EDUCATIONAL QUALITY AND ASSESSMENT PROGRAMME [EQAP]



SOUTH PACIFIC FORM SEVEN CERTIFICATE [SPFSC]

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SOUTH PACIFIC FORM SEVEN CERTIFICATE MATHEMATICS with CALCULUS

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MATHEMATICS with CALCULUS

PREAMBLE AND RATIONALE

This prescription defines the requirements for the South Pacific Form Seven Certificate Mathematics (with Calculus) qualification.

Each of the student outcomes for the course is to be read in conjunction with the Indicator Notes where these are given for each outcome in this prescription.

Students also require knowledge and understanding of outcomes from the national Year 12 or Form 6 qualification or its equivalent, which are related to the specific outcomes of SPFSC Mathematics Calculus.

This prescription is derived from a revision of the South Pacific Board for Educational Assessment

(SPBEA) prescription and the New Zealand National Certificate of Educational Assessment (NCEA) Level 3 Mathematics (with Calculus) Achievement Standards as published by New Zealand Qualifications Authority (NZQA).

The course is designed for students who wish to undertake university studies in mathematics and other related fields.

COURSE AIM

Students undertaking this course are expected to:

- demonstrate mathematical skills, concepts and understanding of the Mathematical Processes, required for Measurement and Calculus, Geometry and Algebra at a level that is equivalent to that required for any Form 7 qualification including the NZ Universities Entrance, NCEA Level 3, University of the South Pacific (USP) Foundation, etc.
- apply these skills, concepts, and understanding to familiar and unfamiliar problems arising in real and simulated situations
- demonstrate the ability to select and use appropriate mathematical techniques for problem solving demonstrate the ability to reason logically and systematically
- demonstrate the ability to communicate mathematical ideas.

PREREQUISITES

Students taking this course are expected to have successfully completed the national Year 12 Senior Secondary Certificate Mathematics course or its equivalent.

GENERAL OBJECTIVES

In a range of meaningful contexts, students will be engaged in thinking mathematically. They will solve problems and model situations that require them to:

- 1. apply algebraic techniques to real and complex numbers.
- 2. use and manipulate trigonometric functions and expressions.
- 3. demonstrate knowledge of advanced concepts and techniques of differentiation.
- 4. demonstrate knowledge of advanced concepts and techniques of integration.
- 5. demonstrate knowledge of basic skills in algebra, differentiation and integration.

CONTENT COMPONENTS

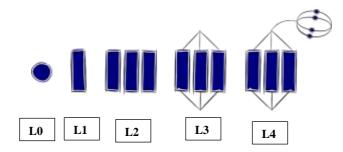
The content of the SPFSC Mathematics with Calculus course is organised under four strands and a number of sub-strands under each strand. These are outlined below:

				External /
Strand		Sub strand		Internal
Number	Strand Title	number	Sub-strand title	Assessment
1.0	Algebra	1.1	Algebra basic skills	External
		1.2	Polynomial and non-linear equation	
		1.3	Complex numbers	
2.0	Trigonometry	2.1	Trigonometry basic skills	Internal
		2.2	Trigonometric equations	External
		2.3	Trigonometric functions to solve problems	Internal
		2.4	Trigonometric identities	Internal
3.0	Differentiation	3.1	Differentiation basic skills	Internal
		3.2	Discontinuity and limits of functions	External
		3.3	Application of differentiation	External
		3.4	Differentiate by sketching to solve problems	Internal
4.0	Integration	4.1	Integration basic skills	Internal
		4.2	Use of variety of integration technique	Internal
		4.3	Volumes of solids of revolution	External
		4.4	Form and interpret solutions	External

UNPACKING LEARNING OUTCOMES

In this prescription, Learning Outcomes are stated at three levels of generality: Major Learning Outcomes (MLOs) are stated at the strand level, Key Learning Outcomes (KLOs) are stated at the sub-strand level, and Specific Learning Outcomes (SLOs) are unpacked from the Key Learning Outcomes. Each SLO is a combination of a cognitive skill and a specific content component. Each SLO is given a skill level, level 1 -4, and this skill level results from the categorisation of the cognitive skill that is embedded in the SLO using the SOLO taxonomy¹.

The SOLO taxonomy provides a simple, reliable and robust model for three levels of understanding – surface deep and conceptual (Biggs and Collis 1982).



At the **prestructural** level (L0) of understanding, the task is inappropriately attacked, and the student has missed the point or needs help to start. The next two levels, unistructural and multistructural are associated with bringing in information (surface understanding). At the **unistructural** level (L1), one aspect of the task is picked up, and student understanding is disconnected and limited. The jump to the multistructural

¹ Structure of Observed Learning Outcomes by Biggs and Collis (1982)

level is quantitative. At the **multistructural** level (L2), several aspects of the task are known but their relationships to each other and the whole are missed. The progression to relational and extended abstract outcomes is qualitative. At the **relational** level (L3), the aspects are linked and integrated, and contribute to a deeper and more coherent understanding of the whole. At the **extended abstract** level (L4), the new understanding at the relational level is re-thought at another conceptual level, looked at in a new way, and used as the basis for prediction, generalisation, reflection, or creation of new understanding (adapted from Hook and Mills 2011). [http://pamhook.com/solo-taxonomy/..]

The progression from Level 1 to Level 4 is exemplified in the progression from *define* \rightarrow *describe* \rightarrow *explain* \rightarrow *discuss* with each succeeding level indicating a *higher level of understanding*, as follows:

- **define** to state a basic definition of a concept [Unistructural or L1]
- **describe** to give the characteristics of, or give an account of, or provide annotated diagrams. [Multistructural or L2]
- **explain** to provide a reason for a relationship an event and its impact, a cause and an effect, as to *how* or *why* something occurs. [Relational or L3]
- **discuss** this means *linking ideas* (descriptions, explanations) to make generalisations or predictions or evaluations. It may involve relating, comparing, analysing, and justifying.
- solve/calculate/compute to carry out a series of algorithms to arrive at a solution [Multistructural (L2) or Relational (L3) or even Extended (L4) depending on the complexity of the algorithm]. If there are two 'loadings' in the calculations (a standard problem) then skill level would be L2, if three 'loadings' (a complex problem) then L3 and four loadings (a more complex problem) for L4.

