MARKER CODE

Student Personal Identification Number

South Pacific Form Seven Certificate

MATHEMATICS WITH STATISTICS
2019

QUESTION and ANSWER BOOKLET

Time allowed: Three hours

(An extra 10 minutes is allowed for reading this paper.)

INSTRUCTIONS

Write your **Student Personal Identification Number (SPIN)** in the space provided on the top right-hand corner of this page.

Answer **ALL QUESTIONS**. Write your answers in the spaces provided in this booklet.

Show all working. Unless otherwise stated, numerical answers correct to **three significant figures** will be adequate.

If you need more space for answers, ask the Supervisor for extra paper. Write your SPIN on all extra sheets used and clearly number the questions. Attach the extra sheets at the appropriate places in this booklet.

Major Learning Outcomes (Achievement Standards)	Skill Level & Number of Questions				Weight/ Time
	Level 1 *Uni-structural*	Level 2 *Multi-structural*	Level 3 *Relational*	Level 4 *Extended Abstract*	
Strand 1: Probability Develop knowledge and skills related to probability in order to solve problems and to investigate situations involving elements of chance.	10	4	2	1	28% 72 min
Strand 2: Modelling Using Graphical Methods Model situations using graphical methods in order to solve problems.	4	4	2	1	22% 57 min
Strand 3: Statistical Investigations Carry out statistical investigations and understand statistical processes.	1	1	1	-	6% 15 min
Strand 4: Numerical and Algebraic Methods Use numeric and algebraic methods to solve problems.	3	2	1	1	14% 36 min
TOTAL	**18**	**11**	**6**	**3**	**70% 180 min**

Check that this booklet contains pages 2-17 in the correct order and that none of these pages are blank.

HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

STRAND 1: PROBABILITY

Assessor's use only

1.1a	Define **sample space**. _____ _____ _____	**Unistructural** 1 ☐ 0 ☐ NR ☐
1.1b	A fair coin and a die are tossed together. What is the probability of getting a head and a 5? _____ _____ _____	**Multistructural** 2 ☐ 1 ☐ 0 ☐ NR ☐
1.2a	What are **complementary events**? _____ _____ _____	**Unistructural** 1 ☐ 0 ☐ NR ☐
1.2b	Given that the probabilities for events A and B are $P(A) = 0.35$, $P(B) = 0.53$ and $\mathrm{P(A\ and\ B)} = 0.42$ Identify whether A and B are independent events. _____ _____ _____ _____	**Unistructural** 1 ☐ 0 ☐ NR ☐
1.3	Event *A* is that a king is drawn and Event *B* is that an ace is drawn from a pack of cards. What type of probability is the probability of drawing a king given that an ace has already been drawn? _____ _____ _____ _____	**Unistructural** 1 ☐ 0 ☐ NR ☐

Assessor's use only

1.4a	State a parameter of a **normal distribution**.	**Unistructural** 1 0 NR
1.4b	State **one** property of a **normal distribution**.	**Unistructural** 1 0 NR
1.4c	The probability that a sunflower plant grows over 1.5 m is 0.25. A random sample of 40 sunflower plants is taken and the height of each is measured and recorded. Find the probability that the number of sunflower plants over 1.5 m high is between 8 and 13 (inclusive) using a normal approximation.	**Extended Abstract** 4 3 2 1 0 NR

Assessor's use only

1.5a Define **random variable**.

Unistructural	
1	
0	
NR	

1.5b Define **standard deviation**.

Unistructural	
1	
0	
NR	

Use the following table to answer questions 1.5c and 1.5d.

1.5c A box contains 5 coins, one worth 25 cents, two worth 10 cents each and two worth 5 cents each. A coin is selected at random and the random variable **X** is the value of the coin that is selected. The probability distribution is given as

X: Value of the coin	5¢	10¢	25¢
Probability	2/5	2/5	1/5

Calculate the expected value of **X**.

