EDUCATIONAL QUALITY AND ASSESSMENT PROGRAMME [EQAP]

Pacific Community
Communauté du Pacifique

SOUTH PACIFIC FORM SEVEN CERTIFICATE [SPFSC]

MATHEMATICS with STATISTICS PRESCRIPTION

GENERAL INFORMATION

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SOUTH PACIFIC FORM SEVEN CERTIFICATE
MATHEMATICS with STATISTICS

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MATHEMATICS WITH STATISTICS

PREAMBLE AND RATIONALE

The SPFSC Mathematics with Statistics course has undergone a review and an alignment. Three main factors contributed to this review.

1. The need to consider international trends in the teaching of Statistics and the subsequent expectations of universities in the Oceania region on what statistical understanding students should possess.
2. The need to raise student achievement by "doing less, but doing it better", which reflects another international trend in education.
3. The influence of changes in technology on the curriculum content.
4. The need for constructive alignment between the intentions and results of teaching and learning.

The main changes made to the prescription were:

- The introduction of the analysis of bivariate data. It was considered that this was an important skill for further study at university across many fields.
- A focus on understanding and using the statistical investigation process in its complete form, as opposed to assessing small parts of it in isolation. This is in alignment with international trends and the need to have students who are more data literate.
- The introduction of three compulsory internal statistical investigations to acknowledge the three different data analysis methods: Bivariate analysis; Time series analysis; and making inferences to compare two populations. Again this is to help develop more data literate students.
- A focus on using methods to model situations and to solve problems rather than looking at sub-skills in isolation. This is in alignment with international trends.
- The removal of the Calculus topic to allow time for a more in-depth study of other topics. It was considered that those students who needed Calculus as a future pathway would need to enter the Mathematics in Calculus course anyway.
- The removal of the Sequences and Series topic was to allow time for more in-depth study of other topics. It was considered that much of the content was a repetition of what was covered in the national Year 12 prescription and provided no future pathways for student enrolled in this course.
- The removal of binomial expansions through the use of the Binomial Theorem. It was considered that this aspect was no longer relevant in the context of this course and that a focus on permutations and combinations in the context of Probability was more suitable.
- The removal of approximating probability distributions and the introduction of combining probability distributions instead. With the changes in technology approximating distributions has less relevance, whereas the ability to deal with more complex probability situations is now seen as being of importance.

- The inclusion of clear learning outcome's at different levels of generality (Major Learning Outcomes, Key Learning Outcomes, Specific Learning Outcomes)

The course content was then reorganized under four specific learning objectives and weightings were allocated to each objective based on teaching and learning time (including time needed for both formative and summative assessment). This, along with the change of emphasis towards understanding and using the statistical investigation process in its completeness, has led to a change in the internal and external assessment weightings.

COURSE AIM

The aim for this course is for students to realise the importance of Statistics and modelling as a means for understanding what is happening in the real world.

PREREQUISITES

This aligned prescription defines the requirements for the South Pacific Form Seven Certificate Mathematics (with Statistics) in terms of unpacked learning outcomes, external examination and internal assessment requirements.

Each of the learning outcomes for the course is to be read in conjunction with the Explanatory Notes given for each outcome in this prescription.

Students also require knowledge and understanding of outcomes from the national Year 12 Mathematics prescription, which are related to the specific outcomes of this prescription.

The course is designed for students who may undertake further studies in a tertiary institution as well as for those students who will complete their formal education at the end of Year 13.

GENERAL OBJECTIVES

Students will be expected to:

1. develop knowledge and skills related to Probability in order to solve problems and to investigate situations involving elements of chance.

2. model situations using graphical methods in order to solve problems

3. carry out statistical investigations and understand statistical processes.

4. use numeric and algebraic methods to solve problems

CONTENT COMPONENTS

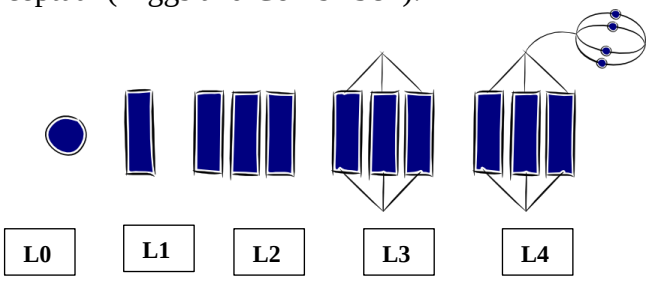
The content of the SPFSC Statistics course is organised under four strands and a number of sub-strands under each strand. These are outlined below:

STRAND NUMBER	STRAND TITLE	SUB STRAND NUMBER	SUB-STRAND TITLE
1.	Probability	1.1	Probability
		1.2	Discrete probability distribution
		1.3	Probability distribution
2.	Modelling using graphical methods	2.1	Piecewise functions
		2.2	Power & Exponential functions
		2.3	Linear programming
3.	Statistical Investigations	3.1	Bivariate data
		3.2	Time series
		3.3	Comparing two population
		3.4	Statistical processes
4.	Numerical and Algebraic methods	4.1	Linear equations
		4.2	Non-linear equations

UNPACKING LEARNING OUTCOMES

In this prescription, Learning Outcomes are stated at three levels of generality: Major Learning Outcomes (MLOs) are stated at the strand level, Key Learning Outcomes (KLOs) are stated at the sub-strand level, and Specific Learning Outcomes (SLOs) are unpacked from the Key Learning Outcomes. Each SLO is a combination of a cognitive skill and a specific content component. Each SLO is given a skill level, level 1 – 4, and this skill level results from the categorisation of the cognitive skill that is embedded in the SLO using the SOLO taxonomy[1].

The SOLO taxonomy provides a simple, reliable and robust model for three levels of understanding – surface deep and conceptual (Biggs and Collis 1982).



At the **prestructural** level (L0) of understanding, the task is inappropriately attacked, and the student has missed the point or needs help to start. The next two levels, unistructural and multistructural are associated with bringing in information (surface understanding). At the **unistructural** level (L1), one aspect of the task is picked up, and student understanding is disconnected and limited. The jump to the multistructural level is quantitative. At the **multistuctural** level (L2), several aspects of the task are known but their relationships to each other and the whole are missed. The progression to relational and extended abstract outcomes is qualitative. At the **relational** level (L3), the aspects are linked and integrated, and contribute to a deeper and more coherent understanding of the whole. At the **extended abstract** level (L4), the new understanding at the relational level is re-thought at another conceptual level, looked at in a new way, and used as the basis for prediction, generalisation, reflection, or creation of new understanding (adapted from Hook and Mills 2011). [http://pamhook.com/solo-taxonomy/..]

The progression from Level 1 to Level 4 is exemplified in the progression from *define* →*describe* → *explain* → *discuss* with each succeeding level indicating a *higher level of understanding*, as follows:

- **define** – to state a basic definition of a concept [Unistructural or L1]
- **describe** – to give the characteristics of, or give an account of, or provide annotated diagrams. [Multistructural or L2]
- **explain** – to provide a reason for a relationship – an event and its impact, a cause and an effect, as to *how* or *why* something occurs. [Relational or L3]
- **discuss** – this means *linking ideas* (descriptions, explanations) to make generalisations or predictions or evaluations. It may involve relating, comparing, analysing, and justifying.
- **solve/calculate/compute** – to carry out a series of algorithms to arrive at a solution [Multistructural (L2) or Relational (L3) or even Extended (L4) depending on the complexity of the algorithm]. If there are two 'loadings' in the calculations (a standard problem) then skill level would be L2, if three 'loadings' (a complex problem) then L3 and four loadings (a more complex problem) for L4.

[1] Structure of Observed Learning Outcomes by Biggs and Collis (1982)

LEARNING OUTCOMES

STRAND 1: Probability

Major Learning Outcome (sta1):

Students are able to develop knowledge and skills related to Probability in order to solve problems and to investigate situations involving elements of chance.

SUB STRAND 1.1: PROBABILITY

Key Learning Outcome (sta1.1):

Students are able to solve probability problems.

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
1.	**define** an *event* in statistics.	1	Sta1.1.1.1
2.	**identify** an example of an event.	1	Sta1.1.1.2
3.	**define** *trial.*	1	Sta1.1.1.3
4.	**define** *outcome.*	1	Sta1.1.1.4
5.	**define** *sample space.*	1	Sta1.1.1.5
6.	**define** *mutually exclusive events.*	1	Sta1.1.1.6
7.	**identify** an example of mutually exclusive events.	1	Sta1.1.1.7
8.	**define** i*ndependent events.*	1	Sta1.1.1.8
9.	**identify** an example of independent events.	1	Sta1.1.1.9
10.	**define** *complementary events.*	1	Sta1.1.1.10
11.	**identify** an example of complementary events.	1	Sta1.1.1.11
12.	**define** *conditional events.*	1	Sta1.1.1.12
13.	**identify** an example of conditional events.	1	Sta1.1.1.13
14.	**define** *inclusive events.*	1	Sta1.1.1.14
15.	**identify** an example of inclusive events.	1	Sta1.1.1.15
16.	**define** *combined events.*	1	Sta1.1.1.16
17.	**identify** an example of combined events.	1	Sta1.1.1.17
18.	**compute** probability of an event.	2	Sta1.1.2.1
19.	**solve** standard mutually exclusive events problems.	2	Sta1.1.2.2
20.	**solve** standard independent events problems.	2	Sta1.1.2.3
21.	**solve** standard complementary events problems.	2	Sta1.1.2.4
22.	**solve** standard conditional event problems.	2	Sta1.1.2.5
23.	**solve** standard problems using tree diagrams techniques.	2	Sta1.1.2.6
24.	**solve** standard problems using venn diagrams techniques.	2	Sta1.1.2.7
25.	**solve** standard problems using theoretical and experimental probability techniques.	2	Sta1.1.2.8
26.	**solve** standard problems using theoretical and experimental probability techniques on exclusive events.	2	Sta1.1.2.9
27.	**solve** standard problems using permutations techniques.	2	Sta1.1.2.10
28.	**solve** standard problems using combinations techniques.	2	Sta1.1.2.11
29.	**solve** standard problems using theoretical and experimental probability techniques on either/or events.	2	Sta1.1.2.12
30.	**solve** standard problems using theoretical and experimental probability techniques on at least one event.	2	Sta1.1.2.13
31.	**solve** standard probability of inclusive events (both conditions are true).	2	Sta1.1.2.14

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
32.	**determine** the probability of complements of events.	2	Sta1.1.2.15
33.	**solve** standard combined event problems.	2	Sta1.1.2.16
34.	**solve** mutually exclusive events problems.	3	Sta1.1.3.1
35.	**solve** complex independent events problems.	3	Sta1.1.3.2
36.	**solve** complex complementary events problems.	3	Sta1.1.3.3
37.	**solve** complex conditional event problems.	3	Sta1.1.3.4
38.	**solve** complex combined event problems.	3	Sta1.1.3.5
39.	**solve** complex problems using tree diagrams techniques.	3	Sta1.1.3.6
40.	**solve** complex problems using venn diagrams techniques.	3	Sta1.1.3.7
41.	**solve** complex problems using tables of counts and relative frequencies techniques.	3	Sta1.1.3.8
42.	**solve** complex problems using theoretical and experimental probability techniques.	3	Sta1.1.3.9
43.	**solve** complex problems using permutations techniques.	3	Sta1.1.3.10
44.	**solve** complex problems using combinations techniques.	3	Sta1.1.3.11
45.	**solve** more complex mutually exclusive events problems.	4	Sta1.1.4.1
46.	**solve** more complex independent events problems.	4	Sta1.1.4.2
47.	**solve** more complex complementary events problems.	4	Sta1.1.4.3
48.	**solve** more complex conditional event problems.	4	Sta1.1.4.4
49.	**solve** more complex combined event problems.	4	Sta1.1.4.5
50.	**solve** more complex problems using tree diagrams techniques.	4	Sta1.1.4.6
51.	**solve** more complex problems using venn diagrams techniques.	4	Sta1.1.4.7
52.	**solve** more complex problems using tables of counts and relative frequencies techniques.	4	Sta1.1.4.8
53.	**solve** more complex problems using theoretical and experimental probability techniques.	4	Sta1.1.4.9
54.	**solve** more complex problems using permutations techniques.	4	Sta1.1.4.10
55.	**solve** more complex problems using combinations techniques.	4	Sta1.1.4.11

SUB STRAND 1.2: DISCRETE PROBABILITY DISTRIBUTION

Key Learning Outcome (sta1.2):
Solve problems involving expected values and variance of random variables from a discrete probability distribution.

