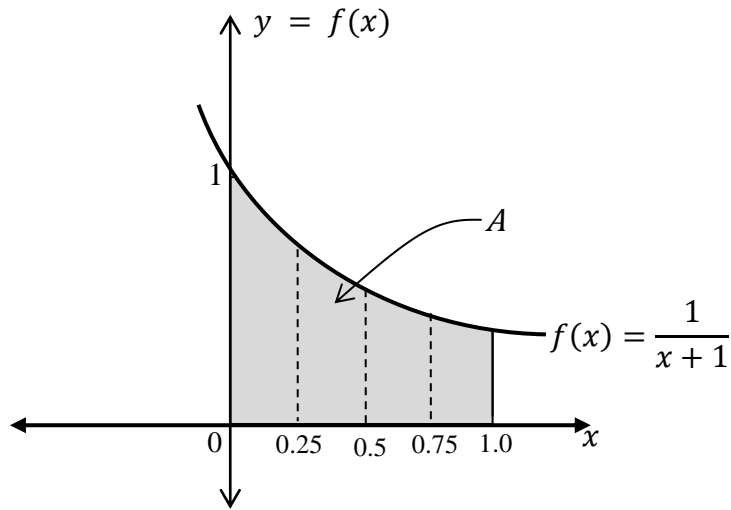
Basic	Level
Excellent	
Weak	
NR	

Basic	Level
Excellent	
Weak	
NR	

Basic	Level
Excellent	
Weak	
NR	

C2

The shaded area A under the curve $f(x) = \frac{1}{x+1}$ is between $x = 0$ and $x = 1$. This area is sub-divided into four rectangles as shown in the diagram.



Use Simpson's rule for $n = 4$:

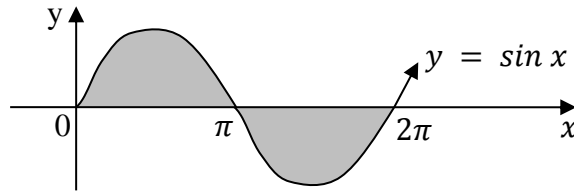
$$\int_a^b f(x)dx = \left(\frac{b-a}{3n}\right) [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)]$$

to calculate an approximate value for the shaded area A .

Basic	Level
Excellent	
Weak	
NR	

Basic	Level
Excellent	
Weak	
NR	

C3 Calculate the area shaded between the function $y = \sin x$ and the x-axis for $0 \leq x \leq 2\pi$.



Basic	Level
Excellent	
Weak	
NR	

C4 The velocity of an object is given by $v = 12t - 3t^2$ m/s. What distance does it travel in the first 4 seconds of its motion?

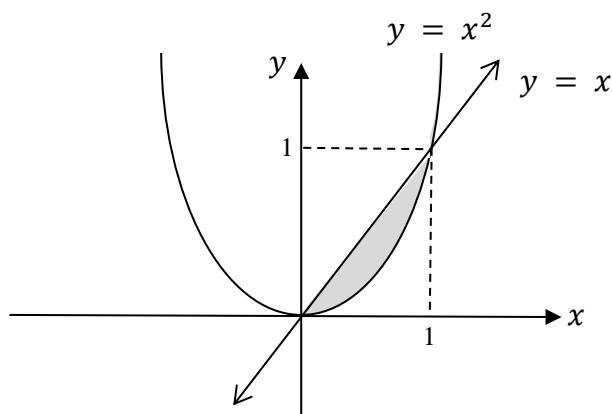
Basic	Level
Excellent	
Weak	
NR	

Basic	Level
Excellent	
Weak	
NR	

C5	Solve the differential equation $\frac{dy}{dx} = y^3 \sin x$ if the point (0, 1) lies on the curve.	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="background-color: #cccccc;">Basic</th> <th style="background-color: #cccccc;">Level</th> </tr> </thead> <tbody> <tr><td>Excellent</td><td></td></tr> <tr><td>Weak</td><td></td></tr> <tr><td>NR</td><td></td></tr> </tbody> </table>	Basic	Level	Excellent		Weak		NR	
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C6	Find the integral $I = \int \sqrt[3]{x} dx$.	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="background-color: #cccccc;">Basic</th> <th style="background-color: #cccccc;">Level</th> </tr> </thead> <tbody> <tr><td>Excellent</td><td></td></tr> <tr><td>Weak</td><td></td></tr> <tr><td>NR</td><td></td></tr> </tbody> </table>	Basic	Level	Excellent		Weak		NR	
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Excellent										
Weak										
NR										

C7

The shaded region in the diagram below is bounded by the graphs of the functions $y = x$ and $y = x^2$.

