

MARKER CODE



Student Personal Identification Number

# South Pacific Form Seven Certificate

## MATHEMATICS WITH STATISTICS

### 2014

### QUESTION and ANSWER BOOKLET

Time allowed: Two and a half hours

#### **INSTRUCTIONS**

Write your **Student Personal Identification Number (SPIN)** in the space provided on the top right hand corner of this page.

Answer **ALL QUESTIONS**. Write your answers in the spaces provided in this booklet.

If you need more space for answers, ask the Supervisor for extra paper. Write your SPIN on all extra sheets used and clearly number the questions. Attach the extra sheets at the appropriate places in this booklet.

Major Learning Outcomes (Achievement Standards)	Skill Level			Weight/ Time
	Band 1 <i>Basic</i>	Band 2 <i>Proficient</i>	Band 3 <i>Advanced</i>	
<b>StaA:</b> Develop knowledge and skills related to Probability in order to solve problems and to investigate situations involving elements of chance.	14 questions	4 questions	2 questions	28% 56 min
<b>StaB:</b> Model situations using graphical methods in order to solve problems.	12 questions	2 questions	2 questions	22% 44 min
<b>StaC:</b> Carry out statistical investigations and understand statistical processes.	4 questions	2 questions	1 questions	11% 22 min
<b>StaD:</b> Use numeric and algebraic methods to solve problems.	7 questions	2 questions	1 questions	14% 28 min
<b>TOTAL</b>	<b>37</b> questions	<b>10</b> questions	<b>6</b> questions	<b>75 %</b> <b>150 min</b>

Check that this booklet contains pages 1-27 in the correct order and that none of these pages is blank.

**YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.**

## SECTION A: SHORT ANSWERS

### Question 1: Major Learning Outcome A

Develop knowledge and skills related to Probability in order to solve problems and to investigate situations involving elements of chance.

*Assessor's use only*

A1a	<p>Two cards are to be drawn from a deck of 52 cards without replacement. What is the probability that both are spade cards?</p> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/>	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="background-color: #cccccc;">Basic</th> <th style="background-color: #cccccc;">Level</th> </tr> </thead> <tbody> <tr> <td>Excellent</td> <td style="width: 20px;"></td> </tr> <tr> <td>Weak</td> <td></td> </tr> <tr> <td>NR</td> <td></td> </tr> </tbody> </table>	Basic	Level	Excellent		Weak		NR							
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A1b	<p>The manager of Clean Carwash finds, on average, 8 cars arrive per hour on Saturdays. Let the random variable <math>X</math> denote the number of cars arriving in 15 minutes interval. What is the mean number of occurrences per 15 minutes?</p> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/>	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="background-color: #cccccc;">Basic</th> <th style="background-color: #cccccc;">Level</th> </tr> </thead> <tbody> <tr> <td>Excellent</td> <td style="width: 20px;"></td> </tr> <tr> <td>Weak</td> <td></td> </tr> <tr> <td>NR</td> <td></td> </tr> </tbody> </table>	Basic	Level	Excellent		Weak		NR							
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Weak																
NR																
A1c	<p>The marks of Statistics exam is known to be normally distributed with a mean of 51 and a standard deviation of 14. If 200 students in Wantok High School take the exam, what is the minimum mark for an A+ grade if 5% of the students get an A+?</p> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/>	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="background-color: #cccccc;">Advanced</th> <th style="background-color: #cccccc;">Level</th> </tr> </thead> <tbody> <tr> <td>Excellent</td> <td style="width: 20px;"></td> </tr> <tr> <td>Moderate</td> <td></td> </tr> <tr> <td>Low</td> <td></td> </tr> <tr> <td>Weak</td> <td></td> </tr> <tr> <td>NR</td> <td></td> </tr> <tr> <td>Exceed</td> <td></td> </tr> </tbody> </table>	Advanced	Level	Excellent		Moderate		Low		Weak		NR		Exceed	
Advanced	Level															
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Exceed																

A1d A committee of five is to be formed from six men and four women. What is the probability that a randomly selected committee has at least two of each gender on the committee?