LEARNING OUTCOMES

STRAND 1.0 ALGEBRA

Major Learning Outcome

Students are able to think mathematically and statistically and will be able to solve problems and model situations that require them to apply algebraic techniques to real and complex numbers.

SUB-STRAND 1.1 Algebra Basic Skills

Key Learning Outcome

Students are able to demonstrate basic algebra skills.

No.	Specific Learning Outcomes Students are able to:	Skill Level	SLO Code
1	simplify linear equations eliminating fractional terms	1	Cal1.1.1.1
2	solve two linear equations simultaneously	1	Cal1.1.1.2
3	solve linear inequations $\frac{x-3}{4} + 1 < \frac{x+1}{-3}$	1	Cal1.1.1.3
4	rearrange a formula to obtain the correct subject	1	Cal1.1.1.4
5	factorise quadratic equations	1	Cal1.1.1.5
6	solve quadratic equations by factorisation	1	Cal1.1.1.6
7	solve quadratic equations using quadratic formula	2	Cal1.1.2.9
8	Factorise a cubic function using factor theorem	<mark>2</mark>	Cal1.1.2.10
9	apply laws of indices to simplify exponential expressions	1	Cal1.1.1.9
10	solve straightforward exponential equations	1	Cal1.1.1.10
11	apply laws of logarithms to simplify logarithmic expressions	1	Cal1.1.1.11
12	solve straightforward logarithmic equations	1	Cal1.1.1.12
13	express a single algebraic fraction as a sum of its partial fractions where denominators of fractions are linear and or repeated	2	Cal1.1.2.1
14	express a single algebraic fraction as a sum of its partial fractions where denominators of fractions include a non-linear function	3	Cal1.1.3.1
15	divide a polynomial by $(x + a)$.	1	Cal1.1.1.13
16	find the remainder to a function for $x = a$ where $(x - a)$ is not a factor.	1	Call.1.1.14
17	use the Binomial Theorem to expand and simplify expressions of the form $(x + y)^x$	1	Call.1.1.14 Call.1.1.15
17	for $n = 3$ or 4.		
18	use the Binomial Theorem to find specific terms and/or their coefficients in expansions where n is greater than $\frac{4}{4}$.	2	Cal1.1.2.11
19	complete the square of reducible quadratics of the form $ax^2 + bx + c$.	1	Cal1.1.1.17
20	complete the square of quadratics of the form $ax^2 + bx + c$.	2	Cal1.1.2.3
21	solve straightforward surd equations and check solutions.	2	Cal1.1.2.4
22	prove a given mathematical statement is true by using the method of Mathematical Induction	2	Cal1.1.2.5
23	simplify sums, differences, and products of surds	1	Cal1.1.1.18
24	simplify quotients of surds (including rationalizing).	2	Cal1.1.2.6
25	simplify sums, differences, and products of complex numbers expressed in rectangular form.	1	Cal1.1.1.19
26	simplify quotients of complex numbers expressed in rectangular form.	2	Cal1.1.2.7
27	convert between rectangular $(a + ib)$ and polar $(rcis \theta)$ forms.	1	Cal1.1.1.20
28	use Argand diagrams to represent complex number in the forms $a + ib$, $rcis \theta$	1	Cal1.1.1.21
29	use Argand diagrams to represent complex number in the forms $a + ib$, $rcis \theta$	2	Cal1.1.2.8

Key Learning Outcome Students are able to form a

nd use polynomial and non-linear equation

No.	Specific Learning Outcomes Students are able to:	Skill Level	SLO Code
1	determine the variables in a contextual problem	1	Cal1.2.1.1
2	form equations based on a contextual problem	2	Cal1.2.2.1
3	solve problems by translating word problems into mathematical expressions	3	Cal1.2.3.1
4	solve problems by translating word problems into mathematical expressions	4	Cal1.2.4.1
5	analyse the existence of solutions in the context of the situation.	3	Cal1.2.3.2
6	analyse the existence of solutions in the context of the situation.	4	Cal1.2.4.2
7	solve linear equations	1	Cal1.2.1.2
8	solve simultaneous equations	3	Cal1.2.3.3
9	solve exponential equations	2	Cal1.2.2.2
10	solve exponential equations	3	Cal1.2.3.4
11	solve exponential equations	4	Cal1.2.4.3
12	solve logarithmic equations	2	Cal1.2.2.3
13	solve logarithmic equations	3	Cal1.2.3.5
14	solve rational equations	3	Cal1.2.3.6
15	solve surd equations	2	Cal1.2.2.4
16	solve surd equations	3	Cal1.2.3.7
17	solve hyperbolic equations	4	Cal1.2.4.4
18	use Remainder and Factor theorems.	2	Cal1.2.2.5
19	use Remainder and Factor theorems.	3	Cal1.2.3.8
20	use Remainder and Factor theorems.	4	Cal1.2.4.5

SUB-STRAND 1.3 Complex numbers

No.	Specific Learning Outcomes	Skill	SLO Code
110.	Students are able to:	Level	SLO Cou
1	use and manipulate simple surds	1	Cal1.3.1.1
2	use and manipulate surds and other irrational numbers.	2	Cal1.3.2.1
3	interpret and use graphical representations of complex numbers, using polar and rectangular form on an Argand diagram.	2	Cal1.3.2.2
4	interpret and use graphical representations of complex numbers, using polar and rectangular form on an Argand diagram.	3	Cal1.3.3.1
5	interpret and use graphical representations of complex numbers, using polar and rectangular form on an Argand diagram.	4	Cal1.3.4.1
6	use and manipulate complex numbers, making links with their graphical representation.	2	Cal1.3.2.3
7	use and manipulate complex numbers, making links with their graphical representation.	3	Cal1.3.3.2
8	use and manipulate complex numbers, making links with their graphical representation.	4	Cal1.3.4.2
9	find roots over the complex number system for polynomial equations with real coefficients, including the special case of the nth roots of a that come from solving equations of the form $z^n = a$, making links with their graphs.	2	Cal1.3.2.4