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
1.	**define** *random variable.*	1	Sta1.2.1.1
2.	**define** *discrete random variable.*	1	Sta1.2.1.2
3.	**define** *continuous random variable.*	1	Sta1.2.1.3
4.	**define** *variance.*	1	Sta1.2.1.4
5.	**define** *standard deviation.*	1	Sta1.2.1.5
6.	**define** *expected value.*	1	Sta1.2.1.6
7.	**define** *probability distribution.*	1	Sta1.2.1.7
8.	**calculate** the expected value of discrete random variables in a standard problem.	2	Sta1.2.2.1
9.	**calculate** the variance of discrete random variables in a standard problem.	2	Sta1.2.2.2
10.	**calculate** the standard deviation of discrete random variables in a standard problem.	2	Sta1.2.2.3
11.	**calculate** the expected value of linear functions of random variables in a standard problem.	2	Sta1.2.2.4
12.	**calculate** the variance of linear functions of random variables in a standard problem.	2	Sta1.2.2.5

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
13.	**calculate** the expected value of sums of independent random variables in a standard problem.	2	Sta1.2.2.6
14.	**calculate** the variance of sums of independent random variables in a standard problem.	2	Sta1.2.2.7
15.	**calculate** and interpret the expected value of discrete random variables in a complex problem.	3	Sta1.2.3.1
16.	**calculate and interpret** the variance of discrete random variables in a complex problem.	3	Sta1.2.3.2
17.	**calculate and interpret** the standard deviation of discrete random variables in a complex problem.	3	Sta1.2.3.3
18.	**calculate and interpret** the expected value of linear functions of random variables in a complex problem.	4	Sta1.2.4.1
19.	**calculate and interpret** the variance of linear functions of random variables in a complex problem.	4	Sta1.2.4.2
20.	**calculate and interpret** the expected value of sums of independent random variables in a complex problem.	4	Sta1.2.4.3
21.	**calculate and interpret** the variance of sums of independent random variables in a complex problem.	4	Sta1.2.4.4

Explanatory Notes

Note: The random variables could come from both given probability distributions and ones that can be developed from tree diagrams.

SUB STRAND 1.3: PROBABILITY DISTRIBUTION

Key Learning Outcome (sta1.3):

Using appropriate probability distributions to model a given situation and solve problems.

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
1.	**define** parameters.	1	Sta1.3.1.1
2.	**identify** properties of the binomial distribution.	1	Sta1.3.1.2
3.	**identify** parameters of a binomial distribution.	1	Sta1.3.1.3
4.	**compute** probabilities associated with a discrete probability distribution.	1	Sta1.3.1.4
5.	**identify** properties of the Poisson distribution.	1	Sta1.3.1.5
6.	**identify** the parameter of a Poisson distribution.	1	Sta1.3.1.6
7.	**state** the properties of a normal distribution.	1	Sta1.3.1.7
8.	**identify** parameters of a normal distribution.	1	Sta1.3.1.8
9.	**compute** parameters [n, p] of a binomial distribution.	2	Sta1.3.2.1
10.	**solve** problems involving number of events using binomial distribution.	2	Sta1.3.2.2
11.	**solve** problems involving probabilities using binomial distribution.	2	Sta1.3.2.3
12.	**solve** problems using binomial distribution.	2	Sta1.3.2.4
13.	**calculate** parameter [λ] for a Poisson distribution.	2	Sta1.3.2.5
14.	**compute** probabilities associated with a Poisson distribution.	2	Sta1.3.2.6
15.	**solve** problems using Poisson distribution.	2	Sta1.3.2.7
16.	**calculate** parameters [n,μ,σ] for a normal distribution.	2	Sta1.3.2.8
17.	**compute** probabilities associated with standard normal distribution.	2	Sta1.3.2.9
18.	**compute** probabilities associated with normal distribution.	2	Sta1.3.2.10
19.	**solve** problems using standard normal distribution.	2	Sta1.3.2.11
20.	**solve** problems using normal distribution.	2	Sta1.3.2.12

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
21.	**calculate** probabilities using formulae and tables.	2	Sta1.3.2.13
22.	**solve** inverse normal type problems.	2	Sta1.3.2.14
23.	**calculate** parameters [n, p] for a binomial distribution.	3	Sta1.3.3.1
24.	**solve** problems using binomial distribution.	3	Sta1.3.3.2
25.	**calculate** parameters for a Poisson distribution.	3	Sta1.3.3.3
26.	**solve** problems using Poisson distribution.	3	Sta1.3.3.4
27.	**calculate** parameters [n,μ,σ] for a normal distribution.	3	Sta1.3.3.5
28.	**solve** problems using normal distribution.	3	Sta1.3.3.7
29.	**calculate** probabilities using formulae and tables.	3	Sta1.3.3.8
30.	**solve** inverse normal type problems.	3	Sta1.3.3.9
31.	**solve** problems involving sum and difference of independent normal distribution random variables.	3	Sta1.3.3.10
32.	**solve** problems involving combined distributions (e.g. a two-step problem requiring first a normal distribution calculation and then a binomial distribution calculation).	3	Sta1.3.3.11
33.	**calculate** parameters [n, p] for a binomial distribution.	4	Sta1.3.4.1
34.	**model** problems using binomial distribution.	4	Sta1.3.4.2
35.	**calculate** parameters for a Poisson distribution.	4	Sta1.3.4.3
36.	**model** problems using Poisson distribution.	4	Sta1.3.4.4
37.	**calculate** parameters [n, μ, σ] for a normal distribution.	4	Sta1.3.4.5
38.	**model** problems using normal (including standard normal) distribution.	4	Sta1.3.4.6
39.	**solve** inverse normal type problems.	4	Sta1.3.4.7
40.	**solve** problems involving sum and difference of independent normal distribution random variables.	4	Sta1.3.4.8
41.	**solve** problems involving combined distributions (e.g. a two-step problem requiring first a normal distribution calculation and then a binomial distribution calculation).	4	Sta1.3.4.9

STRAND 2: Modelling Using Graphical Methods

Major Learning Outcome (sta2):

Students are able to model situations using graphical methods in order to solve problems.

SUB STRAND 2.1: PIECEWISE FUNCTION

Key Learning Outcome (sta2.1):

Understanding simple piecewise functions in order to model data.

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
1.	**state** features of a linear function.	1	Sta2.1.1.1
2.	**state** features of a quadratic function.	1	Sta2.1.1.2
3.	**define** *continuous function*.	1	Sta2.1.1.3
4.	**define** *discontinuous function*.	1	Sta2.1.1.4
5.	**state** properties of a continuous function.	1	Sta2.1.1.5
6.	**state** properties of a discontinuous function.	1	Sta2.1.1.6
7.	**sketch** piecewise functions.	2	Sta2.1.2.1
8.	**determine** equations from graphs of piecewise functions.	2	Sta2.1.2.2
9.	**interpret** piecewise functions which have components that are linear.	2	Sta2.1.2.3
10.	**interpret** piecewise functions which have components that are quadratic.	2	Sta2.1.2.4
11.	**interpret** piecewise functions which have components that are linear and quadratic.	2	Sta2.1.2.5
12.	**determine** values from graphs of a discontinuous function.	2	Sta2.1.2.6
13.	**determine** values from graphs of piecewise functions.	2	Sta2.1.2.7
14.	**interpret** piecewise functions which have components that are linear.	3	Sta2.1.3.1
15.	**interpret** piecewise functions which have components that are quadratic.	3	Sta2.1.3.2
16.	**interpret** piecewise functions which have components that are linear and quadratic.	3	Sta2.1.3.3
17.	**sketch** piecewise functions.	3	Sta2.1.3.4
18.	**determine** equations from graphs of piecewise functions.	3	Sta2.1.3.5
19.	**interpret** piecewise functions which have components that are linear.	4	Sta2.1.4.1
20.	**interpret** piecewise functions which have components that are quadratic.	4	Sta2.1.4.2
21.	**interpret** piecewise functions which have components that are linear and quadratic.	4	Sta2.1.4.3
22.	**sketch** piecewise functions.	4	Sta2.1.4.4
23.	**determine** equations from graphs of piecewise functions.	4	Sta2.1.4.5

SUB STRAND 2.2: POWER & EXPONENTIAL FUNCTIONS

Key Learning Outcome (sta2.2):
Model real data using power and exponential functions in order to solve problems.