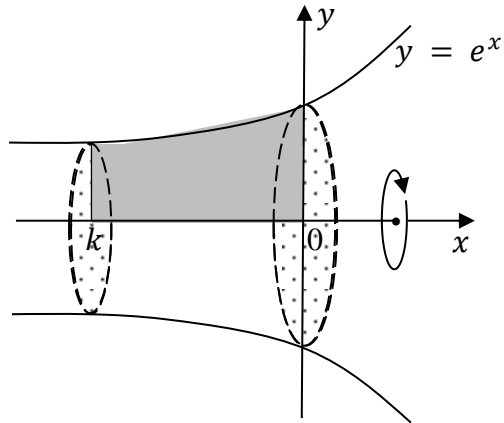


Calculate the volume of the solid formed when the shaded area shown is rotated through 360° about the y-axis.

Proficient	Level
Excellent	
Moderate	
Weak	
NR	

C8

A solid is formed when the shaded region in the diagram from $x = k$ to $x = 0$ is rotated 360° around the x -axis. If the volume of this solid is 0.491 units, calculate the value of k .
 [Hint: Use $V = \pi \int y^2 dx$]



Proficient	Level
Excellent	
Moderate	
Weak	
NR	

C10

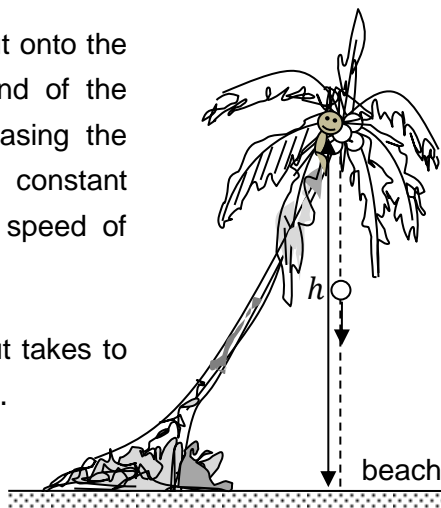
Radioactive carbon-14 (C^{14}) decays at a rate given by $\frac{dN}{dt} = kN$, where N is the amount of the radioactive substance present and k is a constant. It is known that half of any initial amount of the C^{14} will have decayed after 5750 years. Find the value of the constant k .

Proficient	Level
Excellent	
Moderate	
Weak	
NR	

C11

A boy up on a coconut tree drops a coconut onto the sand h metres below. He hears the sound of the impact on the sand 3 seconds after releasing the coconut, which falls under gravity with a constant acceleration of 10 m/s^2 . On this day the speed of sound is 40 m/s .

- i. Show that the time ' t ' that the coconut takes to hit the beach is given by $t = 3 - \frac{h}{40} \text{ s}$.



- ii. Hence determine the value of the height ' h '.

Advanced	Level
Exceed	
Excellent	
Moderate	
Low	
Weak	
NR	

C12

The rate of cooling $\frac{dT}{dt}$ of an object is directly proportional to the difference between the temperature T of the object and the temperature T_s of the surrounding medium, that is:

$$\frac{dT}{dt} = k(T - T_s)$$

where k is a constant. Initially at $t = 0$ hours, the temperature of the object is 120°C while the surrounding air is at the constant temperature of 20°C . After 1 hour the object has cooled to 70°C . How long will the object take to cool to 40°C ?

Advanced	Level
Exceed	
Excellent	
Moderate	
Low	
Weak	
NR	