Multistructural	
2	
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NR	

Assessor's use only

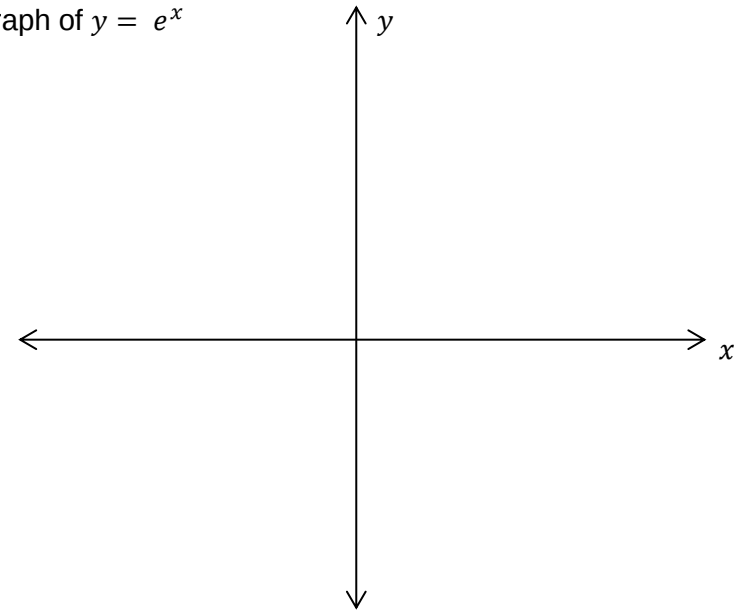
1.5d	Calculate the **standard deviation** of **X.**	**Multistructural** 2 1 0 NR
1.6a	State **one** property of a **binomial distribution**.	**Unistructural** 1 0 NR
1.6b	**Use the following information to answer questions 1.6b and 1.6c.** In a biscuit factory, the mean mass of a cookie is given as 40 g. For quality control, the standard deviation is 2 g. If a cookie is selected at random, what is the probability that the mass will be within 2 g of the mean?	**Multistructural** 2 1 0 NR

Assessor's use only

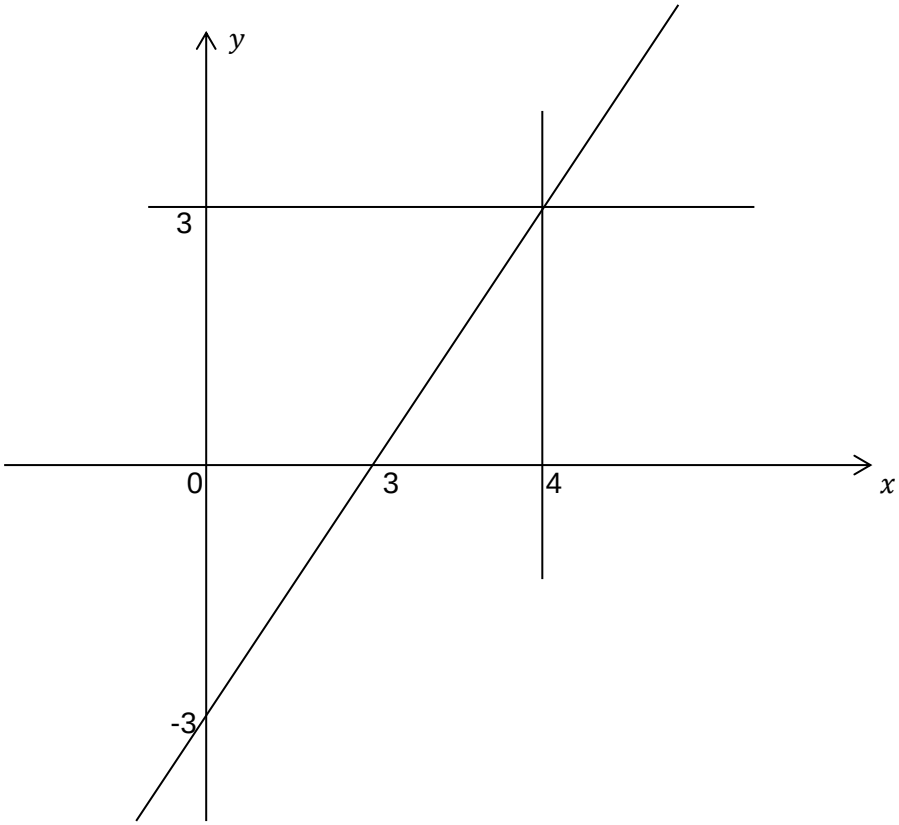
1.6c	Cookies are rejected if they weigh more than 44 g or less than 36 g. How many cookies would you expect to be rejected in a sample of 10,000 cookies?	**Relational** 3 2 1 0 NR
1.7a	Define **discrete random variable**.	**Unistructural** 1 0 NR
1.7b	In a class of 40 students, 19 play football, 20 play netball and 8 play neither of these sports. A student is randomly chosen from the class. Determine the probability that the student plays one and only one of the sports.	**Relational** 3 2 1 0 NR

