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South Pacific Form Seven Certificate MATHEMATICS WITH CALCULUS 2023

QUESTION and ANSWER BOOKLET

Time allowed: Three hours

(An extra 10 minutes is allowed for reading this paper.)

INSTRUCTIONS

- 1. Write your **Student Personal Identification Number (SPIN)** in the space provided on the top right-hand corner of this page.
- 2. Answer ALL QUESTIONS. Write your answers in the spaces provided in this booklet.
- 3. Show all your working. Unless otherwise stated, numerical answers correct to **three significant figures** will be adequate.
- 4. If you need more space for answers, ask the Supervisor for extra paper. Write your SPIN on all extra sheets used and clearly number the questions. Attach the extra sheets at the appropriate places in this booklet.

	Skill Level & Number of Questions			tions	Weight/
Major Learning Outcomes (Achievement Standards)	Level 1 Uni- structural	Level 2 Multi- structural	Level 3 Relational	Level 4 Extended Abstract	Time
Strand 1: Algebra Demonstrate knowledge application and critical evaluation of problems, and model situations involving algebraic techniques for real and complex numbers.	7	1	1	2	20% 60 min
Strand 2: Trigonometry Demonstrate knowledge application and critical evaluation when using and manipulating trigonometric functions as well as applying its relationship to solve problems.	1	3	1	-	10% 30 min
Strand 3: Differentiation Demonstrate knowledge application and critical evaluation of advanced concepts and techniques of differentiation.	1	3	-	2	15% 45 min
Strand 4: Integration Demonstrate knowledge application and critical evaluation of advanced concepts and techniques of integration.	1	2	2	1	15% 45 min
TOTAL	10	9	4	5	60% 180 min

Check that this booklet contains pages 2–21 in the correct order and that none of these pages are blank. A four-page booklet (No. 108/2) containing mathematical formulae and tables is also provided.

STRAND 1:	ALGEBRA	

1.1	Simplify $\frac{2x}{4} - \frac{x}{5}$		
		Unistru	ctural
		1	
		0 NR	
		IVIX	
1.2	Simplify $2xy^2 \times (7x^2y)^3$		
		Unistru	ctural
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1.3	Write $\log 16 + \log 2 - \log 8$ as the log of a single number.		
		Unistru	uctural
		1	
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1.4	Simplify $6\sqrt{12} - 5\sqrt{243}$		
		Unistru	uctural
		1	
		0	
		NR	
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Assessor's use only Solve $\frac{x+1}{4} = \frac{2x}{5}$ 1.5 Unistructural 0 NR Two complex numbers are given as follows: 1.6 z = 3 + i and w = -1 + 2iEvaluate z + 2wUnistructural NR

-		Assessor's	use only
1.7	Simplify 6i ¹¹		
		Unistru	ıctural
		1	
		0	
		NR	
1.8	Simplify 2-3i / 2+i		
		D.O. Jeise	ueti ve l
		Multistr	uctural
		2	
		1	
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1.9	Use the Binomial Theorem to expand and simplify the following:		
	$\left(3x+\frac{1}{y}\right)^4$		
		Relati	ional
		3	
		2	
		1	
		0	
		NR	

1.10	Find the coefficient of x^{10} in the expansion of $(2x - 3x^2)^7$		
	Hint: Use the General Term: $T_{r+1} = \binom{n}{r} a^{n-r} . b^r$		
		Exten Abst	
		4	
		2	
		1	
		0	
		NR	

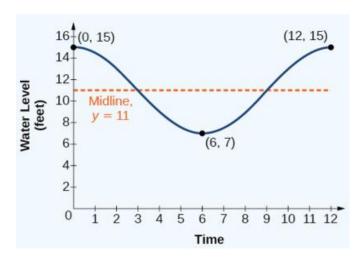
8 Assessor's use only 1.11 Solve the equation $Z^4 = (-1 - \sqrt{3}i)$. Leave your answer in polar form.