10	find roots over the complex number system for polynomial equations with real coefficients, including the special case of the nth roots of a that come from solving equations of the form $z^n = a$, making links with their graphs.	3	Cal1.3.3.3
11	find roots over the complex number system for polynomial equations with real coefficients, including the special case of the nth roots of a that come from solving equations of the form $z^n = a$, making links with their graphs.	4	Cal1.3.4.3
13	find roots of equations of the form $z^n = a + ib$, $z^n = rcis \theta$ where n is a positive integer (includes the use of de Moivre's theorem to solve equations and Argand diagrams to represent relationships between solutions)	3	Cal1.3.3.4
14	find roots of equations of the form $z^n = a + ib$, $z^n = rcis \theta$ where n is a positive integer (includes the use of de Moivre's theorem to solve equations and Argand diagrams to represent relationships between solutions)	4	Cal1.3.4.4

Explanatory Notes for possible context elaborations

- have an understanding of the need and relevance of different number systems leading to the development of *i* and complex numbers
- carry out mathematical operations on expressions incorporating i, eg. Powers of i
- convert between rectangular and polar form
- carry out operations on $rcis\theta$
- use de Moivre's theorem to solve equations of the form $z^{n=}a$ and display these solutions

STRAND 2.0 TRIGNONOMETRY

Major Learning Outcome

Students are able to think mathematically by solving problems and model situations that require them to use and manipulate trigonometric functions and expressions.

SUB-STRAND 2.1 Trigonometry Basic Skills

Key I	Key Learning Outcome				
Stude	Students are able to use the basic trigonometry skills to solve simple problems				
No.	Specific Learning Outcomes Students are able to:	Skill Level	SLO Code		
1	evaluate reciprocal and/or inverse relationships	1	Cal2.1.1.1		
2	find exact values of trig expressions using special triangles	1	Cal2.1.1.2		
3	solve problems that involve manipulating trig expressions using trigonometric forms of Pythagoras theorem	2	Cal2.1.2.1		
4	solve problems that involve manipulating trig expressions using compound angle formula	2	Cal2.1.2.2		
5	solve problems that involve manipulating trig expressions using Double angle formula	2	Cal2.1.2.3		
6	solve problems that involve manipulating trig expressions using sums and products	2	Cal2.1.2.4		
7	display the graphs of inverse and/or reciprocal trigonometric functions (sin x, cos x, tan x) with x in radians or degrees showing the main features of the graphs. Consideration of restrictions on the domain of a function so that its inverse is also a function is required.	2	Cal2.1.2.5		

SUB-STRAND 2.2: Trigonometric functions to solve problems

Key I	Key Learning Outcome				
Stude	Students are able to select and form a trig function to solve problems.				
No.	Specific Learning Outcomes Students are able to:	Skill Level	SLO Code		
1	solve straightforward problems with models involving trigonometric functions of the form; $y = A \sin B (x + C) + D$ where C or D may be zero	3	Cal2.2.3.3		
2	solve straightforward problems with models involving trigonometric functions of the form; $y = A \cos B (x + C) + D$ where C or D may be zero	3	Cal2.2.3.4		
3	solve straightforward problems with models involving trigonometric functions of the form; $y = A \tan B (x + C) + D$ where C or D may be zero	3	Cal2.2.3.5		
4	form an equation for a model and use the model to solve problems; $y = A \sin B (x + C) + D$	3	Cal2.2.3.1		
5	form an equation for a model and use the model to solve problems $y = A \cos B (x + C) + D$	3	Cal2.2.3.2		

Explanatory Notes

For trigonometry functions involves C or D may be zero. Solutions of the problems may require knowledge of amplitude, period and frequency.

SUB-STRAND 2.3: Prove Trigonometric identities

·	Key Learning Outcome				
Students are able to prove trigonometric identities using various formulae					
No.	Specific Learning Outcomes Students are able to:	Skill Level	SLO Code		
1	prove trigonometry identities (involving reciprocal relationships), by solving equations to provide a general solution or a solution within a specified domain.	2	Cal2.3.2.1		
2	prove trigonometry identities (involving Pythagorean identities), by solving equations to provide a general solution or a solution within a specified domain.	2	Cal2.3.2.2		
3	prove trigonometry identities (involving compound angle formulae), by solving equations to provide a general solution or a solution within a specified domain.	3	Cal2.3.3.1		
4	prove trigonometry identities (involving double angle formulae), by solving equations to provide a general solution or a solution within a specified domain.	3	Cal2.3.3.2		
5	prove trigonometry identities (involving sum and product formulae), by solving equations to provide a general solution or a solution within a specified domain.	3	Cal2.3.3.3		

STRAND 3.0 DIFFERENTIATION

Major Learning Outcome

Students are able to solve problems and model situations that require them to demonstrate knowledge of advanced concepts and techniques of differentiation.

SUB-STRAND 3.1 Differentiation Basic Skills

Key Learning Outcome

Students are able to use basic differentiation skills to solve simple problems

No.	Specific Learning Outcomes Students are able to:	Skill Level	SLO Code
1	Identify features of a piecewise function of f(a)	1	Cal3.1.1.1
2	Identify features of a piecewise function if the function is discontinuous	1	Cal3.1.1.2
3	Identify features of a piecewise function if the limit exits	1	Cal3.1.1.3
4	Identify features of a piecewise function if the function is differentiable	1	Cal3.1.1.4
5	Find limits of piecewise functions	1	Cal3.1.1.5
6	Use the first principles to differentiate a function (only for polynomials of degree ≤ 3) using $\lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right)$	3	Cal3.1.3.1
7	Differentiate sums of functions	2	Cal3.1.2.1
8	Differentiate quotients	2	Cal3.1.2.2
9	Differentiate composite functions (chain rule)	2	Cal3.1.2.3
10	Differentiate parametric functions	2	Cal3.1.2.4
11	Finding the second derivatives	2	Cal3.1.2.5
12	Solve problems by finding the maxima or minima with proof for polynomial and rational functions)	3	Cal3.1.3.2
13	Differentiate to find the points of inflection	3	Cal3.1.3.3

