Assessment Schedule 2017
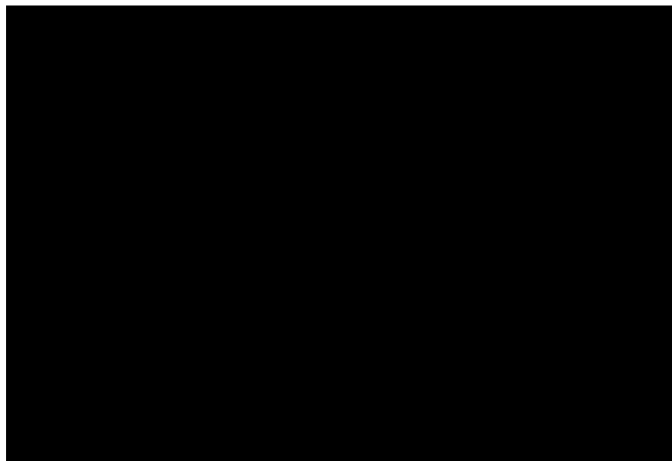
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Strand 1

Question 1: Major Learning Outcome Strand 1

Develop knowledge and skills related to Probability in order to solve problems and to investigate situations involving elements of chance.

Probability						
Item number	**Skill Level**	**Evidence**	**SOLO Skill Levels**			
			1	**2**	**3**	**4**
A1a	**Q1.1ai** **Level 1**	**(i)** An *event* is an outcome or set of outcomes.	Must clearly mention the correct definition.			
	Q1.1aii **Level 1**	**(ii)** The event is {1, 3, 5}.	Correct answer.			
1.1b	**3**	For a win, the profit is $18. For a loss, the profit is -$2. Setting up a probability distribution variable gives 	x	-2	18
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$P(X=x)$	$\frac{9}{11}$	$\frac{2}{11}$ | The expected value is $$E(X) = -2\left(\frac{9}{11}\right) + 18\left(\frac{2}{11}\right) = \frac{18}{11}$$ Hence, on average, the player would expect to gain about $1.63.	One of the following • Identify at least one of the profits. • Setting up probability distribution table (with possibly incorrect numbers).	At least two of the following • Identify at least one of the profits. • Setting up probability distribution table (with possibly incorrect numbers). .	All the following • Identify at least one of the profits. • Setting up probability distribution table (with possibly incorrect numbers). • Compute the expected value. • Making the right interpretation. Allow the possibility that method may be correct but numerical values may be incorrect.	

1.1c	**4**	Clearly, a student cannot fail AND pass at the same time. The two events 'failing the programme' and 'graduating with a Bachelor degree' are mutually exclusive. The probability of either a failure at Bachelor level or a graduate with a Bachelor degree is the sum of their individual probabilities, i.e. 0.014+0.87=0.884.	Mention one of the following. • Probability of failing. • Probability of graduating. • Mutually exclusive events.	Mention at least two of the following. • Probability of failing. • Probability of graduating. • Mutually exclusive events.	Able to tell that these are mutually exclusive events. Give the respective probabilities of failing and graduating based on being mutually exclusive events.	Able to tell that these are mutually exclusive events. Give the respective probabilities of failing and graduating based on being mutually exclusive events. PLUS talking about the generalisation of computing the probability of mutually exclusive events in comparison to events that are not mutually exclusive.
A1d	**Q1.1di Level 1**	**(i)** It means that the occurrence of event X has absolutely no effect on the occurrence of event Y.	Must clearly mention the correct definition.			
	Q1.1dii Level 1	**(ii)** Only A is independent event.	Correct answer.			
1.2ai	**1**	It is the probability that an event will occur given that another event has already occurred.	Must clearly mention the correct definition.			
1.2aii	**1**	It is the probability of a single event without consideration of any other event. It is also called *simple probability*.	Must clearly mention the correct definition.			