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
1.	**state** the general form of a power function.	1	Sta2.2.1.1
2.	**state** the general forms of exponential functions.	1	Sta2.2.1.2
3.	**identify** parts of a power function.	1	Sta2.2.1.4
4.	**identify** parts of an exponential function.	1	Sta2.2.1.5
5.	**use** logs to calculate values of unknowns.	1	Sta2.2.1.6
6.	**draw** graphs of power functions.	1	Sta2.2.1.7
7.	**draw** graphs of exponential functions.	1	Sta2.2.1.8
8.	**solve** power functions.	2	Sta2.2.2.10
9.	**solve** exponential functions.	2	sta2.2.2.1
10.	**use** log-log technique to transform variables to relate power functions of the form $y=ax^n$ (where n is a positive integer) to model real data.	2	sta2.2.2.2
11.	**use** semi-log technique to transform variables to relate exponential (index) functions of the form $y=am^x$ (where m can be any base) or $y=Ae^{kx}$ to model real data.	2	sta2.2.2.3
12.	**solve** problems using the models.	2	sta2.2.2.4
13.	**solve** problems by considering limitations of the model visually.	2	sta2.2.2.5
14.	**use** natural logs to calculate values of unknowns.	2	sta2.2.2.6
15.	**show** that variables are related by a power law.	2	sta2.2.2.7
16.	**show** that variables are related by an exponential law.	2	sta2.2.2.8
17.	**draw** graphs of hyperbolic functions.	2	sta2.2.2.9
18.	**use** log-log technique to transform variables to relate power functions of the form $y=ax^n$ (where ^{n} is a positive integer) to model real data.	3	sta2.2.3.1
19.	**use** semi-log technique to transform variables to relate exponential (index) functions of the form $y= am^x$ (where m can be any base) or $y=Ae^{kx}$ to model real data.	3	sta2.2.3.2
20.	**solve** problems using the models.	3	sta2.2.3.3
21.	**solve** problems by considering limitations of the model visually.	3	sta2.2.3.4
22.	**use** log-log technique to transform variables to relate power functions of the form $y= ax^n$ (where n is a positive integer) to model real data.	4	sta2.2.4.1
23.	**use** semi-log technique to transform variables to relate exponential (index) functions of the form $y= am^x$ (where m can be any base) or $y= y=Ae^{kx}$ to model real data.	4	sta2.2.4.2
24.	**solve** problems using the models.	4	sta2.2.4.3

SUB STRAND 2.3: LINEAR PROGRAMMING

Key Learning Outcome (sta2.3):
Model situations using linear programming techniques to obtain optimal solution.

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
1.	**state** features of inequation functions.	1	sta2.3.1.1
2.	**identify** regions that are represented by inequation functions.	1	sta2.3.1.2
3.	**define** *objective function*.	1	sta2.3.1.3
4.	**define** *constraints.*	1	sta2.3.1.4
5.	**define** *vertex.*	1	sta2.3.1.5
6.	**identify** the objective function (function that is to be maximized or minimized) from a problem.	2	sta2.3.2.1
7.	**identify** the constraints (the inequations that show the restrictions that must be considered) from a problem (NB: in some cases the constraints may be given).	2	sta2.3.2.2
8.	**graph** the constraints to obtain the feasible region and shading the region (shading in or shading out).	2	sta2.3.2.3
9.	**compute** the point where the objective function is maximized or minimized to solve the problem.	2	sta2.3.2.4
10.	**consider** the possibility of multiple solutions when solving a problem.	2	sta2.3.2.5
11.	**identify** the objective function (function that is to be maximized or minimized) from a problem.	3	sta2.3.3.1
12.	**identify** the constraints (the inequations that show the restrictions that must be considered) from a problem (NB: in some cases the constraints may be given)	3	sta2.3.3.2
13.	**graph** the constraints to obtain the feasible region and shading the region (shading in or shading out).	3	sta2.3.3.3
14.	**compute** the point where the objective function is maximized or minimized to solve the problem.	3	sta2.3.3.4
15.	**consider** the possibility of multiple solutions when solving a problem.	3	sta2.3.3.5
16.	**identify** the objective function (function that is to be maximized or minimized) from a problem.	4	sta2.3.4.1
17.	**construct** the constraints (the inequations that show the restrictions that must be considered) from a problem (NB: in some cases the constraints may be given).	4	sta2.3.4.2
18.	**graph** the constraints to obtain the feasible region and shading the region (shading in or shading out).	4	sta2.3.4.3
19.	**consider** the possibility of multiple solutions when solving a problem.	4	sta2.3.4.4
20.	**compute** the point where the objective function is maximized or minimized to solve the problem.	4	sta2.3.4.5

STRAND 3: Statistical Investigations

Major Learning Outcome (sta3):
Students are able to carry out statistical investigations and understand statistical processes.

SUB STRAND 3.1: BIVARIATE DATA

Key Learning Outcome (sta3.1):
Using statistical processes to undertake an investigation of bivariate data.

	Specific Learning Outcomes (SLO): Students are able to	Skill level	SLO code
1.	**define** *bivariate data.*	1	sta3.1.1.1
2.	**state** the Pearson's correlation coefficient, **r**, formula.	1	sta3.1.1.2
3.	**state** the properties of the Pearson's correlation coefficient, **r.**	1	sta3.1.1.3
4.	**state** features of a scatter plot.	1	sta3.1.1.4
5.	**identify** variables of interest from a given data set.	1	sta3.1.1.5
6.	**collect** data of interest from a given data set.	1	sta3.1.1.6
7.	**display** data of interest from a given data set.	1	sta3.1.1.7
8.	**identify** trends of a relationship.	1	sta3.1.1.8
9.	**identify** the strength of a relationship.	1	sta3.1.1.9
10.	**make** statements based on simple data shown on graphs.	2	sta3.1.2.1
11.	**write** relationship questions to be investigated	2	sta3.1.2.2
12.	**state** a purpose for the investigation.	1	sta3.1.1.10
13.	**describe** variables of interest from a given data set.	2	sta3.1.2.4
14.	**collect** data for the investigation.	2	sta3.1.2.5
15.	**display** collected data in an appropriate format.	2	sta3.1.2.6
16.	**compute** the line of best fit for the collected data.	2	sta3.1.2.7
17.	**calculate** the Pearson's correlation coefficient, **r**, of the collected data**.**	2	sta3.1.2.8
18.	**describe** the trend and strength of relationship in an investigation.	2	sta3.1.2.9
19.	**compile** logically, statistical (bivariate) data, results and conclusion.	2	sta3.1.2.10
20.	**display** collected data as a scatter plot.	3	sta3.1.3.1
21.	**draw** the line of best fit.	3	sta3.1.3.2
22.	**calculate** the Pearson's correlation coefficient, **r** of a given data set.	3	sta3.1.3.3
23.	**comment** on the trend and strength of relationship in an investigation.	3	sta3.1.3.4
24.	**develop** conclusions based on the results of the investigation.	3	sta3.1.3.5
25.	**analyze** the collected data by plotting data on a scatter plot; fitting line of best fit; calculating the Pearson's correlation coefficient , r.	4	sta3.1.4.1
26.	**make** statements based on the line of best fit.	4	sta3.1.4.2
27.	**make** statements based on the value of the Pearson's correlation coefficient, **r**.	4	sta3.1.4.3
28.	**make** conclusions based on the investigative question and supported with evidence.	4	sta3.1.4.4
29.	**discuss** trends and strength of relationship with supporting evidence.	4	sta3.1.4.5
30.	**communicate** findings in a report.	4	sta3.1.4.6

SUB STRAND 3.2: TIME SERIES

Key Learning Outcome (sta3.2):
Using statistical processes to undertake an investigation of time series data.

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
1.	**define** *time series*.	1	sta3.2.1.1
2.	**define** *secular trend*.	1	sta3.2.1.2
3.	**define** *periodic or seasonal movements*.	1	sta3.2.1.3
4.	**define** *erratic or residual variation*.	1	sta3.2.1.4
5.	**define** *cyclic variation*.	1	sta3.2.1.5
6.	**define** *moving average*.	1	sta3.2.1.6
7.	**define** the *order* of a moving average.	1	sta3.2.1.7
8.	**define** *moving mean*.	1	sta3.2.1.8
9.	**define** *moving median*.	1	sta3.2.1.9
10.	**state** a purpose for a time series investigation.	1	sta3.2.1.10
11.	**identify** features of a time series distribution.	1	sta3.2.1.11
12.	**select** variables of interest from a given time series data set.	1	sta3.2.1.12
13.	**write** a relationship question on time series to be investigated.	1	sta3.2.1.13
14.	**state** the trend of a relationship.	1	sta3.2.1.14
15.	**state** the strengths of a relationship.	1	sta3.2.1.15
16.	**describe** variables of interest from a given time series data set.	2	sta3.2.2.1
17.	**describe** features of the time series (seasonal and long term trends, cyclic trends, outliers, any unusual features).	2	sta3.2.2.2
18.	**describe** the trend in a time series investigation.	2	sta3.2.2.3
19.	**compute** the trend value.	2	sta3.2.2.4
20.	**compute** the smoothed data using moving means.	2	sta3.2.2.5
21.	**compute** smoothed data using moving medians.	2	sta3.2.2.6
22.	**compile** logically, statistical time series data, results and conclusion.	2	sta3.2.2.7
23.	**analyze** the data - smoothing data (moving medians or moving means and, where appropriate centred moving means); plotting raw and smoothed data on a time series graph; fitting a trend line (linear) based on the smoothed data (either using technology or using visual methods); formulate the equation of the trend line; calculate average seasonal effects	3	sta3.2.3.1
24.	**predict** the moving average.	3	sta3.2.3.2
25.	**use** the trend line and seasonality to make a prediction.	3	sta3.2.3.3
26.	**analyze** the behaviour of smoothing data (moving medians or moving means and, where appropriate centred moving means).	3	sta3.2.3.4
27.	**plot** raw and smoothed data on a time series graph.	3	sta3.2.3.5
28.	**fit** a trend line (linear) based on the smoothed data using technology method.	3	sta3.2.3.6

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
29.	**fit** a trend line (linear) based on the smoothed data using visual methods.	3	sta3.2.3.7
30.	**formulate** the equation of the trend line.	3	sta3.2.3.8
31.	**calculate** average seasonal effects.	3	sta3.2.3.9
32.	**formulate** the equation of the trend line.	3	sta3.2.3.10
33.	**explain** the appropriateness of the prediction with respect to (i) the possibility of other models for the trend (e.g. non-linear, piecewise trends), and (ii) limitations of the prediction.	3	sta3.2.3.11
34.	**explain** the effect of seasonally adjusted values.	3	sta3.2.3.12
35.	**develop** conclusions based the features of the time series (seasonal and long term trends, cyclic trends, outliers, any unusual features).	3	sta3.2.3.13
36.	**develop** conclusions based on the identified trend line and seasonality to make a prediction.	3	sta3.2.3.14
37.	**develop** conclusions based on appropriateness of the prediction with respect to (i) the possibility of other models for the trend (e.g. non-linear, piecewise trends), and (ii) limitations of the prediction; consider and comment on seasonally adjusted values.	3	sta3.2.3.15
38.	**develop** conclusions based on seasonally adjusted values.	3	sta3.2.3.16
39.	**communicate** findings in a report.	3	sta3.2.3.17
40.	**analyze** the data by smoothing data (moving medians or moving means and, where appropriate centred moving means); plotting raw and smoothed data on a time series graph; fitting a trend line (linear) based on the smoothed data (either using technology or using visual methods); formulate the equation of the trend line; calculate average seasonal effects.	4	sta3.2.4.1
41.	**develop** a conclusion by discuss the features of the time series (seasonal and long term trends, cyclic trends, outliers, any unusual features); use the trend line and seasonality to make a prediction; discuss the appropriateness of the prediction with respect to (i) the possibility of other models for the trend (e.g. non-linear, piecewise trends), and (ii) limitations of the prediction; consider and comment on seasonally adjusted values.	4	sta3.2.4.2
42.	**discuss** trends and strength of a time series relationship with supporting evidence.	4	sta3.2.4.3
43.	**communicate** findings in a report.	4	sta3.2.4.4
44.	**draw conclusion and/or make a prediction** based on discussion of findings	4	sta3.2.4.5

SUB STRAND 3.3: COMPARING TWO POPULATIONS

Key Learning Outcome (sta3.3):
Using statistical processes to undertake an investigation of numerical data in order to make a comparison between two populations.