SUB-STRAND 3.2 Discontinuity and limits of functions

Key Learning Outcome

Students are able to identify discontinuity and limits of functions

No.	Specific Learning Outcomes Students are able to:		SLO Code
1	link features of graphs with the limiting behaviour of functions	3	Cal3.2.3.1
2	link features of graphs with the limiting behaviour of functions	4	Cal3.2.4.1
3	use limiting features of functions to sketch graphs	3	Cal3.2.3.2
4	use limiting features of functions to sketch complex graphs	4	Cal3.2.4.2
5	find limits algebraically, graphically and numerically by considering behaviour as x approaches a specific value from above and below	2	Cal3.2.2.1
6	find limits algebraically, graphically and numerically by considering behaviour as x approaches $+\infty$ or $-\infty$	2	Cal3.2.2.2
7	demonstrates understanding of continuity at a point (the limit as x tends to a of $f(x) = f(a)$).	4	Cal3.2.4.3
8	identifies discontinuities graphically or algebraically	2	Cal3.2.2.3
9	link informally concepts of continuity and differentiability	4	Cal3.2.4.4

SUB-STRAND 3.3 Application of differentiation technique

Key I	Learning Outcome		
Stude	nts are able to apply a variety of differentiation technique to functions and	relations	5.
No.	Specific Learning Outcomes Students are able to:	Skill Level	SLO Code
1	use implicit differentiation or parametric differentiation to differentiate functions (functions include: polynomials, Ae^{px} , $ln(ax +b)$, trigonometric functions, power functions such as Ap^{x})	3	Cal3.3.3.1
2	use implicit differentiation or parametric differentiation to differentiate functions (functions include: polynomials, Ae^{px} , $ln(ax +b)$, trigonometric functions, power functions such as Ap^{x})	4	Cal3.3.4.1
3	use a range of differentiation techniques to find an equation of a tangent or normal	3	Cal3.3.3.2
4	use a range of differentiation techniques: Optimization (maxima, minima or point of inflection)	2	Cal3.3.2.2
5	use a range of differentiation techniques: Optimization (maxima, minima or point of inflection)	3	Cal3.3.3.3
6	use a range of differentiation techniques: Optimization (maxima, minima or point of inflection)	4	Cal3.3.4.2
7	use differentiation to find the rates of change	2	Cal3.3.2.3
8	use differentiation to find the rates of change	3	Cal3.3.3.4
9	use differentiation to find the rates of change	4	Cal3.3.4.3
10	apply differentiation to problems in kinematics	2	Cal3.3.2.4
11	apply differentiation to problems in kinematics	3	Cal3.3.3.5
12	apply differentiation to problems in kinematics	4	Cal3.3.4.4

SUB-STRAND 3.4 Differentiate to solve problems by sketching graphs

Key Learning Outcome

Students are able to use differentiation to solve problems involving sketching graphs of polynomials and derivatives.

No.	. Specific Learning Outcomes Students are able to:		SLO Code
1	sketch graphs of polynomials of degree \geq 3 by using differentiation to identify features such as turning points, points of inflection and concavity	3	Cal3.4.3.1
2	sketch the graph of a derived function from a given graph	3	Cal3.4.3.2
3	identify features of given graphs (selection from limits, differentiability, discontinuity, gradients, concavity, turning points, points of inflection)	2	Cal3.4.2.1
4	sketch graphs/derivatives to demonstrate knowledge of the above features.	3	Cal3.4.3.3

STRAND 4.0: INTEGRATION

Major Learning Outcome

Students are able to solve problems and model situations that require them to demonstrate knowledge of advanced concepts and techniques of integration.

SUB-STRAND 4.1: Integration Basic Skills

Key Learning Outcome

Students are able to use basic integration skills to solve simple problems.

No.	Specific Learning Outcomes Students are able to:	Skill Level	SLO Code
1	integrate functions of ax ⁿ , for n real	1	Cal4.1.1.1
2	integrate polynomial functions	1	Cal4.1.1.2

3	integrate exponential functions of the form Ae $f(x)$, where $f(x) = ax + b$	1	Cal4.1.1.3
4	integrate trigonometric functions, including products	2	Cal4.1.2.1
5	integrate rational functions of the type $y = \frac{ax+b}{cx+d}$	2	Cal4.1.2.2
6	find definite integrals	2	Cal4.1.2.3
7	use integration to solve straightforward problems involving areas	2	Cal4.1.2.4
8	use anti-differentiation to form and solve differential equations of first and second order with variables easily separated	3	Cal4.1.3.1

SUB-STRAND 4.2: Use of variety of integration technique

Key Learning Outcome

Students are able to apply a variety of integration and anti-differentiation techniques to functions and relations, using both analytical and numerical methods.

No.	Specific Learning Outcomes Students are able to:	Skill Level	SLO Code
1	Use and apply numerical methods to find areas under the curves functions include polynomials, $x^n, n \in R$, e^x , $\sin(x)$, $\cos(x)$, f'(g(x)).g'(x), $\frac{f'(x)}{f(x)}$	3	Cal4.2.3.1
2	Use and apply numerical methods to find areas between the curves including polynomials, $x^n, n \in R$, e^x , $sin(x)$, $cos(x)$, f'(g(x)).g'(x), $\frac{f'(x)}{f(x)}$	4	Cal4.2.4.1
3	Find area for odd, even and periodic functions	3	Cal4.2.3.2
4	Find area of kinematic equations	3	Cal4.2.3.3

Explanatory Notes:

- integration by substitution restricted only to those involving simple algebraic substitutions
- integration using partial fraction is to be used as an alternative method of integration for students to use where appropriate
- integration by partial fraction to apply only to functions that can also be solved by substitution and of the forms:

(i)
$$\frac{(ax+b)}{(cx+d)^2}$$

(ii)
$$\frac{(ax+b)}{(cx^2+dx+e)}$$
, where (cx ²+dx+e) can easily be factorized.

SUB-STRAND 4.3: Integration: Volumes of solids of revolution

Key Learning Outcome

Students are able to use integration to find the volumes of solids of revolution.