1.2bi	1	It is the square of the standard deviation and used to measure spread.	Must clearly mention the correct definition.			
1.2bii	2	$E(X) = \sum xP(x)$ $= 0(.1) + 1(.2)$ $+ 2(.3) + 3(.2)$ $+ 4(.2) = 2.2$ $Var(X) = E(X^2) - [E(X)]^2$ $= 6.4 - 4.84 = 1.56$	Using the correct formula but wrong answers due to numerical errors. Allow the possibility of using 'almost' correct formula(s).	Correct answers obtained by using correct formulas.		
1.2ci	1	It means that A and B can't occur at the same time.	Must clearly mention the correct definition.			
1.2cii	1	No, because it is possible that a teacher could drive to Smart College.	Must be able to give the correct reason to support the correct answer.			

1.2di Level 2	**(i)** $$E(S + 500) = E(S) + E(500) = E(S) + 500$$ $$= 34{,}000 + 500 = \$34{,}500$$ $Var(S + 500) = Var(S)$ $$SD = \$600$$	One of the following is acceptable. - Giving incorrect answers due to numerical errors. - Incomplete correct answers.	Giving at least two correct answers by using the correct formulas.		
1.2dii level 3	**(ii)** In this case, the effect of the increase is to multiply everyone's salary by 1.02. Hence, the new salary is 1.02S with $$E(1.02S) = 1.02 \times 34{,}000 = \$34{,}680,$$ $$Var(1.02S) = 1.02^2 Var(S) = 1.02^2 \times 600^2,$$ so $SD = \sqrt{1.02^2 \times 600^2} = 1.02 \times 600 = \$612.$	One of the following is acceptable. - Giving incorrect answers due to numerical errors. - Incomplete correct answers.	Giving correct answers without any indication of applying (i).	Being able to apply (i) when answering to give all correct answer.	

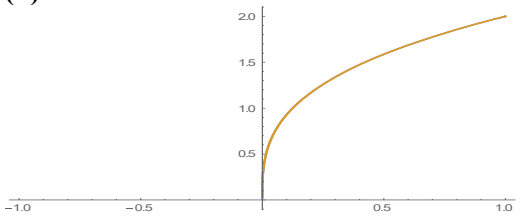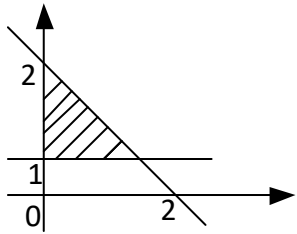|						
1.2e	**1**	• x is a discrete random variable. • The occurrences are random. • The occurrences are independent.	Give at least one of the three conditions.			
B1f	**1.2fi** **Level 2**	**(i)** $$P(x = 2) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{3^2 e^{-3}}{2!} = 0.2240$$	Method is correct but incorrect answer due to numerical errors.	Correct answer using correct method.		
	1.2fii **Level 2**	**(ii)** $P(\text{at least 1 breakdown}) = P(0 \text{ or } 1 \text{ breakdown})$ $\qquad = P(x = 0) + P(x = 1)$ $\qquad = \frac{3^0 e^{-3}}{0!} + \frac{3^1 e^{-3}}{1!} = 0.0498 + 0.1494$ $\qquad = 0.1992$	Method is correct but incorrect answer due to numerical errors.	Correct answer using correct method.		

Strand 2
Model situations using graphical methods in order to solve problems

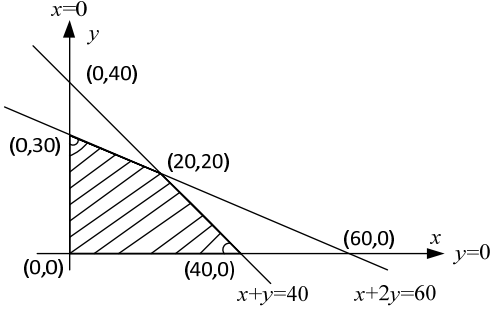| Item number	Skill Level	Evidence	SOLO Skill Levels			
			1	2	3	4
A2a	**2.1ai Level 1**	**(i)** A continuous function has no breaks in its graph.	Correct answer. It is expected that candidates should use graphical means to give a correct answer. .			
	2.1aii Level 1	**(ii)** Linear functions • Are always continuous; • Have general equation $y = mx + b$; • Graph is always a straight line.	At least give one of the listed features.			