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Advanced	Level
Excellent	
Moderate	
Low	
Weak	
NR	
Exceed	

A1e The probability that a runner wins his next race is 0.8 (Event W) and if he does win it his chance of selection for the national team (Event S) will be increased from 0.4 to 0.75. The details are given in the following tree diagram.

```

graph LR
    Root(( )) ---|0.8| W((W))
    Root ---|0.2| Wp((W'))
    W ---|0.75| S1((S))
    W ---|0.25| Sp1((S'))
    Wp ---|0.4| S2((S))
    Wp ---|0.6| Sp2((S'))
    
```

(i) Find the probability that he is selected.

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(ii) If the runner is selected, what is the probability that he did win the race?

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Basic	Level
Excellent	
Weak	
NR	

Basic	Level
Excellent	
Weak	
NR	

## Question 2: Major Learning Outcome B

Model situations using graphical methods in order to solve problems

**Assessor's use only**

A2a

Consider the piecewise function  $f(x) = \begin{cases} 2x + 1, & x \leq 0 \\ x^2, & x > 0 \end{cases}$

(i) Find  $f(0) + f(1)$ .

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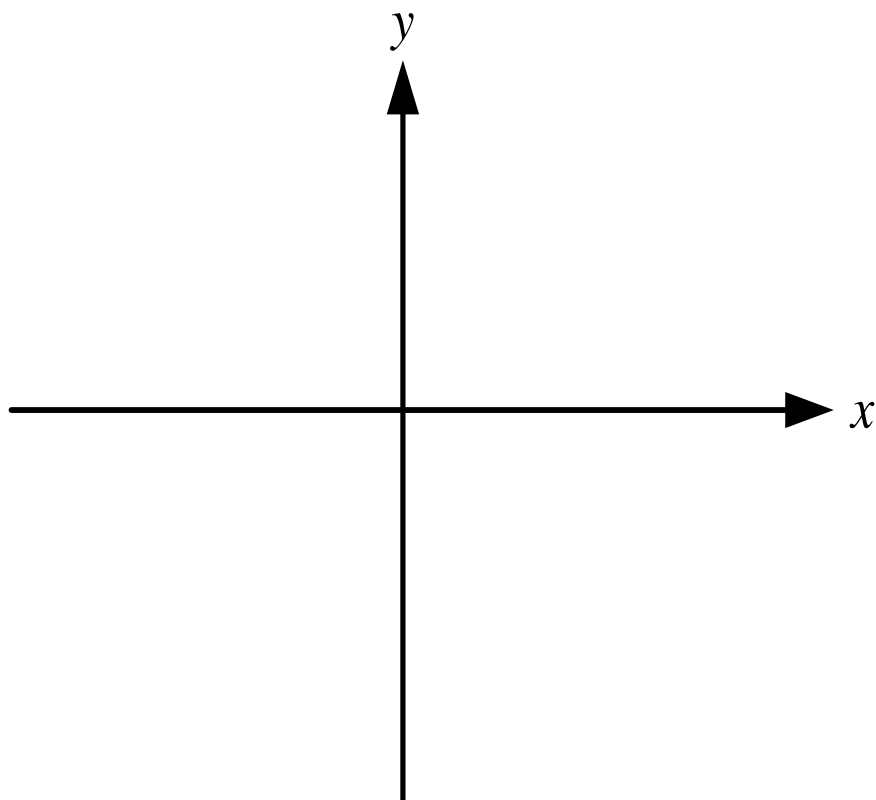


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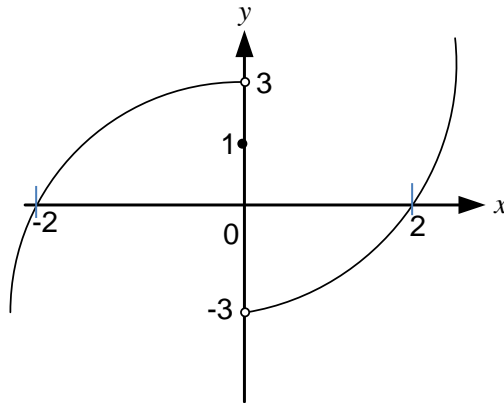
(ii) Sketch the graph of  $f(x)$  clearly marking all relevant intercepts.



