

MARKER CODE



Pacific
Community
Communauté
du Pacifique



Student Personal Identification Number

South Pacific Form Seven Certificate

MATHEMATICS WITH STATISTICS 2016

QUESTION and ANSWER BOOKLET

Time allowed: Two and a half hours

INSTRUCTIONS

Write your **Student Personal Identification Number (SPIN)** in the space provided on the top right hand corner of this page.

Answer **ALL QUESTIONS**. Write your answers in the spaces provided in this booklet.

If you need more space for answers, ask the Supervisor for extra paper. Write your SPIN on all extra sheets used and clearly number the questions. Attach the extra sheets at the appropriate places in this booklet.

Major Learning Outcomes (Achievement Standards)	Skill Level Band			Weight/ Time
	1 <i>Basic</i>	2 <i>Proficient</i>	3 <i>Advanced</i>	
StaA: Develop knowledge and skills related to Probability in order to solve problems and to investigate situations involving elements of chance.	14 items	4 items	2 items	28% 56 min
StaB: Model situations using graphical methods in order to solve problems.	12 items	2 items	2 items	22% 44 min
StaC: Carry out statistical investigations and understand statistical processes.	4 items	2 items	1 item	11% 22 min
StaD: Use numeric and algebraic methods to solve problems.	7 items	2 items	1 item	14% 28 min
TOTAL	37 items	10 items	6 items	75% 150 min

Check that this booklet contains pages 2-24 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

<p>A1d</p>	<p>A box contains 12 light bulbs, of which 5 are defective. Suppose three light bulbs are selected at random from the box. Define F as the event "2 light bulbs are defective". What is the probability that exactly 2 are defective?</p> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/>	<table border="1"> <tr> <th colspan="2">Skill Level 3</th> </tr> <tr> <td>Advanced</td> <td></td> </tr> <tr> <td>Proficient</td> <td></td> </tr> <tr> <td>Basic</td> <td></td> </tr> <tr> <td>Weak</td> <td></td> </tr> <tr> <td>NR</td> <td></td> </tr> </table>	Skill Level 3		Advanced		Proficient		Basic		Weak		NR					
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<p>A1e</p>	<p>Let A and B be events. If $P(A) = 0.6$, $P(B) = 0.3$, and $P(A \cap B) = 0.2$, find:</p> <p>(i) $P(A \cup B)$</p> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <p>(ii) $P(A')$, where A' is the complement of A.</p> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/>	<table border="1"> <tr> <th colspan="2">Skill Level 1</th> </tr> <tr> <td>Basic</td> <td></td> </tr> <tr> <td>Weak</td> <td></td> </tr> <tr> <td>NR</td> <td></td> </tr> </table> <table border="1"> <tr> <th colspan="2">Skill Level 1</th> </tr> <tr> <td>Basic</td> <td></td> </tr> <tr> <td>Weak</td> <td></td> </tr> <tr> <td>NR</td> <td></td> </tr> </table>	Skill Level 1		Basic		Weak		NR		Skill Level 1		Basic		Weak		NR	
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Question 2: Major Learning Outcome B

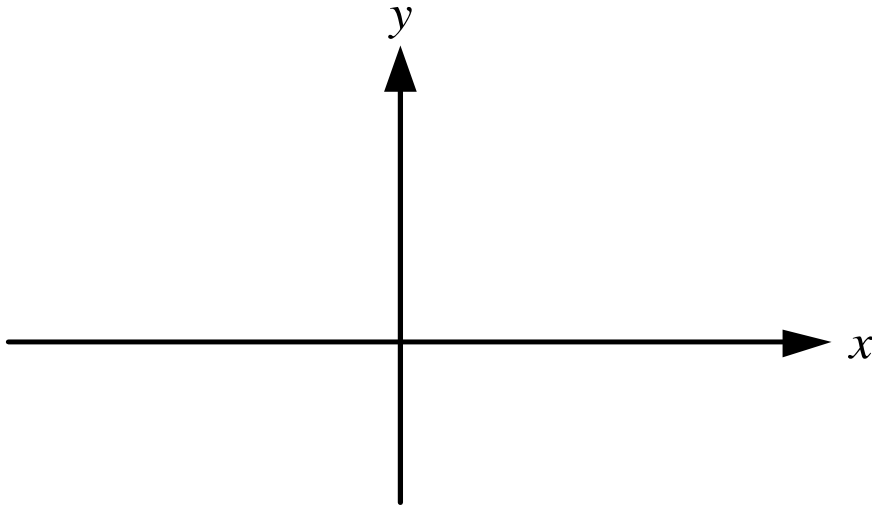
Model situations using graphical methods in order to solve problems

Assessor's use only

A2a Consider the piecewise function

$$f(x) = \begin{cases} x^2, & x > 0 \\ 1, & x \leq 0 \end{cases}$$

- (i) Sketch the graph of $f(x)$, clearly marking all relevant intercepts and points.