No.	Specific Learning Outcomes Students are able to:	Skill Level	SLO Code
1	use integration to find volumes of revolution of simple functions such as: $(ax^p, where p \in rational numbers, exponential, trigonometric functions, around the x and y-axis)$	3	Cal4.3.3.1
2	use integration to find volumes of revolution of simple functions such as: $(ax^p, where p \in rational numbers, exponential, trigonometric functions, around the x and y-axis)$	4	Cal4.3.4.1
3	use of integration to find volumes of revolution of simple functions and around an axis parallel to the x or y axis	3	Cal4.3.3.2

4	use of integration to find volumes of revolution of simple functions and around an axis parallel to the x or y axis	4	Cal4.3.4.2
5	use of integration to find volumes of revolution of area between two functions	3	Cal4.3.3.3
6	use of integration to find volumes of revolution of area between two functions to be based on any of the following; linear, trigonometric, quadratic, exponential, logarithmic, hyperbola	4	Cal4.3.4.3

Explanatory notes

- 1. Simple functions do not involve sums, products, or combinations of the standard functions other than expressions such as $ax^2 + bx + c$ or A e^{*ax*}.
- 2. Complex functions are those that do involve sums, products and combinations of the standard functions.

SUB-STRAND 4.4: Form and interpret solutions

Key Learning Outcome

Students are able to form differential equations and interpret the solutions

No.	Specific Learning Outcomes Students are able to:	Skill Level	SLO Code
1	use anti-differentiation to form and solve differential equations with the rate of change directly or inversely proportional to the variable of interest (functions include: polynomials, e^x , $\ln(x)$, trig functions, x^n , $n \in R$)	3	Cal4.4.3.1
2	use anti-differentiation to form and solve differential equations with the rate of change directly or inversely proportional to the variable of interest (functions include: polynomials, e^x , $\ln(x)$, trig functions, x^n , $n \in R$)	4	Cal4.4.4.1
3	use anti-differentiation to form and solve differential equations of first and second order with variables easily separated (functions include: polynomials, e^x , $\ln(x)$, trig functions, x^n , $n \in R$)	2	Cal4.4.2.1
4	use anti-differentiation to form and solve differential equations of first and second order with variables easily separated (functions include: polynomials, e^x , $\ln(x)$, trig functions, x^n , $n \in R$)	3	Cal4.4.3.2
5	use anti-differentiation to form and solve differential equations of first and second order with variables easily separated (functions include: polynomials, e^x , $\ln(x)$, trig functions, x^n , $n \in R$)	4	Cal4.4.4.2
6	form differential equations of the type with the rate of change directly proportional to the variable of interest (functions include: polynomials, e^x , $\ln(x)$, trig functions, x^n , $n \in R$)	2	Cal4.4.2.2
7	solve differential equations of the first order type with variables easily separated	<mark>3</mark>	Cal4.4.3.3

Explanatory notes

Student applies boundary or initial conditions to solutions of differential equations. Student distinguishes between families of solutions and exact solutions using given boundary or initial conditions and interprets these solutions.

Possible context elaborations

$$\frac{dy}{dx} = ysinx$$
$$\frac{d^2y}{dx^2} = polynomial$$

$$\frac{dy}{dx} = e^x$$
$$\frac{dy}{dx} = \frac{1}{x}$$

Situations which students will be expected to form a model for and solve include:

Growth and decay $\frac{dy}{dx} = ky$

Newton's law of cooling $\frac{dT}{dt} = k(T - T_0)$

Kinematics

Explanatory Notes

Once solutions to differential equations are available students are required to:

- make predictions about a situation based on the solution
- make further calculations with particular solution of differential equation to check accuracy of model
- comment on limitations of their model in relation to uncontrolled factors, etc.

ASSESSMENT

Assessment in Mathematics Calculus course is in two parts:

- 1. External Assessment (EA) is worth 70%.
- 2. Internal Assessment (IA) is worth 30%

The principal, or his/her nominee, will certify that the prescription requirements have been fulfilled.

Suggested Teaching Time and Weightings

	Outcomes	External / internal	Approximate weighting	Suggested Time
1	Algebra: Students will be able to apply algebraic techniques to real and complex numbers.	External	20%	7 weeks
2	Trigonometry: Students will be able to use trigonometric functions as well as apply its relationship to solve problems.	External	10%	6 weeks
	Demonstrate basic skills and use trigonometric functions to solve simple problems. Prove trigonometric identities	Internal	15%	
3	Differentiation: Students will be able to demonstrate knowledge of advanced concepts and techniques of differentiation.	External	20%	8 weeks
	Demonstrate basic skills of differentiation and solve problems by sketching.	Internal	8%	
4	Integration: Students will be able to demonstrate knowledge of advanced concepts and techniques of integration.	External	20%	7 weeks
	Demonstrate basic skills and use a variety of integration technique to solve problems	Internal	7%	
	Total		100%	28 weeks

Assessment Blueprint

	Assessment	SF	SKILL LEVEL/ SCORE			
	Туре	Level 1	Level 2	Level 3	Level 4	
Strand		SS = 1	SS = 2	SS = 3	SS = 4	Weight
1. Algebra	EA					20
	EA					10
2. Trigonometry	IA					15
	EA					20
3. Differentiation	IA					8
4 Integration	EA					20
4. Integration	IA					7
Total number of items		20	15	10	5	
Total skill score		20	30	30	20	100

External Assessment

The prescription contains the Major Learning Outcome (MLO), the Key Learning Outcome (MLO) and the Specific Learning Outcomes. Candidates would be expected to know, understand and apply any mathematics related to the specific outcomes. Examination questions, which require specific mathematical knowledge, will be based on these specific outcomes.

The external assessment will contribute 70% of the final grade. The table below gives an approximate weighting for each outcome in the examination. This is based on the suggested time, based on a 5 hour week, to be spent on each topic. The examination paper will contain four sections, and will be given **3** hours for completion during the examination period.

	Topic/Section	Outcomes	Percentage per section	Suggested Exam Time
		In a range of meaningful contexts, students are able to solve problems and model situations that require them to:		
1	Algebra:	Apply algebraic techniques to real and complex numbers.	20%	50 mins
2	Trigonometry	Use trigonometric functions as well as apply its relationship to solve problems.	10%	30mins
3	Differentiation:	Demonstrate knowledge of advanced concepts and techniques of differentiation.	20%	50 mins
4	Integration:	Demonstrate knowledge of advanced concepts and techniques of integration.	20%	50 mins

Note: There is a 10 minute reading/checking time factored into the examination.