A2b	2.1bi Level 1	**(i)** The general form of a power function is $y = ax^n$, where x and y are variables and n is a constant.	Correct answer.			
	2.1bii Level 1	**(ii)** 	Correct answer/sketch.			
2.1c	2		The sketch is appealing but lacks all the expected details. Allow the possibility where shading may be incorrect but the lines are correctly drawn with the right intercepts.	Correct sketch with all the relevant details particularly the intercepts and shading.		

| 2.2a | 3 | | The graph appears to 'partially' correct. No clear indication of the 'open' and 'closed' circles as per information given in the piecewise function. | Most details are present in the sketch except at most two finer details are missing. | Correct graph containing all the relevant information clearly marked and presented. | |

2.2b	**4**	Need to solve $P(t)=0$. That is, $$4e^{0.13t} - 0.2e^{0.26t} = 0$$ $$4e^{0.13t} = 0.2e^{0.26t}$$ $$20e^{0.13t} = e^{0.26t}$$ $$\ln 20 + \ln e^{0.13t} = \ln e^{0.26t}$$ $$\ln 20 + 0.13t = 0.26t$$ $$0.13t = \ln 20$$ $$t = \frac{\ln 20}{0.13} = 23.0441$$	Stating the equation to be solved which is $P(t) = 0$. There is no clear indication of how to proceed any further.	• Stating the equation to be solved which is $P(t) = 0$. • Partially correctly manipulation but couldn't proceed any further due to poor knowledge properties of logarithm and exponential function.	Correct answer is obtained through correct method.	Not only giving the correct answer but also being able to interpret it to fully answer the question.
2.2c	**3**	The model fails when $x = 0$.	Mentioning one of the following is acceptable. • It is a hyperbola. • There is a point of discontinuity. • $x = 0$ is inside the interval. • There is an asymptote at $x = 0$.	Mentioning at least two of the following is acceptable. • It is a hyperbola. • There is a point of discontinuity. • $x = 0$ is inside the interval. • There is an asymptote at $x = 0$.	Being able to identify that the hyperbola has a point of discontinuity in the given interval and that point happens to be the point of interest $x = 0$.	
B2d	**2.2di Level 3**	**(i)** Profit $$P(x,y) = \$3x + \$4y$$ Constraints $$x + y \le 40$$	Mentioning one of the following is acceptable. • Profit function • Any of the four constraints	Mentioning at least two of the following is acceptable. • on • Any of the four constraints	Being able to state the profit function together with all the constraints.	

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		$x + 2y \leq 60$ $x \geq 0$ $y \geq 0$				
	2.2dii **Level 3**	**(ii)** The values of P at the corner points of the set of feasible points are (0,0): P=0 (0,30): P=$120 (40,0): P=$120 (20,20): P=$140 (Max Profit)	One of the following is acceptable. • A clear sketch of the feasible region with all the relevant corners. • A clear sketch of a feasible region but no corner points indicated. • Incorrect corners are chosen because of incorrect sketch.	At least two of the following are acceptable. • A clear sketch of the feasible region with all the relevant corners. • A clear sketch of a feasible region but no corner points indicated. • Incorrect corners are chosen because of incorrect sketch.	Correct method used to arrive at the correct answer drawing the right conclusion regarding the maximum profit.	

Strand 3

Carry out statistical investigations and understand statistical processes.