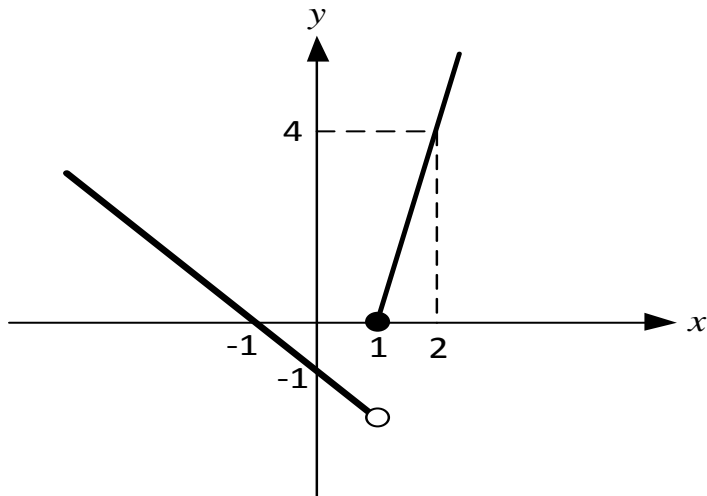


- (ii) Compute the value of $2 \times f(0) + f(1)$.

Skill Level 1	
Basic	
Weak	
NR	

Skill Level 1	
Basic	
Weak	
NR	

A2b Consider the piecewise function $f(x)$ whose graph is shown below.



Write the equation of the function.

Skill Level 1

Basic

Weak

NR

A2c The variables x and y are related by the function $y = 3.2x^{1.4}$. Find x if $y = 8.1$.

Skill Level 1

Basic

Weak

NR

A2d The variables p and q are related by the exponential function $q = 56e^{-0.4p}$. Find p if $q = 32.5$.

Skill Level 1	
Basic	
Weak	
NR	

A2e Lavulavu Area School is making bula shirts and aloha dresses to raise funds. Production figures are given below in the following table.

	Number made	Length (m) of material required for each item	Time (hr) required to make each item	Profit
Bula shirts	x	1.4	1	12
Aloha dresses	y	0.7	1.5	8

They have at most 112 meters of materials and no more than 120 hours available. There are already orders for 15 bula shirts and 20 aloha dresses.

Write down the four inequations conveyed by the information above.

Skill Level 2	
Proficient	
Basic	
Weak	
NR	

Question 3: Major Learning Outcome C

Carry out statistical investigations and understand statistical processes.

Assessor's use only

A3a When certain tropical fish eggs are fertilised, the percentage of them that hatch depends on the temperature of the surrounding water. The following data were obtained from randomly chosen lots of eggs.

Temperature ($T^{\circ}C$)	10	20	30	40	50	60
Percentage (%) of fertilised eggs hatching	12	21	26	35	41	52

- (i) Draw a scatter diagram for the given data and use it to explain why it is unlikely that all fertile eggs in a lot will hatch.



Skill Level 2	
Proficient	
Basic	
Weak	
NR	

- (ii) Is the sign of the correlation coefficient for this set of data positive or negative? Comment.