Basic	Level
Excellent	
Weak	
NR	

Basic	Level
Excellent	
Weak	
NR	

A2b Refer to the graph of  $y = f(x)$  in the following figure.



Find the value of  $x$  for which  $f(x) = 0$ .

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Basic	Level
Excellent	
Weak	
NR	

A2c Solve  $3^{x+1} = 81$ .

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Basic	Level
Excellent	
Weak	
NR	

A2d Determine whether the given function  $H(x)$  is exponential.

$x$	-1	0	1	2	3
$H(x)$	1/4	1	4	16	64

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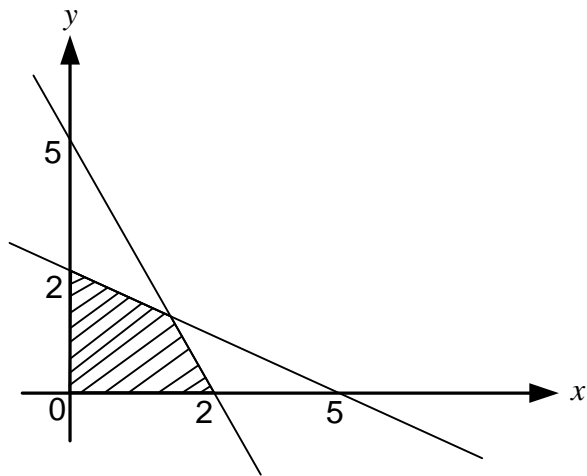
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Basic	Level
Excellent	
Weak	
NR	

A2e Write a system of inequalities that describes the shaded region below.




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Proficient	Level
Excellent	
Moderate	
weak	
NR	

A2f Show in details how the general exponential form  $y = Ab^x$  can be written as a straight line in the form  $Y = c + Xm$ .

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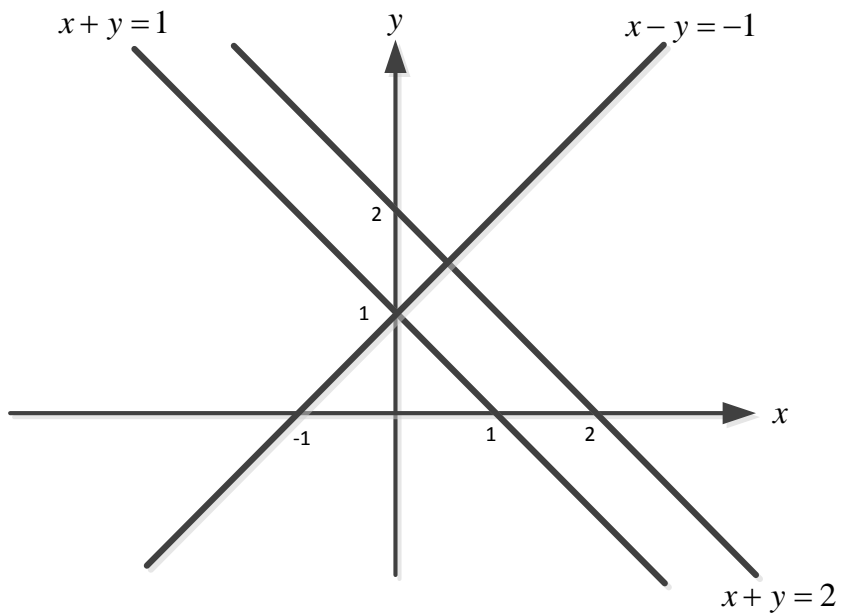
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Advanced	Level
Excellent	
Moderate	
Low	
Weak	
NR	
Exceed	

A2g Clearly shade the feasible region described by the following system:

$$y \geq 0, x \geq -1, x - y \geq -1, x + y \leq 1, x + y \leq 2.$$



Basic	Level
Excellent	
Weak	
NR	

### Question 3: Major Learning Outcome C

Carry out statistical investigations and understand statistical processes.