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
1.	**define** population.	1	sta3.3.1.1
2.	**define** sampling.	1	sta3.3.1.2
3.	**define** sample.	1	sta3.3.1.3
4.	**define** outliers.	1	sta3.3.1.4
5.	**identify** the variable of interest of an investigation (planning).	1	sta3.3.1.5
6.	**identify** the two populations to be investigated (planning).	1	sta3.3.1.6
7.	**identify** sources of variation.	1	sta3.3.1.7
8.	**identify** methods to manage variations.	1	sta3.3.1.8
9.	**state** the different methods of sampling.	1	sta3.3.1.9
10.	**state** a comparison question to be investigated.	1	sta3.3.1.10
11.	**state** a hypothesis statement.	2	sta3.3.2.1
12.	**select** a sampling method.	2	sta3.3.2.2
13.	**plan** methods of collecting and recording data.	2	sta3.3.2.3
14.	**plan** the investigation by identifying the comparison question, variables of interest; the two populations; sources of variation and establishing methods to manage them.	2	sta3.3.2.4
15.	**describe** sources of variation and establishing methods to manage them (e.g. considering outside variables in data collection methods); sampling method selected and methods of collecting and recording data.	2	sta3.3.2.5
16.	**collect** data to be investigated.	2	sta3.3.2.6
17.	**gather** data by collecting and recording data; managing & sorting data; cleaning data (removal of outliers).	2	sta3.3.2.7
18.	**clean** collected data (removal of outliers).	2	sta3.3.2.8
19.	**record** data to be investigated.	2	sta3.3.2.9
20.	**manage** (and sort) collected data.	2	sta3.3.2.10
21.	**describe** sources of variation on the collected data.	2	sta3.3.2.11
22.	**compile** statistical data on two population, its results and conclusion.	2	sta3.3.2.12
23.	**communicate** findings in a report	2	sta3.3.2.13
24.	**display** collected data in an appropriate format.	3	sta3.3.3.1
25.	**analyze** the data by summarizing data with statistical measures; drawing appropriate displays; reflecting on the distribution; formulating a hypothesis.	3	sta3.3.3.2
26.	**analyze** the data - summarizing data with statistical measures	3	sta3.3.3.3
27.	**reflect** on the distribution of the collected data.	3	sta3.3.3.4
28.	**communicate** detailed findings in a report	3	sta3.3.3.5
29.	**establish** methods to manage variations (e.g. considering outside variables in data collection methods).	3	sta3.3.3.6
30.	**make** conclusions based on the results of the investigation.	3	sta3.3.3.7
31.	**make** inferences using confidence interval (difference of two means or proportions).	3	sta3.3.3.8
32.	**write** a conclusion to answer the investigative question.	3	sta3.3.3.9
33.	**discuss** limitations.	3	sta3.3.3.10
34.	**develop** conclusion by interpreting data; making inferences using confidence interval (difference of two means or proportions); writing a conclusion to answer the investigative question and discussing limitations.	3	sta3.3.3.11

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
35.	**make** conclusions based on the results of the investigation and support with evidence.	4	sta3.3.4.1
36.	**make** inferences using confidence interval (difference of two means or proportions).	4	sta3.3.4.2
37.	**write** a conclusion to answer the investigative question.	4	sta3.3.4.3
38.	**discuss** limitations with supporting evidence.	4	sta3.3.4.4
39.	**communicate** findings in a report	4	sta3.3.4.5

SUB STRAND 3.4: STATISTICAL PROCESSES

Key Learning Outcome (sta3.4):
Answer questions related to statistical processes.

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
1.	**define** *margin of error.*	1	sta3.4.1.1
2.	**define** *sample size.*	1	sta3.4.1.2
3.	**define** *point estimate.*	1	sta3.4.1.3
4.	**define** *sample statistics.*	1	sta3.4.1.4
5.	**define** *population parameters.*	1	sta3.4.1.5
6.	**define** *statistical inference.*	1	sta3.4.1.6
7.	**define** *confidence level.*	1	sta3.4.1.7
8.	**state** the central limit theorem.	1	sta3.4.1.8
9.	**compute** the margin of error.	2	sta3.4.2.1
10.	**compute** the sample size (n) using the margin of error.	2	sta3.4.2.2
11.	**compute** the point estimate of the population mean (sample mean).	2	sta3.4.2.3
12.	**compute** the point estimate of the population proportion (sample proportion).	2	sta3.4.2.4
13.	**compute** the standard error in a given ci.	2	sta3.4.2.5
14.	**compute** the degree of confidence for a given ci.	2	sta3.4.2.6
15.	**compute** 90% confidence interval for a population mean.	2	sta3.4.2.7
16.	**compute** 95% confidence interval for a population mean.	2	sta3.4.2.8
17.	**compute** 99% confidence interval for a population mean.	2	sta3.4.2.9
18.	**compute** 90% confidence interval for the difference in means of two population.	2	sta3.4.2.10
19.	**compute** 95% confidence interval for a population proportion.	2	sta3.4.2.11
20.	**compute** 99% confidence interval for a population proportion.	2	sta3.4.2.12
21.	**compute** 90% confidence interval for difference between population means.	2	sta3.4.2.13
22.	**compute** 95% confidence interval for difference between population means.	2	sta3.4.2.14
23.	**compute** 99% confidence interval for difference between population means.	2	sta3.4.2.15
24.	**compute** 90% confidence interval for the difference in proportions.	2	sta3.4.2.16
25.	**compute** 95% confidence interval for the difference in proportions.	2	sta3.4.2.17

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
26.	**compute** 99% confidence interval for the difference in proportions.	2	sta3.4.2.18
27.	**determine** the nature of correlation coefficient for a given set of data or graph.	2	sta3.4.2.19
28.	**express** the confidence interval in a statement.	3	sta3.4.3.1
29.	**explain** how the margin of error changes when certain parameters of the population/sample or confidence interval are altered.	3	sta3.4.3.2
30.	**apply** the central limit theorem to solve problems related to confidence interval.	3	sta3.4.3.3
31.	**apply** point estimates calculations to solve problems related to confidence interval.	3	sta3.4.3.4
32.	**interpret** the 90% confidence interval for a population mean.	3	sta3.4.3.5
33.	**interpret** the 95% confidence interval for a population mean.	3	sta3.4.3.6
34.	**interpret** the 99% confidence interval for a population mean.	3	sta3.4.3.7
35.	**interpret** the 90% confidence interval for a population proportion.	3	sta3.4.3.8
36.	**interpret** the 95% confidence interval for a population proportion.	3	sta3.4.3.9
37.	**interpret** the 99% confidence interval for a population proportion.	3	sta3.4.3.10
38.	**interpret** the 90% confidence interval for the difference in means.	3	sta3.4.3.11
39.	**interpret** the 95% confidence interval for the difference in means.	3	sta3.4.3.12
40.	**interpret** the 99% confidence interval for the difference in means.	3	sta3.4.3.13
41.	**interpret** the 90% confidence interval for the difference in proportions.	3	sta3.4.3.14
42.	**interpret** the 95% confidence interval for the difference in proportions.	3	sta3.4.3.15
43.	**interpret** the 99% confidence interval for the difference in proportions.	3	sta3.4.3.16
44.	**compare and contrast** standard error and margin of error.	3	sta3.4.3.17
45.	**solve** problems related to confidence interval.	3	sta3.4.3.18
46.	**interpret** confidence interval for population means.	3	sta3.4.3.19
47.	**interpret** confidence interval for population proportions.	3	sta3.4.3.20
48.	**interpret** confidence interval for difference between two population means.	3	sta3.4.3.21
49.	**interpret** confidence interval for the difference in proportions.	3	sta3.4.3.22
50.	**explain** the impact of the size of the confidence interval on the margin of error.	3	sta3.4.3.23
51.	**solve** problems related to confidence interval.	4	sta3.4.4.1
52.	**predict** the effect of the margin of error.	4	sta3.4.4.2
53.	**interpret** confidence interval for population means.	4	sta3.4.4.3
54.	**interpret** confidence interval for population proportions.	4	sta3.4.4.4
55.	**evaluate** the validity of claims using confidence interval.	4	sta3.4.4.5

Explanatory Notes

Assessment will involve a selection from:

- performing or commenting on aspects of the processes involved in outcomes 3.1, 3.2, 3.3
- understanding the central limit theorem

- performing calculations related to confidence intervals (mean, proportion, difference of mean, difference of proportion) at the 90%, 95% & 99% level of confidence and interpreting these intervals
- understanding point estimates and margin of error
- calculating the sample size (n) using the margin of error

STRAND 4: Numerical and Algebraic Methods

Major Learning Outcome (sta4):

Students are able to use numeric and algebraic methods to solve problems.

SUB STRAND 4.1: LINEAR EQUATIONS

Key Learning Outcome (sta4.1):

Solve systems of 3 x 3 linear equations; Interpret solutions to linear systems of equations; Solve problems that can be modelled by systems of linear equations.

Specific Learning Outcomes (SLO): Students are able to		Skill level	SLO code
1.	**state** the different types of solutions.	1	sta4.1.1.1
2.	**determine** which method is best to solve a system of linear equations.	1	sta4.1.1.2
3.	**write** linear systems of equations for a given basic situation.	2	sta4.1.2.1
4.	**solve** simple linear systems of equations for a given situation by substitution method.	2	sta4.1.2.2
5.	**solve** simple linear systems of equations for a given situation by elimination method.	2	sta4.1.2.3
6.	**solve** simple linear systems of equations for a given situation by graphing method.	2	sta4.1.2.4
7.	**determine** the nature of solutions of a linear systems of equations.	2	sta4.1.2.5
8.	**determine** the condition for infinitely many solutions to a given set of equations.	2	sta4.1.2.6
9.	**interpret** the nature of solutions of simple linear systems of equations.	2	sta4.1.2.7
10.	**interpret** the nature of solutions of simple linear system of equations.	2	sta4.1.2.8
11.	**solve** linear systems of equations (use of matrices is accepted).	3	sta4.1.3.1
12.	**write** linear systems of equations for a given complex situation.	3	sta4.1.3.2
13.	**solve** application problems by substitution method.	3	sta4.1.3.3
14.	**solve** application problems by elimination method.	3	sta4.1.3.4
15.	**solve** application problems by graphing method.	3	sta4.1.3.5
16.	**interpret** the nature of solutions of linear system of equation with description.	3	sta4.1.3.6
17.	**create** linear systems of equations for real life situation.	4	sta4.1.4.1
18.	**determine** the nature of solutions of complex linear systems of linear equations.	4	sta4.1.4.2
19.	**solve** 3 x 3 linear system of equation based on real life situations (use of matrices is accepted).	4	sta4.1.4.3
20.	**interpret** the nature of solutions with respect to the given situation.	4	sta4.1.4.4

Explanatory Notes

Assessment will focus on using simultaneous equations to solve problems in context:

- solving 3x3 linear systems of equations (use of matrices is accepted).
- forming linear systems of equations for a given situation and solving them in context.
- determining and interpreting the nature of solutions, including
- Unique or dependent solutions
- No solution
- Geometric interpretations

SUB STRAND 4.2: NON-LINEAR EQUATIONS

Key Learning Outcome (sta4.2):

Use bisection method or Newton-Raphson method to solve non-linear equations to a given precision.