Skill Level 2	
Proficient	
Basic	
Weak	
NR	

A4c	<p>Use the Newton-Raphson method to find a solution of the equation $f(x) = 0$, where $f(x) = x^3 - x^2 + x - 1$. Take $x_0 = 0.6$ and evaluate the first two iterates. Give your answers to 4 decimal places.</p> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/>	<table border="1"> <thead> <tr> <th colspan="2">Skill Level 1</th> </tr> </thead> <tbody> <tr> <td>Basic</td> <td></td> </tr> <tr> <td>Weak</td> <td></td> </tr> <tr> <td>NR</td> <td></td> </tr> </tbody> </table>	Skill Level 1		Basic		Weak		NR	
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A4d	<p>The following is the augmented matrix that represents a system of linear equations.</p> $\begin{bmatrix} 0 & 0 & 5 & 5 \\ 0 & 1 & 2 & 1 \\ 1 & 1 & 0 & 2 \end{bmatrix}$ <p>By inspection, how many solutions are there?</p> <p>DO NOT ATTEMPT TO SOLVE THE SYSTEM.</p> <hr/> <hr/> <hr/>	<table border="1"> <thead> <tr> <th colspan="2">Skill Level 1</th> </tr> </thead> <tbody> <tr> <td>Basic</td> <td></td> </tr> <tr> <td>Weak</td> <td></td> </tr> <tr> <td>NR</td> <td></td> </tr> </tbody> </table>	Skill Level 1		Basic		Weak		NR	
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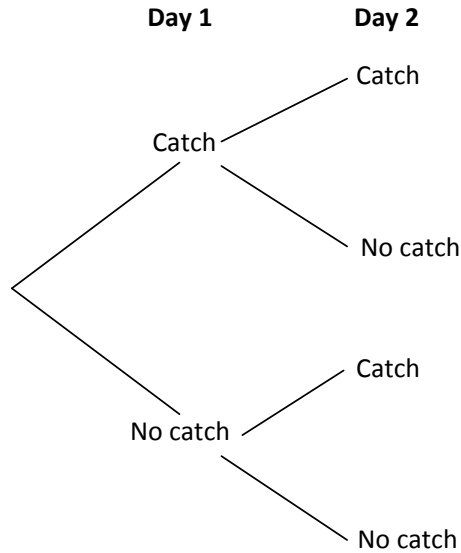
SECTION B: LONG ANSWERS

Question 1: Major Learning Outcome A

Develop knowledge and skills related to Probability in order to solve problems and to investigate situations involving elements of chance.

Assessor's use only

B1a The tree diagram below describes the possible outcomes for a person who goes fishing two days in a row.



If the probability of catching a fish is always $\frac{2}{3}$ whenever one goes fishing, what is the probability that they:

(i) do not catch a fish on either day?

(ii) are successful on the first day and not the second day?

(iii) catch a fish on at least one day?