Extended Abstract		
4		
3		
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STRA	ID 2: TRIGONOMETRY	Assessor'	's use onl
2.1	Evaluate $\csc\left(\frac{\pi}{6}\right)$	Unistra 1 0 NR	uctural
		NR_	
2.2	Solve for x : $\cos(x-20^\circ) = \frac{\sqrt{3}}{2}$ $\text{where } 0^\circ \le x \le 360^\circ.$		
		Multist	ructural
		2	
		1	
		0	
		NR	

		Assessor's use only
2.3	Find the value of $\sec x$ if $\sin x = \frac{\sqrt{3}}{2}$ Use the information in the diagram.	
	(Do not use the calculator for this problem.)	
		Multistructural 2
		1 0
		NR
2.4	Prove that $\sin^2\theta - \cos^2\theta = 2\sin^2\theta - 1$	
		Multistructural
		2 1 0
		NR NR

2.5 The height of the tide in a small beach town is measured along a seawall. Water levels oscillate between 7 feet at low tide and 15 feet at high tide. On a particular day, low tide occurred at 6 AM and high tide occurred at noon. Approximately every 12 hours, the cycle repeats as shown in the graph that is given below:



Write an equation in the form:

$$y = A\cos(Bt \pm C) \pm D$$

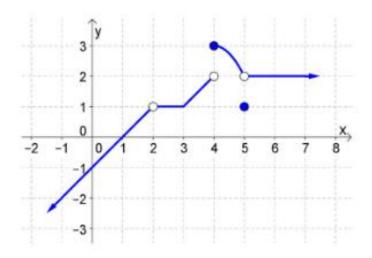
to model the water levels, y as a function of time, t (in hours).

Relational	
3	
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NR	

STRAND 3: DIFFERENTIATION

Assessor's use only

3.1 The graph of f(x) is given below. Use the graph to answer question 3.1.



Evaluate $\lim_{x\to 5} f(x) =$ _____

Unistructural		
1		
0		
NR		

3.2 Use the **quotient rule** to find the derivative of $y = \frac{2x+7}{3x-1}$

Multi-structural	
2	
1	
0	
NR	

		Assessor's use only
3.3	Differentiate $y = (3x + 5)^7$	Assessor's use only
		Multi-structural
		2
		1
		0
		NR
3.4	Calculate $\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$	
		Multistructural
		2
		1
		0
		NR

3.5	The displacement of a particle from the origin is given by:		
	$s(t) = e^{3t} + 2t^3 + 5$		
	where s is in centimetres and t is in seconds.		
	Find the initial acceleration of the particle.		
		Exter	nded
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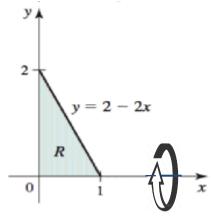
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Assessor's use only 3.6 Cont'd Extended Abstract 4 3 2 1 0 NR

	STRAND 4:	INTEGRATION	Assessor's	use only
4.1	Find $\int 8 x^{-3} dx$			
			Unistru 1 0 NR	ctural
4.2	Evaluate $\int_{-1}^{2} (y^3 - y^3 - y^3)$	- y) dy		
			Multistr	uctural
			2	
			NR	

4.3	A ball is thrown vertically upwards with an initial velocity of 40 m/s from a point 5 m above ground level. The speed of the ball after t seconds is given by the formula:		
	v(t) = (40 - 8t) m/s		
	a. Find an expression for the \mathbf{height} of the ball above ground level at time t .		
		Relati	ional
		3	
		2	
		1	
		0	
		NR	
	b. Find the maximum height reached by the ball at the time when its speed is zero.		
		Multistr	uctural
		2	
		1	
		0	
		NR	

4.4 Let *R* be the region bounded by the curves y = 2 - 2x, y = 0 and x = 0.



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Relational		
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1		
0		
NR		

1.5	A total of 600 goats are introduced to an uninhabited island. From 1 June 2008 to 1 June 2018 the number of goats has increased to 3000. Assume the		
	population growth follows an exponential growth model of the form:		
	$g(t) = Ae^{kt}$		
	where $g(t)$ = quantity at time t, k is the growth constant and A is the initial population size.		
	When will the goat population double?		
		Exten	
		Abst	ract
		3	
		2	
		0	
		NR	

	Assessor's use only
Extra blank page (if needed)	