MARKER CODE		Student Personal Identification Number

South Pacific Form Seven Certificate

MATHEMATICS WITH STATISTICS 2017

QUESTION and ANSWER BOOKLET

Time allowed: Three hours

(An extra 10 minutes is allowed for reading this paper.)

INSTRUCTIONS

Write your **Student Personal Identification Number (SPIN)** in the space provided on the top right hand corner of this page.

Answer **ALL QUESTIONS**. Write your answers in the spaces provided in this booklet.

Show all working. Unless otherwise stated, numerical answers correct to **three significant figures** will be adequate.

If you need more space for answers, ask the Supervisor for extra paper. Write your SPIN on all extra sheets used and clearly number the questions. Attach the extra sheets at the appropriate places in this booklet.

Major Learning Outcomes (Achievement Standards)	Skill Level & Number of questions				Weight/ Time
	Level 1 *Uni-structural*	Level 2 *Multi-structural*	Level 3 *Relational*	Level 4 *Extended Abstract*	
Strand 1: Probability Develop knowledge and skills related to Probability in order to solve problems and to investigate situations involving elements of chance.	10	4	2	1	28% 72 min
Strand 2: Modelling using graphical methods Model situations using graphical methods in order to solve problems.	4	1	4	1	22% 57 min
Strand 3: Statistical Investigations Carry out statistical investigations and understand statistical processes.	-	3	-	-	6% 15 min
Strand 4: Numerical & algebraic methods Use numeric and algebraic methods to solve problems.	4	3	-	1	14% 36 min
TOTAL	**18**	**11**	**6**	**3**	**70% 180 min**

Check that this booklet contains pages 2-18 in the correct order and that none of these pages is blank.

HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

STRAND 1: PROBABILITY

Assessor's use only

1.1ai	What is an ***event***? _____ _____ _____	**Unistructural** 1 | 0 | NR |	
1.1aii	A die whose faces are labelled 1, 2, 3, 4, 5 and 6 is rolled. Identify the event "result is odd". _____ _____ _____	**Unistructural** 1 | 0 | NR |	
1.1b	A player pays \$2 to enter a game in which he has a 2 in 11 chance of winning a \$20 prize. If the dollar profit is X, what is the expected profit per game? _____ _____ _____ _____ _____ _____ _____ _____ _____ _____ _____	**Relational** 3 | 2 | 1 | 0 | NR |	

Assessor's use only

Use the information below to answer Question 1.1c.

The probability of a student failing (not achieving) a certain bachelor's degree programme at Kanikani University is 0.014. The probability of a student to graduate with (achieving) a bachelor's degree in the same university is 0.87.

1.1c What is the probability of a randomly selected student to either achieve or not achieve a bachelor's degree? Provide reasons with your working.

Extended Abstract	
4	
3	
2	
1	
0	
NR	

1.1di What does it mean when event *X* is independent of event *Y*?

Unistructural	
1	
0	
NR	

Use the information below to answer Question 1.1dii.

A basket contains 4 red balls and 5 yellow balls. A ball is drawn at random. Its colour is recorded then returned into the same basket. The exercise is then repeated once.

1.1dii State **one** pair of **independent events**.

Unistructural	
1	
0	
NR	

Assessor's use only

1.2ai Define ***conditional probability***.

Unistructural	
1	
0	
NR	

1.2aii Define ***marginal probability***.

Unistructural	
1	
0	
NR	

1.2bi Define ***variance***.

Unistructural	
1	
0	
NR	

1.2bii Calculate the variance of the discrete random variable X.

x	0	1	2	3	4
$P(X = x)$	0.1	0.2	0.3	0.2	0.2

Multistructural	
2	
1	
0	
NR	

Assessor's use only

1.2ci

What does it mean for two events A and B to be **mutually exclusive**?

Unistructural	
1	
0	
NR	

1.2cii

For Question 1.2cii, circle the letter of the best answer.

Which of the following pair of events is mutually exclusive?

A.	A person is a teacher at Smart College. A person drives to Smart College every day.
B.	A person turns left. A person turns right.
C.	A person turns left. A person scratches his head.

Unistructural	
1	
0	
NR	

Assessor's use only

Use the information below to answer Questions 1.2di and 1.2dii.

The mean salary at Wanavu Fisheries is $34,000 per annum with a standard deviation of $600. All employees got a $500 per annum rise to celebrate and mark the occasion when the company won the national trade award for being the highest exporter of the year in the fisheries sector.

1.2di What will be the new mean and standard deviation of the salaries?

Multistructural	
2	
1	
0	
NR	

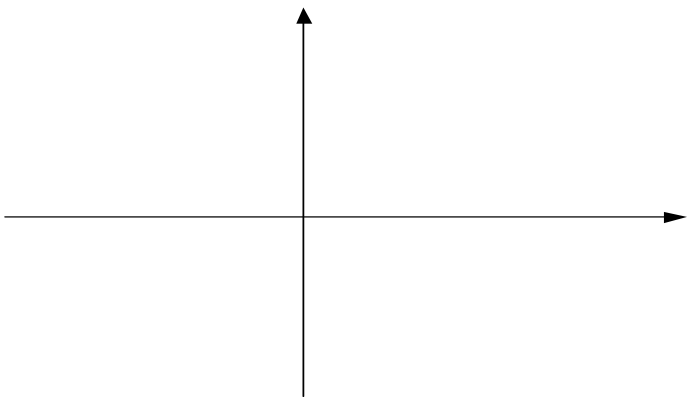
1.2dii If instead of a flat rise, each employee had an income increase of 2%, what would be the new mean and standard deviation of the salaries?