A standard list of formulae will be incorporated in the examination paper. Additional formulae may be provided where appropriate. A copy of the formulae list is attached as an appendix to this prescription.

The use of calculators and computers is an integral part of mathematics and candidates will be required to supply their own calculators for use in the examination. It is recommended that students purchase a graphics calculator. The use of calculators must conform to the Board's Rules and Procedures governing electronic calculators. A copy of the Board's Rules and Procedures governing the use of electronic calculators in all of the Board's examinations is given in the appendix to this prescription.

At the start of the year, candidates should be given a copy of the assessment statement to be used. The assessment statement should clearly indicate how candidates will be assessed and the standard of work required by candidates in order to achieve a particular level.

Internal Assessment

The two IA tasks are both Common Assessment Tasks (CATs), and they assess the specific learning outcomes (SLOs) indicated. The task instructions for each task are provided below.

IA TASK 1 Instructions

Task 1 is worth 15%. The task is based on Strand 2: Trigonometry

The key learning outcome that is assessed in this task is the student's ability to demonstrate basic as well as more complex skills in trigonometry, the use of trigonometric functions to solve simple problems as well as proving trigonometric identities.

The Specific Learning Outcomes that are the focus of Task 1 are listed and shaded under Strand 2. This task is a Common Assessment Task from EQAP, so teachers will be responsible for the implementation and scoring of this task within the specification provided below. The scoring rubric will also be provided from EQAP.

Task Specifications:

- 1. The task is a group task, and the teacher is to allocate students into their respective **groups of two or three**. A group of 4 may be necessary in some situations but it will end up being a large group, so teachers are cautioned about organising students into groups of 4s.
- The SLOs that are to be targeted in this Task are highlighted in the three sub-strands of Strand 2. Teachers are hereby advised to create the items in the task to suit those learning outcomes. There are 15 items in the task to meet the 15 highlighted outcomes. This is very important for teachers to note.
- 3. There will be **two parallel versions of Task 1** and teachers are to give one version only to each group. In this way, not all groups will be working on exactly the same task.
- 4. Students are to be given time in-class to work on their task. At the end of each class time, the teacher is to collect the students' work and return the same to them to work on during the next class time. In this way, the efforts of students is more meaningful and fairer. Teachers are to make independent decisions about allocation of scores for students who are absent during lessons in which groups are working on their tasks.
- 5. It is expected that students spend about 5 6 lessons on this task.
- 6. Teachers are to refer to the scoring rubric provided from EQAP and follow this closely when scoring students' responses to Task 1.

IA TASK 2 Instructions

Task 2 is worth 15%. The task is based on Strands 3 and 4.

Strand 3: Differentiation (8%)

Demonstrate basic skills of differentiation and solve problems by sketching.

Strand 4: Integration (7%)

Demonstrate basic skills and use a variety of integration technique to solve problems

This is to encourage students to master the basics so that they can move forward and challenge themselves to reach higher standards overall. The philosophy behind this is that if students master the basic skills, they are more likely to be able to reach a deeper level of understanding of the topics.

Task 2 is again a Common Assessment Task. This task is a **common test** given to students at the same time, under strict test conditions. The test and the scoring rubric will be provided from EQAP and have the following features:

- 1. The common test is based on Strands 3 (Differentiation) and 4 (Integration);
- 2. The learning outcomes to be assessed in this test are highlighted in the sub-strands of Sub-strands 3.1, 3.4, 4.1, and 4.2;
- 3. There are 14 highlighted learning outcomes, so it is expected that the test is to have 14 items;
- 4. Students are to be given about **one hour** to complete the test. More time may be given for students who may need more time;
- 5. Details for the task are to be included in the IA Program that the teacher is to send to EQAP for approval;
- 6. The exact day and week of the test is suggested in the 'Calendar', 'Verification Manual' and 'IA Due Dates' from EQAP, and this has accommodated the coverage of strands 3 and 4; and
- 7. There is only one test covering both Differentiation and Integration.

At the beginning of each year, each school presenting students for the South Pacific Form Seven Certificate Mathematics (with Calculus) assessment must complete an Internal Assessment Program and forward to EQAP by the date set down by the Director. The form must clearly indicate the time (start and completion date) for each task. In the case of the CATs, the start and end date is the same as it is a test.

It is recommended that at the start of the year students are given copies of the learning outcomes and the task description or the IA Programme. The IA Program and copies of all assessment tasks and assessment schedules used, as well as a sample of candidate responses to all internal assessment work undertaken, must be available for verification during the IA verification visit.

The moderation of Internal Assessment will be done in accordance with EQAP policy as specified from time to time.

APPENDICES

Appendix 1: TASK 1 Scoring Schedule / Scoring Rubric

Task 1: TRIGONOMETRY (15%)

#	SLO Code	Skill Score	Evidence of correct		Student Res	sponse Level	
"	520 0000	~	response	4	3	2	1
1	Cal2.1.1.2	1					
2	Cal2.1.2.1	2					
3	Cal2.1.2.2	2					
4	Cal2.1.2.3	2					
5	Cal2.1.2.4	2					
6	Cal2.2.2.1	2					
7	Cal2.2.2.2	2					
8	Cal2.2.2.3	2					
9	Cal2.2.3.1	3					
10	Cal2.2.3.2	3					
11	Cal2.3.2.1	2					
12	Cal2.3.2.2	2					
13	Cal2.3.3.1	3					
14	Cal2.3.3.2	3					
15	Cal2.3.3.3	3					