Skill Level 1	
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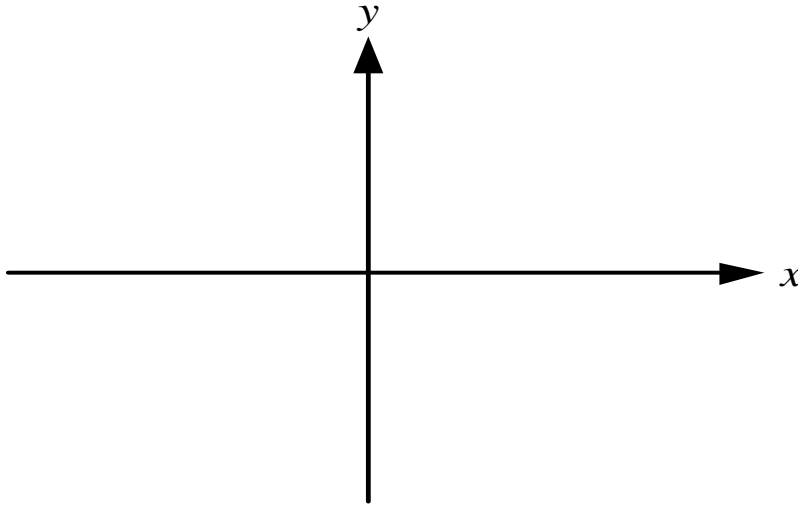
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<p>B1b</p>	<p>A person is selected at random from a population. Define events A and B to be:</p> <p style="padding-left: 40px;">A = person is a nonsmoker B = person smokes at least 5 cigarettes per day</p> <p>Are these events mutually exclusive? Why?</p> <hr/> <hr/> <hr/>	<table border="1" style="width: 100%;"> <tr> <th colspan="2" style="background-color: #cccccc;">Skill Level 1</th> </tr> <tr> <td style="width: 80%;">Basic</td> <td></td> </tr> <tr> <td>Weak</td> <td></td> </tr> <tr> <td>NR</td> <td></td> </tr> </table>	Skill Level 1		Basic		Weak		NR			
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<p>B1c</p>	<p>Suppose a coin is tossed, a six-sided die is rolled, and a card is drawn randomly from a deck of 52 cards. Let events A, B, and C be:</p> <p style="padding-left: 40px;">A = coin is a tail; B = die is a 3; C = card is a heart.</p> <p>Assuming independence, what is the probability that all three events occur?</p> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/>	<table border="1" style="width: 100%;"> <tr> <th colspan="2" style="background-color: #cccccc;">Skill Level 2</th> </tr> <tr> <td style="width: 80%;">Proficient</td> <td></td> </tr> <tr> <td>Basic</td> <td></td> </tr> <tr> <td>Weak</td> <td></td> </tr> <tr> <td>NR</td> <td></td> </tr> </table>	Skill Level 2		Proficient		Basic		Weak		NR	
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<p>B1d</p>	<p>A researcher is studying the takahe, a species of bird, which nests on the ground. He finds that, on average, there are three nests per hectare in regions where the bird is found. Let the random variable N be the number of nests in a given area which has a Poisson distribution.</p> <p>(i) If the researcher searches an area of one hectare, find the probability that he will find at least one nest. Show your reasoning.</p> <hr/> <hr/> <hr/> <hr/>	<table border="1" style="width: 100%;"> <tr> <th colspan="2" style="background-color: #cccccc;">Skill Level 1</th> </tr> <tr> <td style="width: 80%;">Basic</td> <td></td> </tr> <tr> <td>Weak</td> <td></td> </tr> <tr> <td>NR</td> <td></td> </tr> </table>	Skill Level 1		Basic		Weak		NR			
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<p>B1d</p>	<p>(ii) If the researcher searches an area of two hectares, find the probability that he will find at least one nest. Show your reasoning.</p> <hr/> <hr/> <hr/> <hr/>	<table border="1"> <tr> <th colspan="2">Skill Level 2</th> </tr> <tr> <td>Proficient</td> <td></td> </tr> <tr> <td>Basic</td> <td></td> </tr> <tr> <td>Weak</td> <td></td> </tr> <tr> <td>NR</td> <td></td> </tr> </table>	Skill Level 2		Proficient		Basic		Weak		NR									
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<p>B1e</p>	<p>The leaves of a species of grape vine have normally distributed length with a mean of 10 cm and a standard deviation of 3 cm.</p> <p>(i) What is the probability a randomly selected leaf has a length which lies between 11.8 cm and 13.6 cm?</p> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <p>(ii) What length are 67% of leaves less than?</p> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/>	<table border="1"> <tr> <th colspan="2">Skill Level 1</th> </tr> <tr> <td>Basic</td> <td></td> </tr> <tr> <td>Weak</td> <td></td> </tr> <tr> <td>NR</td> <td></td> </tr> </table> <table border="1"> <tr> <th colspan="2">Skill Level 2</th> </tr> <tr> <td>Proficient</td> <td></td> </tr> <tr> <td>Basic</td> <td></td> </tr> <tr> <td>Weak</td> <td></td> </tr> <tr> <td>NR</td> <td></td> </tr> </table>	Skill Level 1		Basic		Weak		NR		Skill Level 2		Proficient		Basic		Weak		NR	
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<p>B1f</p>	<p>In the country of Tikong, the probability of a citizen being an immigrant is 0.3. The probability of a person being married given they are an immigrant is 0.7. The probability of someone being an immigrant and having a university education is 0.08. The probability of a citizen having children is 0.36.</p> <p>(i) Find the probability of a person having a university education given they are an immigrant.</p> <hr/> <hr/> <hr/> <hr/>	<table border="1"> <tr> <th colspan="2">Skill Level 1</th> </tr> <tr> <td>Basic</td> <td></td> </tr> <tr> <td>Weak</td> <td></td> </tr> <tr> <td>NR</td> <td></td> </tr> </table>	Skill Level 1		Basic		Weak		NR											
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B2b Clearly identify and shade the region satisfying the inequations $x \geq 0$, $y \leq 4$, and $y \geq x - 2$. Clearly mark all relevant intercepts.



Skill Level 1	
Basic	
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B2c The variable x and y are related by the rule $y = 8.8/x^4$.

(i) Show that a power law relates x and y .