Relational	
3	
2	
1	
0	
NR	

1.2e	State **one** condition that must be satisfied in order to apply Poisson probability distribution to a random variable. _____ _____ _____ _____	**Unistructural** 1 0 NR
1.2fi	**Use the information below to answer Questions 1.2fi and 1.2fii.** A washing machine in a Laundromat breaks down at an average of three times per month. Use the Poisson probability distribution formula to answer questions that follow. Find the probability that during the month the machine will have exactly **two** breakdowns. _____ _____ _____ _____ _____ _____	**Multistructural** 2 1 0 NR
1.2fii	Find the probability that during the month the machine will have at most **one** breakdown. _____ _____ _____ _____ _____ _____ _____ _____ _____	**Multistructural** 2 1 0 NR

STRAND 2: MODELLING USING GRAPHICAL METHODS

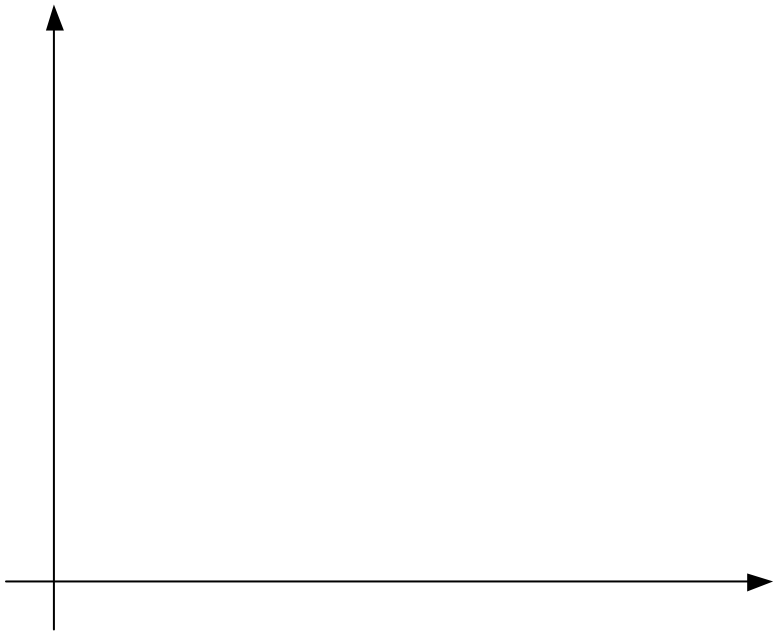
Assessor's use only

2.1ai	What is a **continuous function**? _____ _____ _____	**Unistructural** 1 | 0 | NR |	
2.1aii	Give **one** particular feature of a linear function. _____ _____ _____ _____	**Unistructural** 1 | 0 | NR |
2.1bi	State the general form of a power function, and clearly indicate the relevant variables and constants. _____ _____ _____ _____	**Unistructural** 1 | 0 | NR |
2.1bii	Sketch the graph of the function $y = 2x^{\frac{1}{3}}$, for $x \geq 0$, clearly showing all relevant intercepts. 	**Unistructural** 1 | 0 | NR |

Assessor's use only

2.1c Graph the following constraints then shade the feasible region by shading in or shading out.

$$x + y \leq 2, \qquad y \geq 1, \qquad x \geq 0.$$



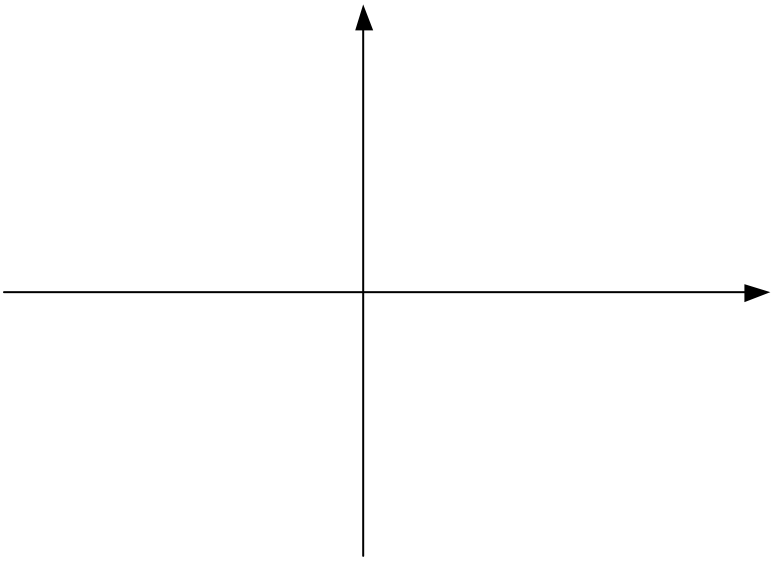
Multistructural	
2	
1	
0	
NR	

Assessor's use only

2.2a Sketch the graph of

$$f(x) = \begin{cases} x + 1, & x < 0 \\ 2, & 0 \leq x < 2 \\ x^2 - 4, & 2 \leq x \end{cases}$$

clearly marking all relevant points.