*Assessor's use only*

A3a	<p>A 95% confidence interval for the difference <math>D</math> between the mean of two populations is <math>(0.3443, 0.4856)</math>. Is there a significant difference between the means of the two populations at the 95% level of confidence? Give reasons.</p> <hr/> <hr/> <hr/> <hr/>	<table border="1"> <thead> <tr> <th>Proficient</th> <th>Level</th> </tr> </thead> <tbody> <tr> <td>Excellent</td> <td></td> </tr> <tr> <td>Moderate</td> <td></td> </tr> <tr> <td>Weak</td> <td></td> </tr> <tr> <td>NR</td> <td></td> </tr> </tbody> </table>	Proficient	Level	Excellent		Moderate		Weak		NR	
Proficient	Level											
Excellent												
Moderate												
Weak												
NR												
A3b	<p><math>(0.102, 0.181)</math> is a 99% confidence interval for the proportion of people who listen to Friendly FM. How should this statement be interpreted?</p> <hr/> <hr/> <hr/> <hr/>	<table border="1"> <thead> <tr> <th>Proficient</th> <th>Level</th> </tr> </thead> <tbody> <tr> <td>Excellent</td> <td></td> </tr> <tr> <td>Moderate</td> <td></td> </tr> <tr> <td>Weak</td> <td></td> </tr> <tr> <td>NR</td> <td></td> </tr> </tbody> </table>	Proficient	Level	Excellent		Moderate		Weak		NR	
Proficient	Level											
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### Question 4: Major Learning Outcome D

Use numeric and algebraic methods to solve problems.

*Assessor's use only*

A4a	<p>Give a geometric interpretation of an inconsistent <math>3 \times 3</math> system of linear equations.</p> <hr/> <hr/> <hr/>	<table border="1"> <thead> <tr> <th>Basic</th> <th>Level</th> </tr> </thead> <tbody> <tr> <td>Excellent</td> <td></td> </tr> <tr> <td>Weak</td> <td></td> </tr> <tr> <td>NR</td> <td></td> </tr> </tbody> </table>	Basic	Level	Excellent		Weak		NR	
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A4b	<p>Why is it that the bisection method fail when applied to <math>f(x) = 1/x</math> on the closed interval <math>[-1, 1]</math>?</p> <hr/> <hr/> <hr/>	<table border="1"> <thead> <tr> <th>Basic</th> <th>Level</th> </tr> </thead> <tbody> <tr> <td>Excellent</td> <td></td> </tr> <tr> <td>Weak</td> <td></td> </tr> <tr> <td>NR</td> <td></td> </tr> </tbody> </table>	Basic	Level	Excellent		Weak		NR	
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Weak										
NR										
A4c	<p>The following is the augmented matrix of a system of linear equations.</p> $\begin{bmatrix} 1 & 4 & 2 & 23 \\ 1 & 1 & 1 & 15 \\ 2 & 8 & 4 & 46 \end{bmatrix}$ <p>By inspection, how many solutions are there?</p> <p>DO NOT ATTEMPT TO SOLVE THE SYSTEM.</p> <hr/> <hr/> <hr/>	<table border="1"> <thead> <tr> <th>Basic</th> <th>Level</th> </tr> </thead> <tbody> <tr> <td>Excellent</td> <td></td> </tr> <tr> <td>Weak</td> <td></td> </tr> <tr> <td>NR</td> <td></td> </tr> </tbody> </table>	Basic	Level	Excellent		Weak		NR	
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A4d	<p>Comment on a possible scenario where the Newton-Raphson method may fail to converge to a root.</p> <hr/> <hr/> <hr/> <hr/>	<table border="1"> <thead> <tr> <th>Basic</th> <th>Level</th> </tr> </thead> <tbody> <tr> <td>Excellent</td> <td></td> </tr> <tr> <td>Weak</td> <td></td> </tr> <tr> <td>NR</td> <td></td> </tr> </tbody> </table>	Basic	Level	Excellent		Weak		NR	
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NR										

## SECTION B: LONG ANSWERS

### Question 1: Major Learning Outcome A

Develop knowledge and skills related to Probability in order to solve problems and to investigate situations involving elements of chance.