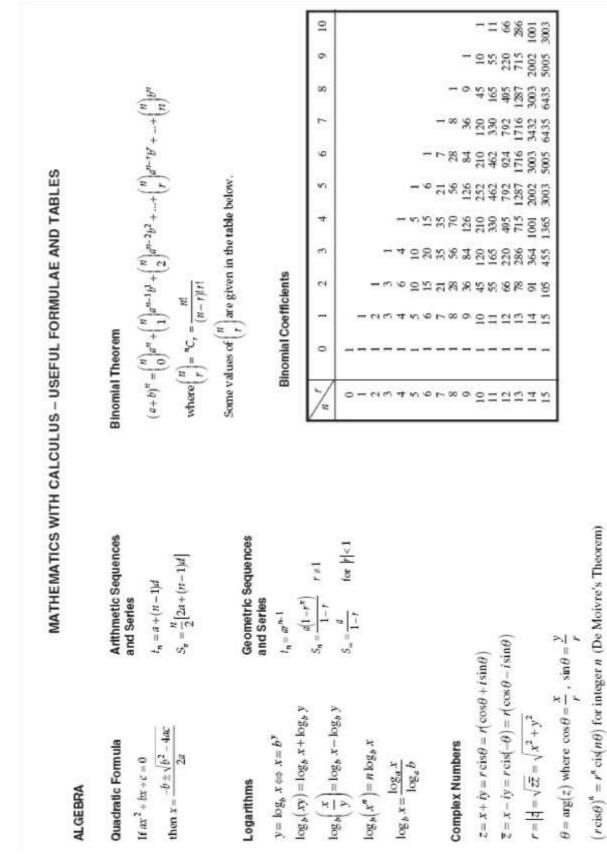
<u>NB</u>: 1. For classroom lesson activities prior to the CAT, teachers are to complete the scoring rubric above by filling in the appropriate descriptors for each level of student responses.

Appendix 2: TASK 2 Scoring Schedule / Scoring Rubric

TASK 2: DIFFERENTIATION and INTEGRATION (15%)

#	SLO Code	Skill Score	Evidence of correct		Student Res	ponse Level	
	blo code	Score	response	4	3	2	1
1	Cal3.1.1.3	1					
2	Cal3.1.2.1	2					
3	Cal3.1.2.2	2					
4	Cal3.1.2.3	2					
5	Cal3.1.2.4	2					
6	Cal3.1.3.3	3					
7	Cal3.4.3.1	3					
8	Cal3.4.3.3	3					
9	Cal4.1.1.2	1					
10	Cal4.1.1.3	1					
11	Cal4.1.2.1	2					
12	Cal4.1.2.3	2					
13	Cal4.1.2.4	2					
14	Cal4.2.4.1	4					

<u>NB</u>: 1. For classroom lesson activities prior to the CAT, teachers are to complete the scoring rubric above by filling in the appropriate descriptors for each level of student responses.



Appendix 3: List of useful formulae and tables

CALCULUS		COORDINATE GEOMETRY	MEASUREMENT
Differentiation	Integration	Straight Line	Triangle
y <u>dy</u> dx	y Jy dt	Gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$	$area = \frac{1}{2}ab \sin C$
	$k = \frac{k - c}{x^{n+c}}$	Equation $y - y_1 = m(x - x_1)$ Distance $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$	Trapezium area = $\frac{1}{2}(a+b)h$
e ^{ett} de ^{ett} sin x cos x cos x - sin x tan x sec ² x		Antopour $m = \left(\frac{2}{2}, \frac{1}{2}\right)$ Circle $x^2 + y^2 = r^2$ or $\left(r\cos\theta, r\sin\theta\right)$ be control 0.0 and reduce.	Circle circumference = $2\pi r$ area = πr^2
sec.x scc.rtun.r conc.x -conc.rcot.r cot.x -coseC ² .x	$\frac{f'(x)}{f(x)}$ $\ln[f(x)] + c$	$(x-a)^2 + (y-b)^2 = r^2$ has centre (a,b) and radius r	Sector
Product Rule $(fg) = f.g' + g.f'$		Parabola $y^2 = 4ax$ or $\left(at^2, 2at\right)$	area = $\frac{2}{2}r^{+}\theta$ arc length = $r\theta$
or if $y = uv$ then $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$		Ellipse $\frac{x^2}{x^2} + \frac{y^2}{y^2} = 1$ or $(a\cos\theta, b\sin\theta)$	Cylinder volume = $\pi r^2 h$ curved surface area = $2\pi rh$
Quotient Rule $\begin{pmatrix} f \\ g \end{pmatrix} = \frac{g \cdot f - f \cdot g'}{g^2}$ or if $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{v \frac{dx}{dx} - u \frac{dv}{dx}}{v^2}$	Composite Function or Chain Rule (f(g))' = f'(g)g' or if $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$	Hyperbola Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ or $(a \sec \theta, b \tan \theta)$ asymptotes $y = \pm \frac{b}{a} x$	Cons volume $= \frac{1}{3}\pi r^2 h$ curved surface a rea $= \pi rl$ where l = slant height Sphere volume $= \frac{4}{3}\pi r^3$

NUMERICAL METHODS Trapezium Rule $\int_{a}^{b} f(x) dx = \frac{1}{2} i h \Big[y_0 + y_e + 2 \Big(y_1 + y_2 + \dots + y_{e-1} \Big) \Big]$ where $h = \frac{b-a}{n}$ and $y_r = f(x_r)$ where $h = \frac{1}{2} i h \Big[y_0 + y_a + 4 \Big(y_1 + y_3 + \dots + y_{e-1} \Big) + 2 \Big(y_2 + y_4 + \dots + y_{n-2} \Big) \Big]$ Simpson's Rule $\int_{a}^{b} f(x) dx = \frac{1}{2} i h \Big[y_0 + y_a + 4 \Big(y_1 + y_3 + \dots + y_{n-1} \Big) + 2 \Big(y_2 + y_4 + \dots + y_{n-2} \Big) \Big]$	where $h = \frac{b-a}{n}$, $y_r = f(x_r)$ and <i>n</i> is even.	
Double Angles $\sin 2A = 2\sin A \cos A$ $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$ $\cos 2A = \cos^2 A - \sin^2 A$ $= 2\cos^2 A - 1$ $= 1-2\sin^2 A$	Products 2 sin A cos B = sin(A + B) + sin(A - B) 2 cos A sin B = sin(A + B) - sin(A - B) 2 cos A cos B = cos(A + B) + cos(A - B) 2 sin A sin B = cos(A - B) - cos(A + B) 2 sin C + sin D = 2 sin $\frac{C + D}{2}$ cos(A + B) sin C + sin D = 2 sin $\frac{C + D}{2}$ cos(A + B) sin C - sin D = 2 cos $\frac{C + D}{2}$ cos(A + B) cos C + cos D = 2 cos $\frac{C + D}{2}$ cos $\frac{C - D}{2}$ cos C - cos D = -2 sin $\frac{C + D}{2}$ sin $\frac{C - D}{2}$	General Solutions If $\sin \theta = \sin \alpha$ then $\theta = n\pi + (-1)^{\pi} \alpha$ If $\cos \theta = \cos \alpha$ then $\theta = 2n\pi \pm \alpha$ If $\tan \theta = \tan \alpha$ then $\theta = n\pi + \alpha$ where <i>n</i> is any integer
TRIGONOMETRY $cosec \theta = \frac{1}{\sin \theta}$ $sec \theta = \frac{1}{\sin \theta}$ $cot \theta = \frac{1}{\tan \theta}$ $tan \theta = \frac{2}{\cos \theta}$ $tan \theta = \frac{2}{\cos \theta}$	Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ Sin A = $\frac{b}{\sin C}$ Costne Rule $c^2 = a^2 + b^2 - 2ab \cos C$ $c^2 = a^2 + b^2 - 2ab \cos C$ Identities $c^2 = a^2 + b^2 - 2ab \cos C$ $c^2 = a^2 + b^2 - 2ab \cos C$ $c^2 = a^2 + b^2 - 2ab \cos C$ $c^2 = a^2 + b^2 - 2ab \cos C$	Compound Angles $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