Specific Learning Outcomes (SLO): Students are able to		**Skill level**	**SLO code**
1.	**determine** the presence of roots of a hyperbolic function between two x values.	1	sta4.2.1.1
2.	**determine** the nature of the roots after using the Newton-Raphson to improve the approximation to a given precision.	1	sta4.2.1.2
3.	**state** the advantages of using the Newton-Raphson method.	1	sta4.2.1.3
4.	**state** the disadvantages of using the Newton-Raphson method.	1	sta4.2.1.4
5.	**state** the advantages of using the Bisection method.	1	sta4.2.1.5
6.	**state** the disadvantages of using the Bisection method.	1	sta4.2.1.6
7.	**differentiate** simple power functions in order to use the Newton-Raphson Method (this involves derivative of polynomials only. If needed, derivatives of other functions will be given).	2	sta4.2.2.4
8.	**use** the Newton-Raphson method to solve the equation f(x).	2	sta4.2.2.1
9.	**use** the Bisection Method to solve the equation f(x).	2	sta4.2.2.2
10.	**use** the Newton-Raphson and Bisection method to solve the equation f(x).	2	sta4.2.2.3
11.	**differentiate** power functions in order to use the Newton-Raphson Method.	3	sta4.2.3.4
12.	**use** the Newton-Raphson method to solve the equation f(x).	3	sta4.2.3.1
13.	**use** the Bisection method to solve the equation f(x).	3	sta4.2.3.2
14.	**use** the Newton-Raphson and Bisection method to solve the equation f(x).	3	sta4.2.3.3
15.	**differentiate** polynomial functions to use the Newton-Raphson Method.	4	sta4.2.4.5
16.	**use** the Newton-Raphson and Bisection Method to solve the equation f(x).	4	sta4.2.4.1
17.	**solve** a non-linear equation using the Newton-Raphson method with a given starting value.	4	sta4.2.4.2
18.	**solve** a non-linear equation using the Bisection Method with a given starting interval.	4	sta4.2.4.3
19.	**use** a suitable method to find an approximate solution to a non-linear equation.	4	sta4.2.4.4

Explanatory Notes

Assessment will focus on:

- *use of the Newton-Raphson or the Bisection Method (or both) to improve the approximation to a given precision.*
- *differentiate simple power functions in order to use the Newton-Raphson Method (this involves derivatives of polynomials only. If needed, derivatives of other functions will be given)*

NB: Students will be expected to be familiar with both the Newton-Raphson and the bisection methods.

The starting values will be provided.

ASSESSMENT

Assessment in this Mathematics with Statistics course is in two parts - external assessment and internal assessment. The respective weightings are

1. **External Assessment (EA) : 70%**
2. **Internal Assessment (IA) : 30%**

The principal, or his/her nominee, will certify that the prescription requirements have been fulfilled.

Suggested Teaching Time and Weightings

OBJECTIVES/OUTCOMES	EXTERNAL/ INTERNAL	SUGGESTED TIME (TEACHING, LEARNING & ASSESSING)	APPROXIMATE WEIGHTING
1. Students are able to develop knowledge and skills related to Probability in order to solve problems and to investigate situations involving elements of chance.	*External*	*8 weeks*	*28 %*
2. Students are able to model situations using graphical methods in order to solve problems	*External*	*6 weeks*	*22 %*
3. Students are able to carry out statistical investigations and understand statistical processes.	*External/ internal*	*10 weeks*	*(6% external / 30% internal) 36 %*
4. Students are able to use numeric and algebraic methods to solve problems	*External*	*4 weeks*	*14 %*
Total		*28 weeks*	*100%*

Assessment Blueprint

Strand	Assessment Type	SKILL LEVEL/ SCORE				
		1	2	3	4	Weight
1. Probability	EA					**28**
2. Modelling using graphical methods	EA					**22**
3. Statistical investigations	EA					**6**
	IA					**30**
4. Numerical and algebraic methods	EA					**14**
Total number of items		**20**	**15**	**10**	**5**	
Total skill score		**20**	**30**	**30**	**20**	**100**

External Examination

This will be a **three-hour** written examination and will be out of **70%**.

The external written examination will assess learning outcomes from all four strands/sections in the following proportions:

Topic/Section		Objectives	Percentage per section	Suggested Exam Time
1	Probability	Develop knowledge and skills related to Probability in order to solve problems and to investigate situations involving elements of chance.	28%	70 mins
2	Modelling using graphical methods.	Model situations using graphical methods in order to solve problems.	22%	60 mins
3	Statistical investigations	Carry out statistical investigations and understand statistical processes.	6%	15 mins
4	Numerical and algebraic methods	Use numeric and algebraic methods to solve problems	14%	35 mins

Learning outcomes that have been assessed using internal assessment will not be assessment in the external assessment.

Students would be expected to know, understand and apply any mathematical skill related to the outcomes. Examination questions, which require specific mathematical knowledge, will be based on the outcomes. However, students can be assessed the same learning outcomes using an unfamiliar context to assess students' application of the mathematical concepts in the real world context.

Emphasis or the number of questions in the examination will reflect the teaching time and weightings allocated to each topic.

A standard list of formulae will be incorporated in the examination paper. Additional formulae may be provided where appropriate. A copy of the formulae list is attached as Appendix 1 in this prescription.

The use of calculators and computers is an integral part of mathematics. Students will be required to supply their own calculators for use in the examination. The use of calculators must conform to the EQAP Rules and Procedures governing electronic calculators.

Internal Assessment

There are three internal assessment tasks and these include:

Task 1: Statistical Investigations- Bivariate Project (10%)

Task 2: Statistical Investigations- Time Series Project (10%)

Task 3: Statistical Investigations- Comparison Project (10%)

These assessments will complement the marks that are provided by the written examination. They emphasize those specific outcomes in the prescription for which assessment by an external examination is less appropriate, i.e. carrying out statistical investigations.

To meet the internally assessed component requirement, students will submit three pieces of written or word processed work, reporting on the statistical investigations undertaken by the student. The three investigations needing to be studied are briefly described below.

The total amount of time spent on such work should be commensurate with its weight in the final assessment, i.e. about 10% of the yearly course time for the teaching, learning and assessment of investigation 3.1, about 10% of the yearly course time for the teaching, learning and assessment of investigation 3.2, and about 10% of the yearly course time for the teaching, learning and assessment of investigation 3.3.

The use of computer technology in the investigations is strongly encouraged, including the use of appropriate statistical packages, or simple graphing packages, where available.

At the beginning of each year, each school presenting students for the South Pacific Form Seven Certificate Mathematics (with Statistics) assessment must complete an Internal Assessment Summary Form **(SAT-IA)** and forward to EQAP by the date set down by the Director. The form must clearly indicate the time (start and completion date) each investigation is intended to be undertaken.

At the start of the year students should be given a copy of the assessment statement to be used. The assessment statement and copies of all assessment tasks and assessment schedules used, as well as a sample of candidate responses to all internal assessment work undertaken, must be available for verification on request until 30 November of the year of the examination.

The moderation of Internal Assessment will be done in accordance with EQAP policy as specified from time to time.

A possible assessment/scoring schedule for the respective investigations are included in ***Appendix 3, 4 and 5***.

Explanatory Notes on Investigation 1 – Bivariate Project

The investigation will involve:

- choosing variables of interest from a given data set
- posing a relationship question to be investigated
- analyzing the data
  - plotting data on a scatter plot
  - fitting a line of best fit (either using technology or using the method of two medians)
  - calculating the Pearson's correlation coefficient, **r** (NB: if students do not have access to computer technology, teachers may need to consider restricting the amount of data that is to be analysed)
- developing a conclusion
  - interpreting the trend
  - interpreting the strength of the relationship (correlation)
  - writing a conclusion to answer the investigative question
  - discuss causality
  - discussing limitations
- communicating findings in a report

Explanatory Notes on Investigation 2 – Time Series Project

The investigation will involve:

- choosing variables of interest from an existing data set
- stating a purpose for the investigation
- analyzing the data
  - smoothing data (moving medians or moving means and, where appropriate centred moving means)
  - plotting raw and smoothed data on a time series graph
  - fitting a trend line (linear) based on the smoothed data (either using technology or using visual methods)
  - formulate the equation of the trend line.
  - calculate average seasonal effects
- developing a conclusion
  - discuss the features of the time series (seasonal and long term trends, cyclic trends, outliers, any unusual features)
  - use the trend line and seasonality to make a prediction
  - discuss the appropriateness of the prediction with respect to:
    - The possibility of other models for the trend (e.g. Non-linear, piecewise trends)
    - Limitations of the prediction
  - consider and comment on seasonally adjusted values
- communicating findings in a report

Explanatory Notes on Investigation 3 – Comparison Project

The investigation will involve:

- posing a comparison question to be investigated
- planning the investigation
  - identifying the variable of interest
  - identifying the two populations
  - identifying sources of variation and establishing methods to manage them (e.g. Considering outside variables in data collection methods)
  - choosing a sampling method
  - planning methods of collecting and recording data
- gathering the data
  - collecting and recording data
  - managing & sorting data
  - cleaning data (removal of outliers)
- analyzing the data
  - summarizing data with statistical measures
  - drawing appropriate displays
  - reflecting on the distribution
  - formulating a hypothesis
- developing a conclusion
  - interpreting data
  - making inferences using confidence interval (difference of two means or proportions).
  - writing a conclusion to answer the investigative question
  - discussing limitations
- communicating findings in a report

APPENDICES

Appendix 1: Formulas and Tables

MATHEMATICS WITH STATISTICS — USEFUL FORMULAE AND TABLES

Straight Line

Gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Equation $y - y_1 = m(x - x_1)$

Quadratic Equations

If $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Bisection Method

$x_2 = \dfrac{x_0 + x_1}{2}$

Newton-Raphson Method

$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

Differentiation

$y = f(x)$	$\dfrac{dy}{dx} = f'(x)$
x^n	nx^{n-1}
$\ln x$	$\dfrac{1}{x}$
e^{ax}	ae^{ax}

Differential Equation

If $\dfrac{dy}{dx} = ky$ then $y = Ae^{kx}$

Arithmetic Sequences and Series

$t_n = a + (n-1)d$

$S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$

Geometric Sequences and Series

$t_n = ar^{n-1}$

$S_n = \dfrac{a(1-r^n)}{1-r}$ for $r \neq 1$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Exponential Series

$e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots$ (for all x)

Logarithms

$\ln x = \log_e x$

If $y = b^x$ then $\log_b y = x$

If $y = e^x$ then $x = \ln y = \log_e y$

$\log_b x + \log_b y = \log_b xy$

$\log_b x - \log_b y = \log_b \dfrac{x}{y}$

$\log_b x^n = n \log_b x$

Binomial Theorem

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + \binom{n}{n}b^n$$

where $\binom{n}{r} = {}^nC_r = \dfrac{n!}{(n-r)!\,r!}$ Note: ${}^nP_r = \dfrac{n!}{(n-r)!}$

Some values of $\binom{n}{r}$ are given in the table below.