*Assessor's use only*

<b>B1a</b>	<p>The events <math>X</math> and <math>Y</math> are such that <math>P(X) = 0.5</math>, <math>P(Y) = 0.7</math>, and <math>P(X \cap Y) = 0.2</math>. Find:</p> <p>(i) <math>P(X \cup Y)</math></p> <hr style="border: 0.5px solid black;"/> <hr style="border: 0.5px solid black;"/> <p>(ii) <math>P(Y')</math></p> <hr style="border: 0.5px solid black;"/> <hr style="border: 0.5px solid black;"/> <p>(iii) <math>P(X Y)</math></p> <hr style="border: 0.5px solid black;"/> <hr style="border: 0.5px solid black;"/> <hr style="border: 0.5px solid black;"/>	<table border="1" style="width: 100%; border-collapse: collapse; margin-bottom: 10px;"> <thead> <tr> <th style="width: 50%; text-align: center;">Basic</th> <th style="width: 50%; text-align: center;">Level</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">Excellent</td> <td style="width: 20px;"></td> </tr> <tr> <td style="text-align: center;">Weak</td> <td></td> </tr> <tr> <td style="text-align: center;">NR</td> <td></td> </tr> </tbody> </table> <table border="1" style="width: 100%; border-collapse: collapse; margin-bottom: 10px;"> <thead> <tr> <th style="width: 50%; text-align: center;">Basic</th> <th style="width: 50%; text-align: center;">Level</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">Excellent</td> <td></td> </tr> <tr> <td style="text-align: center;">Weak</td> <td></td> </tr> <tr> <td style="text-align: center;">NR</td> <td></td> </tr> </tbody> </table> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 50%; text-align: center;">Basic</th> <th style="width: 50%; text-align: center;">Level</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">Excellent</td> <td></td> </tr> <tr> <td style="text-align: center;">Weak</td> <td></td> </tr> <tr> <td style="text-align: center;">NR</td> <td></td> </tr> </tbody> </table>	Basic	Level	Excellent		Weak		NR		Basic	Level	Excellent		Weak		NR		Basic	Level	Excellent		Weak		NR	
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B1b The probability that Jane failing History (Event H) is 0.3 while the probability that she fails English (Event E) is 0.4. The probability that she fails both subjects is 0.2

(i) Are events H and E mutually exclusive? Explain.

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(ii) Determine the probability that Jane passes both History and English.

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Basic	Level
Excellent	
Weak	
NR	

Proficient	Level
Excellent	
Moderate	
Weak	
NR	

<p>B1c</p>	<p>Melanesian Limited finds that 40% of the items produced in their plant are defective. During a random inspection, the floor manager selected six items from the production line. Find the probability that the floor manager finds:</p> <p>(i) Two defectives.</p> <hr/> <hr/> <hr/> <hr/> <hr/> <p>(ii) At least two defectives.</p> <hr/> <hr/> <hr/> <hr/> <hr/>	<table border="1"> <thead> <tr> <th>Basic</th> <th>Level</th> </tr> </thead> <tbody> <tr> <td>Excellent</td> <td></td> </tr> <tr> <td>Weak</td> <td></td> </tr> <tr> <td>NR</td> <td></td> </tr> </tbody> </table> <table border="1"> <thead> <tr> <th>Proficient</th> <th>Level</th> </tr> </thead> <tbody> <tr> <td>Excellent</td> <td></td> </tr> <tr> <td>Moderate</td> <td></td> </tr> <tr> <td>Weak</td> <td></td> </tr> <tr> <td>NR</td> <td></td> </tr> </tbody> </table>	Basic	Level	Excellent		Weak		NR		Proficient	Level	Excellent		Moderate		Weak		NR	
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B1d Consider the following probability distribution of the random variable  $X$ .

$x$	0	1	2	3	4
$P(X = x)$	1/16	3/16	7/16	3/16	2/16

Find:

(i)  $E[X]$ .