<table>
<tr><th colspan="7">Statistical Investigation</th></tr>
<tr><th rowspan="2">Item number</th><th rowspan="2">Skill Level</th><th rowspan="2">Evidence</th><th colspan="4">SOLO Skill Levels</th></tr>
<tr><th>1</th><th>2</th><th>3</th><th>4</th></tr>
<tr>
<td>3.1ai</td>
<td>2</td>
<td>

(i)

The coefficient is

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}SS_{yy}}}$$
$$= \frac{485.2143}{\sqrt{(1972.929)(238.3571)}}$$
$$= 0.707561.$$

</td>
<td>Method is correct but due to numerical errors, arrives at an incorrect answer.</td>
<td>Correct answer using the correct formula/method.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.1aii</td>
<td>2</td>
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(ii)

External marks rise as school marks rise. There is not a lot of scatter about the trend line.

</td>
<td>Partially correct interpretation.

Allow the possibility that the partial correct interpretation was based on an incorrect answer.</td>
<td>Correct interpretation based on a correct answer.</td>
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3.2a	**2**	For 95% confidence, $z = 1.96$. A 95% CI is $$1.13 - 1.96 \cdot \frac{0.04}{\sqrt{60}} < \mu < 1.13 + 1.96 \cdot \frac{0.04}{\sqrt{60}}$$ $$1.12 < \mu < 1.14$$	One of the following is acceptable. • Identifying the correct z value. • Correct construction formula used. • May be halfway to the arriving at the correct answer but due to numerical error, incorrect answer is obtained.	Correct CI interval is obtained by using correct formula with the correct information.	

Strand 4

Use numeric and algebraic methods to solve problems.

			Numerical and Algebraic Methods			
Item number	Skill Level	Evidence	SOLO Skill Levels			
			1	2	3	4
4.1a	1	Unique solution Infinitely many solutions No solution	Must present all three.			
4.1b	1	$f(x)$ has no root since $f(x) = 0$ has no (real) solution.	Must mention at least one of the following. • There is no root in the given interval. • The graph of f is asymptotic at $x = -1$. • There is no real solution of the equation $f(x) = 0$.			
4.1c	1	• Easy to use and implement. • Converges quite quickly when the interval in relatively small.	Must mention at least one of the two.			

4.1d	**1**	Elimination method - adding the two equations would eliminate x so we could first solve for *y*.	Must mention suitable method. There is no need to give any reasons why.			

| 4.2a | 2 | Two equations must be the same.

Multiply the second equation by 4 to get

$$\begin{aligned} 8x &+ ay &= k \\ 8x &+ 4y &= 4 \end{aligned}$$

Thus $a = 4$ and $k = 4$. | Attempt is made and partially correct answer is given. | Correct values are given. | | |

| 4.2b | 4 | The system is

$a + b + c = 24.5$
$2.5a = c$
$2c + a + b = 37$

$a + b + c = 24.5$
$a + b + 2c = 37$
$2.5a - c = 0$

$(a, b, c) = (5, 7, 12.5)$ | One of the three equations is correct. | At least two of the three equations are correct. | All three equations are correctly given and acceptable method is used to solve the system of equation. There may some complications due to numerical errors but the method is correct. | Correct answer is obtained using the correct method.

Geometrical interpretation is also given which means that these three planes meet at a single point. |

4.2c	**2**	We can't apply the Bisection method because $f(x)$ is not continuous on $[0,1]$, i.e. f has a point of discontinuity at $x = \frac{1}{4}$.	One of the following is acceptable. • Discontinuity of f in the given interval. • Existence of an asymptote at $x = \frac{1}{4}$	Being able to discuss the continuity requirement of the Bisection method and because f is not continuous on the given interval therefore the method can't be applied.		
4.2d	**2**	$x_1 = -0.2722$ $x_2 = -0.2926$ x	Able to apply the method successfully and a partially correct answer is obtained due to numerical errors. Allow the possibility that only one of the two is correct whereas the the other approximation may be incorrect.	The two approximations are correctly obtained using the correct method of calculation.		