Binomial Coefficients

n \ r	0	1	2	3	4	5	6	7	8	9	10
0	1										
1	1	1									
2	1	2	1								
3	1	3	3	1							
4	1	4	6	4	1						
5	1	5	10	10	5	1					
6	1	6	15	20	15	6	1				
7	1	7	21	35	35	21	7	1			
8	1	8	28	56	70	56	28	8	1		
9	1	9	36	84	126	126	84	36	9	1	
10	1	10	45	120	210	252	210	120	45	10	1
11	1	11	55	165	330	462	462	330	165	55	11
12	1	12	66	220	495	792	924	792	495	220	66
13	1	13	78	286	715	1287	1716	1716	1287	715	286
14	1	14	91	364	1001	2002	3003	3432	3003	2002	1001
15	1	15	105	455	1365	3003	5005	6435	6435	5005	3003

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Expectation Algebra

$$E(aX + b) = aE(X) + b$$

$$Var(aX + b) = a^2 Var(X)$$

$$E(aX + bY) = aE(X) + bE(Y)$$

$$Var(aX + bY) = a^2 Var(X) + b^2 Var(Y),$$

(if X, Y are independent)

Mean and Variance of Data

$$\bar{x} = \frac{\Sigma fx}{n} \qquad s^2 = \frac{\Sigma fx^2 - \frac{(\Sigma fx)^2}{n}}{n}$$

Mean and Variance of a Random Variable

$$\mu = E(X) = \Sigma x.P(X = x) \qquad \sigma^2 = Var(X) = E(X^2) - [E(X)]^2$$

Distribution of Sample Statistics

Statistic	Mean	Standard Deviation
Sample Mean	$E(\bar{X}) = \mu$	$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$ (std. error of the mean)
Sample Proportion	$E(P) = \pi$	$\sigma_P = \sqrt{\frac{\pi(1-\pi)}{n}}$ (std. error of the proportion)
Difference of Means (of two independent samples)	$E(\bar{X}_1 - \bar{X}_2) = \mu_1 - \mu_2$	$\sigma_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

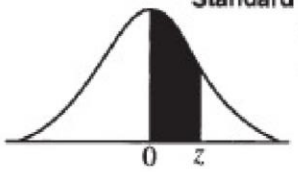
Confidence Intervals

Mean $\bar{X} - z.\sigma_{\bar{X}} < \mu < \bar{X} + z.\sigma_{\bar{X}}$

Proportion $P - z.\sigma_P < \pi < P + z.\sigma_P$

Difference of two means $(\bar{X}_1 - \bar{X}_2) - z.\sigma_{\bar{X}_1 - \bar{X}_2} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + z.\sigma_{\bar{X}_1 - \bar{X}_2}$

Standard Normal Distribution



$$\left(Z = \frac{X - \mu}{\sigma}\right)$$

Each entry gives the probability that the standardised normal random variable Z lies between 0 and z.

z	0	1	2	3	4	5	6	7	8	9	Differences 1	2	3	4	5	6	7	8	9
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359	4	8	12	16	20	24	28	32	36
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754	4	8	12	16	20	24	28	32	36
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141	4	8	12	15	19	22	27	31	35
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517	4	8	11	15	19	22	26	30	34
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879	4	7	11	14	18	22	25	29	32
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224	3	7	10	14	17	21	24	27	31
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549	3	6	10	13	16	19	23	26	29
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852	3	6	9	12	15	18	21	24	27
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133	3	6	8	11	14	17	19	22	25
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389	3	5	8	10	13	15	18	20	23
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621	2	5	7	9	12	14	16	18	21
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830	2	4	6	8	10	12	14	16	19
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015	2	4	5	7	9	11	13	15	16
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177	2	3	5	6	8	10	11	13	14
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319	1	3	4	6	7	8	10	11	13
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441	1	2	4	5	6	7	8	10	11
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545	1	2	3	4	5	6	7	8	9
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633	1	2	3	3	4	5	6	7	8
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706	1	1	2	3	4	4	5	6	6
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767	1	1	2	2	3	4	4	5	5
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817	0	1	1	2	2	3	3	4	4
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857	0	1	1	2	2	2	3	3	4
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890	0	1	1	1	2	2	2	3	3
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916	0	0	1	1	1	2	2	2	2
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936	0	0	1	1	1	1	1	2	2
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952	0	0	0	1	1	1	1	1	1
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964	0	0	0	0	1	1	1	1	1
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974	0	0	0	0	0	1	1	1	1
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981	0	0	0	0	0	0	0	0	1
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986	0	0	0	0	0	0	0	0	1
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990	0	0	0	0	0	0	0	0	0
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993	0	0	0	0	0	0	0	0	0
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995	0	0	0	0	0	0	0	0	0
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997	0	0	0	0	0	0	0	0	0
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998	.4998	0	0	0	0	0	0	0	0	0
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	0	0	0	0	0	0	0	0	0
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	0	0	0	0	0	0	0	0	0
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	0	0	0	0	0	0	0	0	0
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.5000	.5000	.5000	0	0	0	0	0	0	0	0	0
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	0	0	0	0	0	0	0	0	0

Binomial Distribution

Each entry gives the probability that a binomial random variable X, with the parameters n and π, has the value x.

$$\left(P(X=x) = \binom{n}{x} \pi^x (1-\pi)^{n-x} \qquad \mu = n\pi, \qquad \sigma = \sqrt{n\pi(1-\pi)} \right)$$

n	x	π = 0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5
4	0	0.8145	0.6561	0.5220	0.4823	0.4096	0.3164	0.2401	0.1975	0.1785	0.1296	0.0915	0.0625
	1	0.1715	0.2916	0.3685	0.3858	0.4096	0.4219	0.4116	0.3951	0.3845	0.3456	0.2995	0.2500
	2	0.0135	0.0486	0.0975	0.1157	0.1536	0.2109	0.2646	0.2963	0.3105	0.3456	0.3675	0.3750
	3	0.0005	0.0036	0.0115	0.0154	0.0256	0.0469	0.0756	0.0988	0.1115	0.1536	0.2005	0.2500
	4		0.0001	0.0005	0.0008	0.0016	0.0039	0.0081	0.0123	0.0150	0.0256	0.0410	0.0625
5	0	0.7738	0.5905	0.4437	0.4019	0.3277	0.2373	0.1681	0.1317	0.1160	0.0778	0.0503	0.0313
	1	0.2036	0.3281	0.3915	0.4019	0.4096	0.3955	0.3602	0.3292	0.3124	0.2592	0.2059	0.1563
	2	0.0214	0.0729	0.1382	0.1608	0.2048	0.2637	0.3087	0.3292	0.3364	0.3456	0.3369	0.3125
	3	0.0011	0.0081	0.0244	0.0322	0.0512	0.0879	0.1323	0.1646	0.1811	0.2304	0.2757	0.3125
	4		0.0005	0.0022	0.0032	0.0064	0.0146	0.0284	0.0412	0.0488	0.0768	0.1128	0.1563
	5			0.0001	0.0001	0.0003	0.0010	0.0024	0.0041	0.0053	0.0102	0.0185	0.0313
6	0	0.7351	0.5314	0.3771	0.3349	0.2621	0.1780	0.1176	0.0878	0.0754	0.0467	0.0277	0.0156
	1	0.2321	0.3543	0.3993	0.4019	0.3932	0.3560	0.3025	0.2634	0.2437	0.1866	0.1359	0.0938
	2	0.0305	0.0984	0.1762	0.2009	0.2458	0.2966	0.3241	0.3292	0.3280	0.3110	0.2780	0.2344
	3	0.0021	0.0146	0.0415	0.0536	0.0819	0.1318	0.1852	0.2195	0.2355	0.2765	0.3032	0.3125
	4	0.0001	0.0012	0.0055	0.0080	0.0154	0.0330	0.0595	0.0823	0.0951	0.1382	0.1861	0.2344
	5		0.0001	0.0004	0.0006	0.0015	0.0044	0.0102	0.0165	0.0205	0.0369	0.0609	0.0938
	6					0.0001	0.0002	0.0007	0.0014	0.0018	0.0041	0.0083	0.0156
7	0	0.6983	0.4783	0.3206	0.2791	0.2097	0.1335	0.0824	0.0585	0.0490	0.0280	0.0152	0.0078
	1	0.2573	0.3720	0.3960	0.3907	0.3670	0.3115	0.2471	0.2048	0.1848	0.1306	0.0872	0.0547
	2	0.0406	0.1240	0.2097	0.2344	0.2753	0.3115	0.3177	0.3073	0.2985	0.2613	0.2140	0.1641
	3	0.0036	0.0230	0.0617	0.0781	0.1147	0.1730	0.2269	0.2561	0.2679	0.2903	0.2918	0.2734
	4	0.0002	0.0026	0.0109	0.0156	0.0287	0.0577	0.0972	0.1280	0.1442	0.1935	0.2388	0.2734
	5		0.0002	0.0012	0.0019	0.0043	0.0115	0.0250	0.0384	0.0466	0.0774	0.1172	0.1641
	6			0.0001	0.0001	0.0004	0.0013	0.0036	0.0064	0.0084	0.0172	0.0320	0.0547
	7						0.0001	0.0002	0.0005	0.0006	0.0016	0.0037	0.0078
8	0	0.6634	0.4305	0.2725	0.2326	0.1678	0.1001	0.0576	0.0390	0.0319	0.0168	0.0084	0.0039
	1	0.2793	0.3826	0.3847	0.3721	0.3355	0.2670	0.1977	0.1561	0.1373	0.0896	0.0548	0.0313
	2	0.0515	0.1488	0.2376	0.2605	0.2936	0.3115	0.2965	0.2731	0.2587	0.2090	0.1569	0.1094
	3	0.0054	0.0331	0.0839	0.1042	0.1468	0.2076	0.2541	0.2731	0.2786	0.2787	0.2568	0.2188
	4	0.0004	0.0046	0.0185	0.0260	0.0459	0.0865	0.1361	0.1707	0.1875	0.2322	0.2627	0.2734
	5		0.0004	0.0026	0.0042	0.0092	0.0231	0.0467	0.0683	0.0808	0.1239	0.1719	0.2188
	6			0.0002	0.0004	0.0011	0.0038	0.0100	0.0171	0.0217	0.0413	0.0703	0.1094
	7					0.0001	0.0004	0.0012	0.0024	0.0033	0.0079	0.0164	0.0313
	8							0.0001	0.0002	0.0002	0.0007	0.0017	0.0039
9	0	0.6302	0.3874	0.2316	0.1938	0.1342	0.0751	0.0404	0.0260	0.0207	0.0101	0.0046	0.0020
	1	0.2985	0.3874	0.3679	0.3489	0.3020	0.2253	0.1556	0.1171	0.1004	0.0605	0.0339	0.0176
	2	0.0629	0.1722	0.2597	0.2791	0.3020	0.3003	0.2668	0.2341	0.2162	0.1612	0.1110	0.0703
	3	0.0077	0.0446	0.1069	0.1302	0.1762	0.2336	0.2668	0.2731	0.2716	0.2508	0.2119	0.1641
	4	0.0006	0.0074	0.0283	0.0391	0.0661	0.1168	0.1715	0.2048	0.2194	0.2508	0.2600	0.2461
	5		0.0008	0.0050	0.0078	0.0165	0.0389	0.0735	0.1024	0.1181	0.1672	0.2128	0.2461
	6		0.0001	0.0006	0.0010	0.0028	0.0087	0.0210	0.0341	0.0424	0.0743	0.1160	0.1641
	7				0.0001	0.0003	0.0012	0.0039	0.0073	0.0098	0.0212	0.0407	0.0703
	8						0.0001	0.0004	0.0009	0.0013	0.0035	0.0083	0.0176
	9								0.0001	0.0001	0.0003	0.0008	0.0020
10	0	0.5987	0.3487	0.1969	0.1615	0.1074	0.0563	0.0282	0.0173	0.0135	0.0060	0.0025	0.0010
	1	0.3151	0.3874	0.3474	0.3230	0.2684	0.1877	0.1211	0.0867	0.0725	0.0403	0.0207	0.0098
	2	0.0746	0.1937	0.2759	0.2907	0.3020	0.2816	0.2335	0.1951	0.1757	0.1209	0.0763	0.0439
	3	0.0105	0.0574	0.1298	0.1550	0.2013	0.2503	0.2668	0.2601	0.2522	0.2150	0.1665	0.1172
	4	0.0010	0.0112	0.0401	0.0543	0.0881	0.1460	0.2001	0.2276	0.2377	0.2508	0.2384	0.2051
	5	0.0001	0.0015	0.0085	0.0130	0.0264	0.0584	0.1029	0.1366	0.1536	0.2007	0.2340	0.2461
	6		0.0001	0.0012	0.0022	0.0055	0.0162	0.0368	0.0569	0.0689	0.1115	0.1596	0.2051
	7			0.0001	0.0002	0.0008	0.0031	0.0090	0.0163	0.0212	0.0425	0.0746	0.1172
	8					0.0001	0.0004	0.0014	0.0030	0.0043	0.0106	0.0229	0.0439
	9							0.0001	0.0003	0.0005	0.0016	0.0042	0.0098
	10	(all other entries < 0.0001)									0.0001	0.0003	0.0010