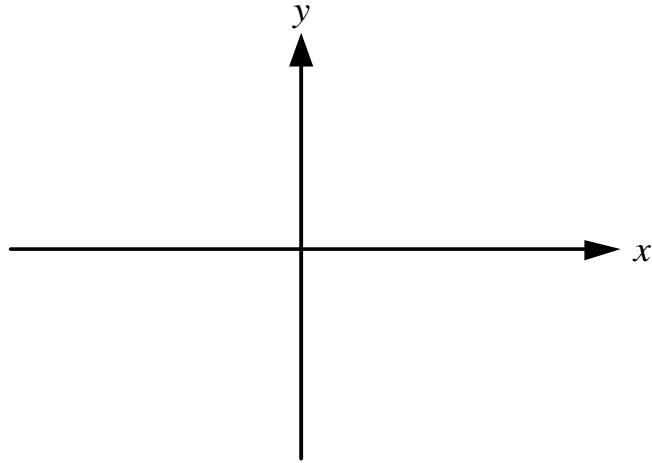
(ii) Comment on any limitation if $y = 8.8/x^4$ is to be adopted as a model.

Skill Level 1	
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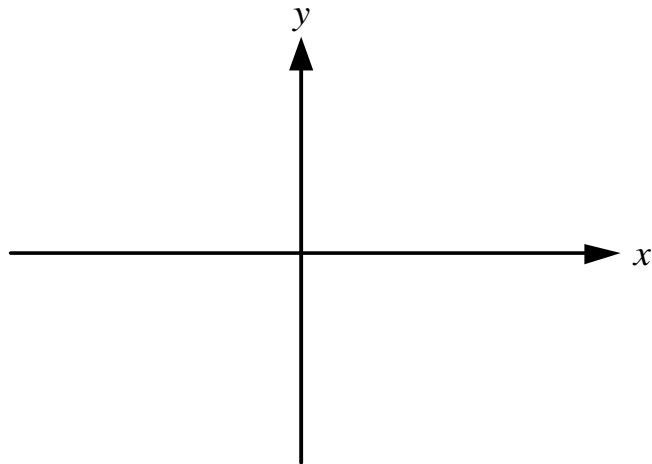
Skill Level 1	
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Weak	
NR	

B2d Draw the graphs of the following functions and clearly indicate all relevant intercepts and points.

(i) $y = 8e^{-0.3x}$



(ii) $x^{2/5}y = 1$, for $0 \leq x \leq 3$



Skill Level 1	
Basic	
Weak	
NR	

Skill Level 1	
Basic	
Weak	
NR	

Question 3: Major Learning Outcome C

Carry out statistical investigations and understand statistical processes.

Assessor's use only

B3a Consider the confidence interval for the population proportion π given by

$$0.6 - 2.576 \sqrt{\frac{0.6 \times 0.4}{300}} < \pi < 0.6 + 2.576 \sqrt{\frac{0.6 \times 0.4}{300}}$$

of those who voted for the People's Party in a recent election.

(i) What is the sample proportion?

(ii) What is the standard error?

(iii) What is the degree of confidence?

(iv) What does the confidence interval for π say?

Skill Level 1	
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Weak	
NR	

Skill Level 1	
Basic	
Weak	
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Skill Level 1	
Basic	
Weak	
NR	

Skill Level 1	
Basic	
Weak	
NR	

Question 4: Major Learning Outcome D

Use numeric and algebraic methods to solve problems.

Assessor's use only

B4a	<p>The following system of linear equations.</p> $\begin{aligned}x - y + 3z &= 5 \\x - y + 3z &= 6 \\x + y + z &= 1\end{aligned}$ <p>By inspection, comment on the nature of the solution.</p> <p>DO NOT SOLVE THE SYSTEM.</p> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/>	<table border="1"> <thead> <tr> <th colspan="2">Skill Level 2</th> </tr> </thead> <tbody> <tr> <td>Proficient</td> <td></td> </tr> <tr> <td>Basic</td> <td></td> </tr> <tr> <td>Weak</td> <td></td> </tr> <tr> <td>NR</td> <td></td> </tr> </tbody> </table>	Skill Level 2		Proficient		Basic		Weak		NR	
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B4e By inspection, what is the value of k so that the following system has infinitely many solutions.

$$\begin{aligned}3x + 4y - 5z &= 6 \\6x + 8y - 10z &= 3k\end{aligned}$$

Skill Level 1

Basic

Weak

NR