MAC – IA

SOUTH PACIFIC FORM SEVEN CERTIFICATE

Internal Assessment Summary Form MATHEMATICS WITH CALCULUS

Country: _____ School: _____

Task	Task Description/Focus	Start Date	End Date	Weighting
1. Trigonometry				15%
				15%
2. Differentiation &				
Integration				
				200/
	Total			30%

Note: 1. Be specific about dates, not just Week 3 Term 1, etc.

- 2. Assessment Schedules/Scoring Rubrics for the tasks will be provided by EQAP. Teachers must use these when scoring students' work.
 - 3. All IA Score Capture Sheets will be provided by EQAP to schools.

Teacher's Name and Signature:	Date:
Principal's Name and Signature:	Date:

A full IA program is to be submitted together with this IA Summary Form.

Appendix 5: Electronic calculators in examinations

Written or printed materials (including information and routines stored in the programmable memory of calculators) are prohibited.

Due to rapid changes in calculator technology, the Board will regularly review its policy on the use of calculators in examinations. However, every effort is made to ensure that schools are given adequate notice of policy changes.

The following policy aims to compromise between rewarding the appropriate use of technology while giving consideration to associated equity issues. The Board has a responsibility to ensure fairness and equity to all candidates.

Examination setters are aware of calculator technology, and take calculator capability into account in the design of examination questions and marking schedules.

General Policy

The EQAP *Assessment and Certification Rules and Procedures for Secondary Schools* allow candidates to use a calculator in any of its examinations provided that the calculator is silent, hand-held, non-printing and contains its own power source. However, calculators may not be used to pass information to other candidates, bring information into the examination, or as a dictionary/translator.

EQAP encourages examiners to set papers that examine understanding of concepts in such a way that the use of sophisticated calculators is not a significant advantage. Examination questions may require details of working steps to be shown to ensure that candidates understand the key concepts being assessed.

EQAP's policy on calculators in examinations allows the legitimate use of most types of calculator, including graphical and programmable calculators. The intention of the policy is to support the directions of curriculum development and encourage the appropriate use of calculator technology. The policy does not allow the use of calculators to contravene other examination rules and procedures.

The exception to the above paragraph is the use of any calculator that has symbolic algebraic manipulation capability. These will continue to be prohibited in all of the Board's examinations as they may offer candidates who use them a significant advantage over other candidates.

The following models have currently been identified as having this capability:

- Texas Instruments T189
- Texas Instruments T192
- Texas Instruments T192 Plus
- Casio CFX 9970G
- Casio Algebra FX 2.0
- Hewlett Packard HP48G
- Hewlett Packard HP48GX
- Hewlett Packard HP49G

EQAP may from time to time publish more detailed rules for the use of calculators, or further add to the list of prohibited calculators.

Information for Students

The *Instructions to Candidates* booklet, which is issued to all candidates prior to the examination period, summarises the above rules and procedures and also states:

Calculators

Candidates are recommended to take a calculator into the examination room for subjects where they have used a calculator during the year. For subjects where a scientific calculator has been used during the year, this should be taken to the examination.

Candidates bringing more sophisticated calculators into an examination room may be subjected to additional scrutiny by supervisors.

Any possible misuse of calculators during an examination will be handled through the Board's standard procedures for investigating possible misconduct in examinations.

ADVISORY SECTION

Suggested Texts

This is a list of only some Mathematics texts that are available and have been used for teaching the course for University Bursary Mathematics with Calculus. It is important teachers use this as only a guide and check current book lists available through publishers and book retailers.

A. Suggested Text

1. Delta Mathematics -	Barton, D.; Johnson, W. & Laird S
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B. Supplementary Texts

- 1. Year 13 Study Guide, <u>Mathematics with Calculus</u> Sidebotham, T. ESA
- 2. Longman Write-on, <u>Notes Calculus</u> _ Barton, D
- 3. Bursary Calculus Questions from the last 8 bursary papers with suggested answers.

Really Useful Resources Box 19-939 Woolston Christchurch

4. Study Pass reference notes, Year 13 Calculus - info@studypass.co.nz

Websites

5. Calculus Website

- i). <u>www.bbc.uk/education/asguru/maths/intro.shtml</u> is part of <u>www.bbc.co.uk</u>
- *ii)* <u>www.ies.co.jp/math/java/calc/index/html</u> is part of <u>www.ies.co.jp/math/indexeng.html</u>
- iii) www.ies.co.jp/math/java/comp/index.html is part of www.ies.co.jp/math/
- iv) www.ies.co.jp/math/java/conics/index.html is part of www.ies.co.jp/math/
- v) <u>www.unc.edu/~rowlett/units.html</u> is part of <u>www.unc.edu/</u>
- vi) www.mathforum.org/pow/
- vii)www.ies.co.jp/math/java/misc/index.html is part of www.ies.co.jp/math/indexeng.html
- viii) www.btinternet.com/~rfbarrow/

The End