Poisson Distribution

Each entry gives the probability that a Poisson random variable X, with parameter λ, has the value x.

$$\left(P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!} \qquad \mu = \lambda, \qquad \sigma = \sqrt{\lambda} \right)$$

x \ λ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066	0.3679
1	0.0905	0.1637	0.2222	0.2681	0.3033	0.3293	0.3476	0.3595	0.3659	0.3679
2	0.0045	0.0164	0.0333	0.0536	0.0758	0.0988	0.1217	0.1438	0.1647	0.1839
3	0.0002	0.0011	0.0033	0.0072	0.0126	0.0198	0.0284	0.0383	0.0494	0.0613
4		0.0001	0.0003	0.0007	0.0016	0.0030	0.0050	0.0077	0.0111	0.0153
5				0.0001	0.0002	0.0004	0.0007	0.0012	0.0020	0.0031
6							0.0001	0.0002	0.0003	0.0005
7										0.0001

x \ λ	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0	0.3329	0.3012	0.2725	0.2466	0.2231	0.2019	0.1827	0.1653	0.1496	0.1353
1	0.3662	0.3614	0.3543	0.3452	0.3347	0.3230	0.3106	0.2975	0.2842	0.2707
2	0.2014	0.2169	0.2303	0.2417	0.2510	0.2584	0.2640	0.2678	0.2700	0.2707
3	0.0738	0.0867	0.0998	0.1128	0.1255	0.1378	0.1496	0.1607	0.1710	0.1804
4	0.0203	0.0260	0.0324	0.0395	0.0471	0.0551	0.0636	0.0723	0.0812	0.0902
5	0.0045	0.0062	0.0084	0.0111	0.0141	0.0176	0.0216	0.0260	0.0309	0.0361
6	0.0008	0.0012	0.0018	0.0026	0.0035	0.0047	0.0061	0.0078	0.0098	0.0120
7	0.0001	0.0002	0.0003	0.0005	0.0008	0.0011	0.0015	0.0020	0.0027	0.0034
8			0.0001	0.0001	0.0001	0.0002	0.0003	0.0005	0.0006	0.0009
9							0.0001	0.0001	0.0001	0.0002

x \ λ	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0
0	0.1108	0.0907	0.0743	0.0608	0.0498	0.0408	0.0334	0.0273	0.0224	0.0183
1	0.2438	0.2177	0.1931	0.1703	0.1494	0.1304	0.1135	0.0984	0.0850	0.0733
2	0.2681	0.2613	0.2510	0.2384	0.2240	0.2087	0.1929	0.1771	0.1615	0.1465
3	0.1966	0.2090	0.2176	0.2225	0.2240	0.2226	0.2186	0.2125	0.2046	0.1954
4	0.1082	0.1254	0.1414	0.1557	0.1680	0.1781	0.1858	0.1912	0.1944	0.1954
5	0.0476	0.0602	0.0735	0.0872	0.1008	0.1140	0.1264	0.1377	0.1477	0.1563
6	0.0174	0.0241	0.0319	0.0407	0.0504	0.0608	0.0716	0.0826	0.0936	0.1042
7	0.0055	0.0083	0.0118	0.0163	0.0216	0.0278	0.0348	0.0425	0.0508	0.0595
8	0.0015	0.0025	0.0038	0.0057	0.0081	0.0111	0.0148	0.0191	0.0241	0.0298
9	0.0004	0.0007	0.0011	0.0018	0.0027	0.0040	0.0056	0.0076	0.0102	0.0132
10	0.0001	0.0002	0.0003	0.0005	0.0008	0.0013	0.0019	0.0028	0.0039	0.0053
11			0.0001	0.0001	0.0002	0.0004	0.0006	0.0009	0.0013	0.0019
12					0.0001	0.0001	0.0002	0.0003	0.0004	0.0006
13								0.0001	0.0001	0.0002
14										0.0001

x \ λ	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0
0	0.0150	0.0123	0.0101	0.0082	0.0067	0.0055	0.0045	0.0037	0.0030	0.0025
1	0.0630	0.0540	0.0462	0.0395	0.0337	0.0287	0.0244	0.0207	0.0176	0.0149
2	0.1323	0.1188	0.1063	0.0948	0.0842	0.0746	0.0659	0.0580	0.0509	0.0446
3	0.1852	0.1743	0.1631	0.1517	0.1404	0.1293	0.1185	0.1082	0.0985	0.0892
4	0.1944	0.1917	0.1875	0.1820	0.1755	0.1681	0.1600	0.1515	0.1428	0.1339
5	0.1633	0.1687	0.1725	0.1747	0.1755	0.1748	0.1728	0.1697	0.1656	0.1606
6	0.1143	0.1237	0.1323	0.1398	0.1462	0.1515	0.1555	0.1584	0.1601	0.1606
7	0.0686	0.0778	0.0869	0.0959	0.1044	0.1125	0.1200	0.1267	0.1326	0.1377
8	0.0360	0.0428	0.0500	0.0575	0.0653	0.0731	0.0810	0.0887	0.0962	0.1033
9	0.0168	0.0209	0.0255	0.0307	0.0363	0.0423	0.0486	0.0552	0.0620	0.0688
10	0.0071	0.0092	0.0118	0.0147	0.0181	0.0220	0.0262	0.0309	0.0359	0.0413
11	0.0027	0.0037	0.0049	0.0064	0.0082	0.0104	0.0129	0.0157	0.0190	0.0225
12	0.0009	0.0013	0.0019	0.0026	0.0034	0.0045	0.0058	0.0073	0.0092	0.0113
13	0.0003	0.0005	0.0007	0.0009	0.0013	0.0018	0.0024	0.0032	0.0041	0.0052
14	0.0001	0.0001	0.0002	0.0003	0.0005	0.0007	0.0009	0.0013	0.0017	0.0022
15			0.0001	0.0001	0.0002	0.0002	0.0003	0.0005	0.0007	0.0009
16						0.0001	0.0001	0.0002	0.0002	0.0003
17	(all other entries < 0.0001)							0.0001	0.0001	0.0001

Appendix 2: Verb Taxonomy

BLOOMS TAXONOMY	SOLO TAXONOMY	SKILL LEVEL SCORE	SKILL LEVEL BAND	DESCRIPTORS
Knowledge	Unistructural • students make simple and obvious connections between pieces of information	1	1	Mastery of the basic knowledge and skills that are fundamental for proficient work.
Comprehension	Multistructural • a number of connections are made, but not the metaconnections between them	2		
Application	Relational • students see the significance of how the various pieces of information relate to one another	3	2	Solid academic performance for the given learning outcome and competency over challenging subject matter including subject-matter knowledge, application of such knowledge to real world situations.
Synthesis, Analysis, Evaluation	Extended Abstract • at this level students can make connections beyond the scope of the problem or question, to generalise or transfer learning into a new situation	4	3	Presumes mastery of both the Basic and Proficient levels and represents superior academic performance.

Appendix 3: IA Task 1 Assessment Schedule

A. BIVARIATE (10%)

QUESTION #	SLO CODE	SKILL LEVEL	EVIDENCE	STUDENT RESPONSE LEVEL			
				Level 1	Level 2	Level 3	Level 4
C1a	sta3.1.1.10	1	Purpose stated	Purpose stated.			
C1b	sta3.1.2.4	2	Variables identified and clearly described	Variables identified.	Variables clearly described		.
C1c	sta3.1.4.1	4	Scatter plot clear, appropriate line of best fit fitted, and correlation coefficient correctly stated	Scatter plot partially correctly.	Any one correctly provided; scatter plot, appropriate line of best fit, or correlation coefficient.	Any two appropriately provided; scatter plot, appropriate line of best fit, or correlation coefficient are	Scatter plot clear, appropriate line of best fit fitted, and correlation coefficient correctly stated.
C1d	sta3.1.2.9	2	Trend and strength of relationship described.	Trend or strength of relationship correctly identified.	Trend and strength of relationship correctly described.		
C1e	sta3.1.3.5	3	Clear conclusion showing interpretation of trend in context and strength of relationship, with an in-depth discussion on causality and limitations of the model.	Trend or strength of relationship correctly identified	Clear interpretation of trend in context and strength of relationship.	Clear conclusion showing interpretation of trend in context and strength of relationship, with an in-depth discussion on causality and limitations of the model.	