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(ii)  $\text{Var}(4X + 2)$ .

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Basic	Level
Excellent	
Weak	
NR	

Proficient	Level
Excellent	
Moderate	
Weak	
NR	

B1e The mean number of students entering the ABC Bookshop during one minute is 10. Given that this situation can be modelled by a Poisson distribution, find the probability of:

(i) Exactly eight students entering during one minute.

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Basic	Level
Excellent	
Weak	
NR	

(ii) Fewer than two entering during one minute.

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Basic	Level
Excellent	
Weak	
NR	

(iii) At least three entering during one minute.

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Proficient	Level
Excellent	
Moderate	
Weak	
NR	

B1f

The probability distribution of the random variable  $X$  is given by the following function.

$$P(X = x) = \frac{k}{x}, \quad x = 1, 2, 3, 4.$$

Find:

(i) The value of  $k$ .

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(ii)  $P(2 \leq X \leq 4)$ .

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Basic	Level
Excellent	
Weak	
NR	

Basic	Level
Excellent	
Weak	
NR	







B2c The variable  $x$  and  $y$  are related by the rule  $y^3x^2 = 6$ .

(i) Show that  $y^3x^2 = 6$  can be written in the form  $y = ax^n$ . Find  $a$  and  $n$ .

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(ii) Calculate  $x$  when  $y = 0.62$ .

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Basic	Level
Excellent	
Weak	
NR	

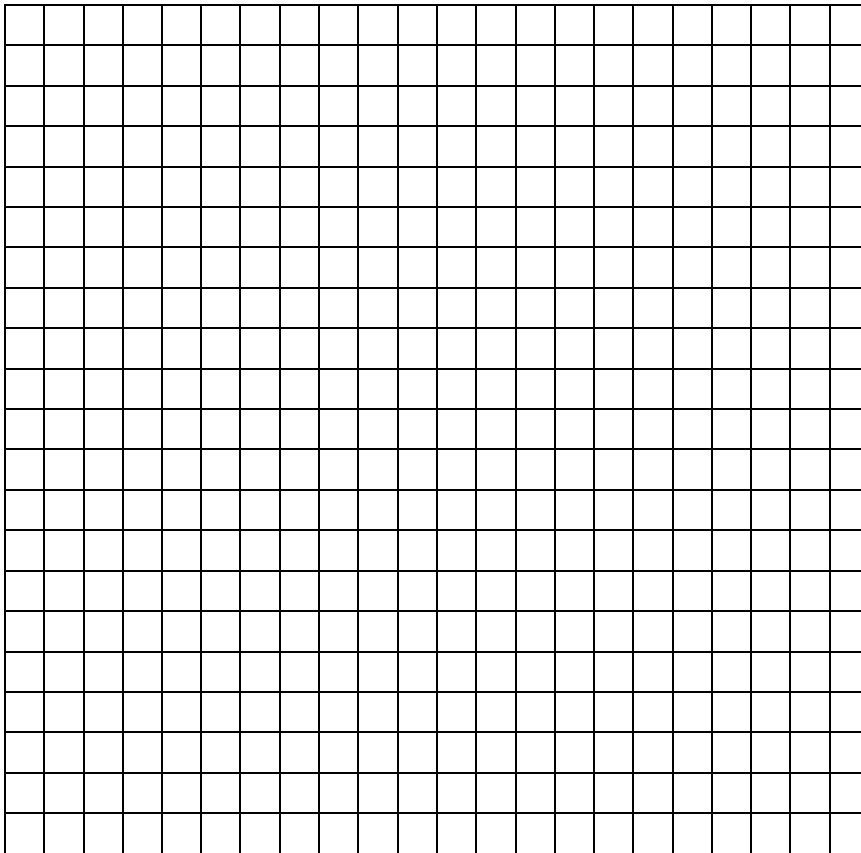
Basic	Level
Excellent	
Weak	
NR	

B2d Consider the points (1.5, 0.77), (2.7, 0.27), (3.6, 0.16).