Relational	
3	
2	
1	
0	
NR	

2.2b

A population of creatures is given in millions by $P(t) = 4e^{0.13t} - 0.2e^{0.26t}$, where t is in years. The population starts to decrease after a certain time.

When does the population become extinct?

Assessor's use only

Extended Abstract	
4	
3	
2	
1	
0	
NR	

2.2c

Sketch the graph of $y = 4.5x^{-1}$ when considering values of x in the closed interval $[-3, 3]$ and comment on the limitation of $y = 4.5x^{-1}$as a model.

Graph:

Comment:

Relational	
3	
2	
1	
0	
NR	

Assessor's use only

Use the information below to answer Questions 2.2di and 2.2dii.

Mike's Famous Toy Trucks manufactures two kinds of toy trucks---standard model and a deluxe model. In the manufacturing process each standard model requires 2 hours of grinding and 2 hours of finishing, and each deluxe model needs 2 hours of grinding and 4 hours of finishing. The company has two grinders and three finishers, each of who works at most 40 hours per week. Each standard model toy truck brings a profit of \$3 and each deluxe model a profit of \$4. We assume that every truck made will be sold.

Let: x = Number of standard models made

y = Number of deluxe models made

2.2di Identify and give the profit function and the constraints.

Relational	
3	
2	
1	
0	
NR	

Assessor's use only

2.2dii Compute the point where the profit function is maximised.

Relational	
3	
2	
1	
0	
NR	

STRAND 3: STATISTICAL INVESTIGATIONS

Assessor's use only

Use the information below to answer questions 3.1ai and 3.1aii.

The following sets of mathematics marks are those obtained by a group of Form 7 students in two different exams. The top row gives the mark in the Term 3 school exam. The bottom row gives the mark in the external exam.

School (x)	70	44	43	65	53	65	48	59	61	86	69	70	61	45
External (y)	65	52	53	63	53	62	51	58	58	59	56	60	56	55

3.1ai Calculate the correlation coefficient r between the two sets of marks.

Note that

$SS_{xy} = \sum xy - \frac{\sum x \sum y}{n} = 485.2143$, $SS_{yy} = \sum y^2 - \frac{(\sum y)^2}{n} = 238.3571$, and

$SS_{xx} = \sum x^2 - \frac{(\sum x)^2}{n} = 1972.929$

Multistructural	
2	
1	
0	
NR	

3.1aii Comment on the trends shown by the two sets of marks.

Multistructural	
2	
1	
0	
NR	

Assessor's use only

3.2a A sample of 60 bags of flour was found to have a mean mass of 1.13 kg and a standard deviation of 0.04 kg.

Find the 95% confidence interval for the mean weight of all such bags.

Multistructural	
2	
1	
0	
NR	

STRAND 4: NUMERICAL AND ALGEBRAIC METHODS

Assessor's use only

4.1a	State the possible different types of solutions in solving a system of linear equations. _____ _____ _____	**Unistructural** 1 0 NR
4.1b	Determine if $f(x) = \frac{3}{x+1}$ has a root on the closed interval $[2, 5]$. _____ _____ _____ _____ _____	**Unistructural** 1 0 NR
4.1c	State **one** advantage of using the Newton-Raphson method. _____ _____ _____ _____	**Unistructural** 1 0 NR
4.1d	By inspection, what method is best to solve the following system of linear equations? $x + y = 1$ $2y - x = 5$ DO NOT ATTEMPT TO SOLVE THE SYSTEM. _____	**Unistructural** 1 0 NR

Assessor's use only

4.2a

Find a value of a and k such that the following system of linear equations is consistent with infinitely many solutions.

$$\begin{array}{rcrcl} 8x & + & ay & = & k \\ 2x & + & y & = & 1 \end{array}$$

Multistructural	
2	
1	
0	
NR	

4.2b

There are three numbers x, y and z. The largest is two and a half times bigger than the smallest number. The sum of all three numbers is 24.5. Double the largest plus the other two numbers is 37.

Form a system of **three** linear equations and find the **three** numbers.

Extended Abstract	
4	
3	
2	
1	
0	
NR	

Assessor's use only

4.2c Consider the function $f(x) = \frac{1}{4x-1}$.

Comment on why the Bisection method cannot be applied to $f(x)$ on interval $[0, 1]$.

Multistructural	
2	
1	
0	
NR	

4.2d Consider $f(x) = 2x^2 - 4x + 1$. Use the Newton-Raphson method with an initial value of $x_0 = -0.1$ to give two approximations x_1 and x_2.

Multistructural	
2	
1	
0	
NR	

THE END