C1f	sta3.1.2.10	2	Logical presentation and communication of ideas.	Ideas are identified but not communicated logically.	Clear, logical, effective communication of ideas		

Appendix 4: IA Task 2 Assessment Schedule

B. TIME SERIES (10%)

QUESTION #	SLO CODE	SKILL LEVEL	EVIDENCE	STUDENT RESPONSE LEVEL			
				Level 1	Level 2	Level 3	Level 4
C2a	sta3.2.1.10	1	Purpose stated.	Purpose of an investigation is stated.			
C2b	sta3.2.2.1	2	Variables identified and clearly described.	Variables identified only.	Variables identified and clearly described		
C2c	sta3.2.3.1	3	Table, graphs of raw and smoothed data with trend line, equation of trend line, and calculation of average seasonal effects all correctly done.	Data presented (as a table or graph) or calculation is partially correct.	Data presented in a table along with correct smoothed values and graph of raw and smooth data presented plus trend line fitted.	Table, graphs of raw and smoothed data with trend line, equation of trend line, and calculation of average seasonal effects all correctly done.	
C2d	sta3.2.2.3	2	Trend clearly described.	Trend indentified.	Trend clearly described.		

C2e	sta3.2.4.5	4	Clear conclusion with appropriate discussions on features, prediction and seasonally adjusted values of the time series.	A feature or correct prediction is identified.	Some features described and a correct prediction is made.	All features described, a correct prediction is made and discussed the appropriateness of the prediction.	Clear conclusion with appropriate discussions on features, prediction and seasonally adjusted values of the time series.
C2f	sta3.2.2.7	2	Logical presentation and communication of ideas.	Ideas are identified but not communicated logically.	Clear, logical, effective communication of ideas		

Appendix 5: IA Task 3 Assessment Schedule

C: COMPARISON PROJECT (10%)

QUESTION #	SLO CODE	SKILL LEVEL	EVIDENCE	STUDENT RESPONSE LEVEL			
				Level 1	Level 2	Level 3	Level 4
C3a	sta3.3.2.4	2	Comparison question clearly stated identifying variable of interests, populations, sources of variation and establishing methods to manage them.	Comparison question identified with either two of the following missing; variables, population, sources of variation, methods.	Comparison question, variables of interests, populations, sources of variation and methods clearly identified.		
C3b	sta3.3.2.5	2	Sampling method clearly described, sources of variation considered, and clear indication of how data will be collected and recorded.	Sampling method identified and/or how data will collected and recorded mentioned.	Sampling method clearly described, sources of variation considered, and clear indication of how data will be collected and recorded.		
C3c	sta3.3.2.7	2	Data collected, clearly recorded and cleaned where appropriate.	Data collected but not clearly recorded or cleaned.	Data collected, clearly recorded and cleaned where appropriate.		

C3d	sta3.3.3.2	3	Correctly summarized the data with appropriate measures of central tendency and spread; correctly used appropriate displays; described features evident; commented on a possible hypothesis.	Presented data through an appropriate display, an appropriate measure of central tendency and an appropriate measure of spread.	Correctly summarized the data with appropriate measures of central tendency and spread; correctly used appropriate displays; AND correctly described features evident OR made a valid comment on a possible hypothesis.	Correctly summarized the data with appropriate measures of central tendency and spread; correctly used appropriate displays; described features evident; commented on a possible hypothesis.	
C3e	sta3.3.3.9	3	Clear conclusion that answers the investigative question, making a clear and justified inference back in the population, discussing any limitations in their data study, and discussing any possible further investigation that the data may have highlighted.	Conclusion given that answers the investigative question.	Conclusion given that answers the investigative question, making an inference back in the population supported by either describing any limitations in their data study or any possible further investigation that the data may have highlighted.	Clear conclusion that answers the investigative question, making a clear and justified inference back in the population, discussing any limitations in their data study, and discussing any possible further investigation that the data may have highlighted.	
C3f	sta3.3.2.13	2	Excellent presentation and communication of ideas.	Ideas are identified but not communicated logically.	Clear, logical, effective communication of ideas		

Appendix 6: IA Summary Form

SAT – IA

SOUTH PACIFIC FORM SEVEN CERTIFICATE
Internal Assessment Summary Form

MATHEMATICS WITH STATISTICS

Country: _____ **School:** _____

Task	Task Description/Focus	Start Date	End Date	Weighting
1. Bivariate project				10%
2. Time Series project				10%
3. Comparison project				10%
	Total			30%

Note: 1. Be specific about dates, not just Week 3 Term 1, etc.

2. Suggested assessment schedules for the tasks are provided in Appendix 3, 4 and 5. Teachers are encouraged to use these schedules when guiding students with their IA tasks.

Teacher's Name and Signature: _____ _____

Principal's Name and Signature: _____ _____

ADVISORY SECTION

Recommended Texts & Resources

This is a list of only some Mathematics texts that are available and have been used for teaching courses for Mathematics with Statistics. It is important teachers use this as only a guide and check current book lists available through publishers and book retailers.

A. Suggested Text

1. Sigma Mathematics - Barton, D. Longman, 1998 (2nd Edition)
2. Sigma Mathematics - Barton, D. Longman, 2006 (3rd Edition)

B. Supplementary Texts

1. Sigma mathematics Workbook - David Barton, 2007
2. Achieving in Statistics - W. Geldof, 2006
3. Statistics Workbook - Lakeland, R & Nugent, C, Nulake, 1998
4. Year 13 Study Guide, *Mathematics with Statistics* - Barrett, ESA
5. Longman write-on Notes – *Statistics* - Barton, D.
6. Bursary Statistics – Questions from the last 8 bursary papers with suggested answers.

   Really Useful Resources
   Box 19-939
   Woolston
   Christchurch

7 Schaum's Outlines, Probability and Statistics, 2nd education, Speigal et al, McGraw Hill – a metal teacher resource.

8 Study Pass reference notes: Year 13 Statistics, info@studypass.co.nz

9. Time Series resources - Stats NZ - Yearbook
   - PCINFOS online. - Met service
   - weather stats - Business Indicators
   - share index (http://www.world-exchanges.org/)
   - exchange rates - The internet
10. Some Statistics Websites:
    - NZ Ministry of Education resources www.tki.org.nz
    - Censusatschools data, analysis tools and resources www.censusatschool.org.nz
    - University of Auckland www.stat.auckland.ac.nz
    - Secretariat of the Pacific Community website www.spc.int
    - Tonga Statistics department www.spc.int/prism/country/to/stats/ - Statistics New Zealand www.stats.govt.nz

Sample Teaching Programme

It is important that teachers recognize that while this course is divided up into 4 specific objectives, the teaching programme should not necessarily have each objective taught as one big chunk.

Below is one example of a teaching programme that splits the objectives into smaller topics of learning. This is merely an example of what could be done, it is not intended that schools will be expected to follow this format. Schools need to be flexible to the needs of their school and community when planning the teaching order of this prescription.

Teaching Programme Example – Form 7 Mathematics with Statistics

Week	Learning Outcome	Prescription Reference	Assessment
Term I			
1 - 3	Solve probability problems.	1.1	
4 - 5	Solve problems involving expected values and variance of random variables from discrete probability distributions.	1.2	
6 - 8	Use statistical processes to undertake an investigation of bivariate data.	3.1	Internal assessment #1
9 - 10	Understanding simple piecewise functions in order to model data.	2.1	
11 - 13	Model real data using power and exponential functions in order to solve problems.	2.2	
Term II			
1 - 3	Using appropriate probability distributions to model a given situation and solve problems.	1.3	
4	Exam Revision		
5	**Exam Time**		**Exam**
6 - 8	Use statistical processes to undertake an investigation of time series data.	3.2	Internal assessment #2
9 - 10	Simultaneous equations.	4.1	
11 - 14	Use statistical processes to undertake an investigation of numerical data in order to make a comparison between two populations.	3.3	Internal assessment #3
Term III			
1 - 2	Model situations using linear programming techniques to obtain an optimum solution.	2.3	
3 - 4	Use the bisection or Newton-Raphson method to solve non-linear equations to a given precision.	4.2	
5 - 6	Answer questions related to statistical processes.	3.4	
7	Revision.		
8	**Exam Time**		**Exam**
9	Go over Exam.		
10 - 11	Revision/exam preparation.		

Issues & Recommendations for Assessment Governing Bodies

1. *Technology:*

   At the present time the prevalent calculators being used by students are scientific ones. A growing trend in Asian markets is to produce low cost scientific calculators with extra chips that allow some algebraic manipulation. There is also potential for graphical calculators to become more prevalent as the prices of these drop. It is important that the examiner has an awareness of what calculator technology is being used by students and that question styles are adapted so that students who do not have the more advanced technology are not penalized. This is particularly of importance in the topics covered by learning objectives 2 and 4, where the use of a graphics calculator could potentially advantage a student both in simplifying a problem and in saving them a lot of time. While the temptation may be to just ban such technology, we cannot ignore its development and also the maintenance of an exclusion list becomes impractical as the market starts to become flooded with so many new brands. The onus on ensuring fairness must stand with the examiner structuring questions in a way that minimizes the advantages of one calculator over another.

   It is also important that students are able to access suitable computer technology and statistical packages to deal with the analysis of data. To understand data students need to be able to deal with a suitable volume of data and a wide variety of variables. It is impractical for students to manage that volume of data by hand and suitable packages need to be provided. The use of such packages also will help keep students up to date with international trends in data analysis.
   Teachers will also need professional development in using such packages.

2. *Development of Statistical Thinking:*

   Statistics and the development of statistical thinking in our students is becoming of increasing importance in our world. We are now in a data driven world. Technology has allowed us to deal with large amounts of data to get clearer pictures to assist societies in our economic and social development. We need students who understand the whole statistical process, not just the mechanics. They need to be able to gather, manage, analyze and interpret data, while considering variation. These understandings take time, so it is important that a greater emphasis on Statistics is given throughout all years of secondary education. This is a trend that is starting to occur internationally and it is important that the Pacific Islands region gives it some careful consideration.

3. *Professional Development*

   There has been a new topic added to the scheme, analysis of bivariate data. Teachers will need resources and professional development in this area. There is also a changing trend internationally starting to come about on how Statistics is taught. In the past, Statistics prescriptions were based on a mathematical perspective on statistics. Currently the emphasis has changed to developing an understanding of how statistics are dealt with in the real world. There is less importance placed on the actual performance of calculations (which become trivialized with technology) and more importance placed on understanding what the measures calculated are actually telling us about the data and how it relates back to the population. With this change in thinking it is very important that teachers are provided support and professional development to help them understand the impact of this change on what has to happen in the teaching and learning of Statistics in the classroom.

The End