(i) Fill in the following table.

ln $x$			
ln $y$			

(ii) It is known that  $x$  and  $y$  are related by a formula of the type  $\ln y = \ln A + B \ln x$ .  
Use a suitable graph to give estimates of  $A$  and  $B$ .




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Basic	Level
Excellent	
Weak	
NR	

Basic	Level
Excellent	
Weak	
NR	

B2e

Without graphing, determine which of the points  $P_1(4, -3)$ ,  $P_2(2, -6)$ , and  $P_3(8, -3)$  are part of the graph of the following system of linear equations.

$$\begin{aligned} x + 2y &\leq 8 \\ 2x - y &\geq 4 \end{aligned}$$

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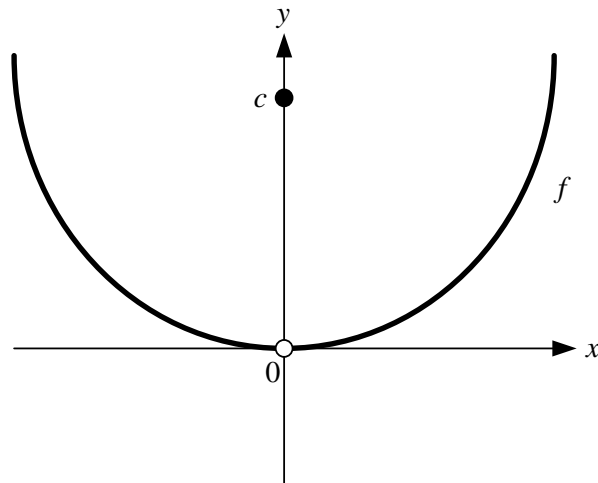
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Proficient	Level
Excellent	
Moderate	
weak	
NR	
Exceed	

B2f

Give the equation of the function  $f(x)$  whose graph is given below and that  $f(-2) = f(2) = 4$ .




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Basic	Level
Excellent	
Weak	
NR	

**Question 3: Major Learning Outcome C**

Carry out statistical investigations and understand statistical processes.

**Assessor's use only**

B3a Consider the confidence interval

$$60 - 1.645 \frac{17}{\sqrt{30}} < \mu < 60 + 1.645 \frac{17}{\sqrt{30}}$$

for the population mean of a special variety of giant taro. The population standard deviation is not known.

(i) What is the sample mean?

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Basic	Level
Excellent	
Weak	
NR	

(ii) How many are there in the sample?

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Basic	Level
Excellent	
Weak	
NR	

(iii) What is the degree of confidence of the interval?

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Basic	Level
Excellent	
Weak	
NR	

(iv) What is the margin of error?

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Basic	Level
Excellent	
Weak	
NR	



**Question 4: Major Learning Outcome D**

Use numeric and algebraic methods to solve problems.

*Assessor's use only*

B4a

Determine the value of  $k$  for which the following system of linear equations has no solution.

$$\begin{aligned} 2x + 5y &= 10 \\ x - ky &= -1 \end{aligned}$$

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Proficient	Level
Excellent	
Moderate	
Weak	
NR	

B4b Solve the following system of linear equations.

You must show all relevant working.

$$\begin{array}{rcccccl} x & + & y & + & z & = & 4 \\ -x & + & 2y & & & = & 5 \\ & & 5y & - & z & = & 7 \end{array}$$

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Advanced	Level
Excellent	
Moderate	
Low	
Weak	
NR	
Exceed	





B4d

Let  $f(x) = 2x^3 - 1.8x^2 + 2x - 1.8$ . By taking  $x_0 = 1.1$ , find the first two iterates obtained by using Newton-Raphson Method to solve  $f(x) = 0$ . Give answers correct to 4 decimal places.

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Basic	Level
Excellent	
Weak	
NR	

B4e A quadratic function has the form  $f(x) = ax^2 + bx + c$ , where  $a, b, c$  are constants. Suppose  $f(1) = 2$ ,  $f(2) = 4$ , and  $f(3) = 6$ . Without solving any equation, explain how you would go about finding the constants  $a, b, c$ .

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Basic	Level
Excellent	
Weak	
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THE END