STRAND 2: MODELLING USING GRAPHICAL METHODS

Assessor's use only

2.1	Define **discontinuous function**.	**Unistructural** 1 0 NR
2.2	Give **one** feature of a **quadratic function**.	**Unistructural** 1 0 NR
2.3a	Draw the graph of $y = e^x$ 	**Unistructural** 1 0 NR
2.3b	Solve $3^{x+2} = 81$	**Multistructural** 2 1 0 NR

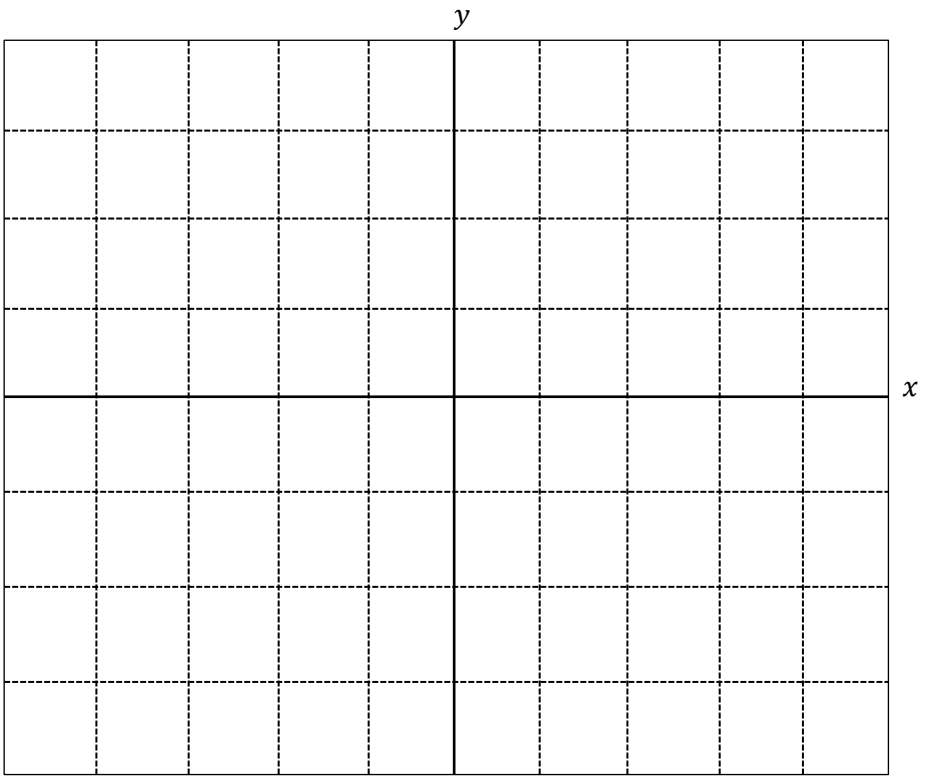
Assessor's use only

2.4a	State **one** feature of an **inequation function**. _____ _____ _____ _____	**Unistructural** 1 0 NR
2.4b	Clearly shade the region whose boundaries are $x \geq 0$, $y \leq 3$, and $y \geq x - 3$  	**Multistructural** 2 1 0 NR

Assessor's use only

2.5 Draw the graph of the piecewise function $f(x)$ in the specified domain values.

$$f(x) = \begin{cases} x & x < -1 \\ x - 2 & -1 \leq x < 1 \\ x^2 - 1 & x \geq 1 \end{cases}$$



Extended Abstract	
4	
3	
2	
1	
0	
NR	

Assessor's use only

2.6a	A population of creatures is given in millions by the equation $P(t) = 4e^{0.055t} - 0.2e^{0.086t}$, where t is in years. The population starts to decrease after a certain time. When does the population become extinct?	**Relational** 3 2 1 0 NR
2.6b	Use natural logarithm to solve the equation $2^x = 12$	**Multistructural** 2 1 0 NR

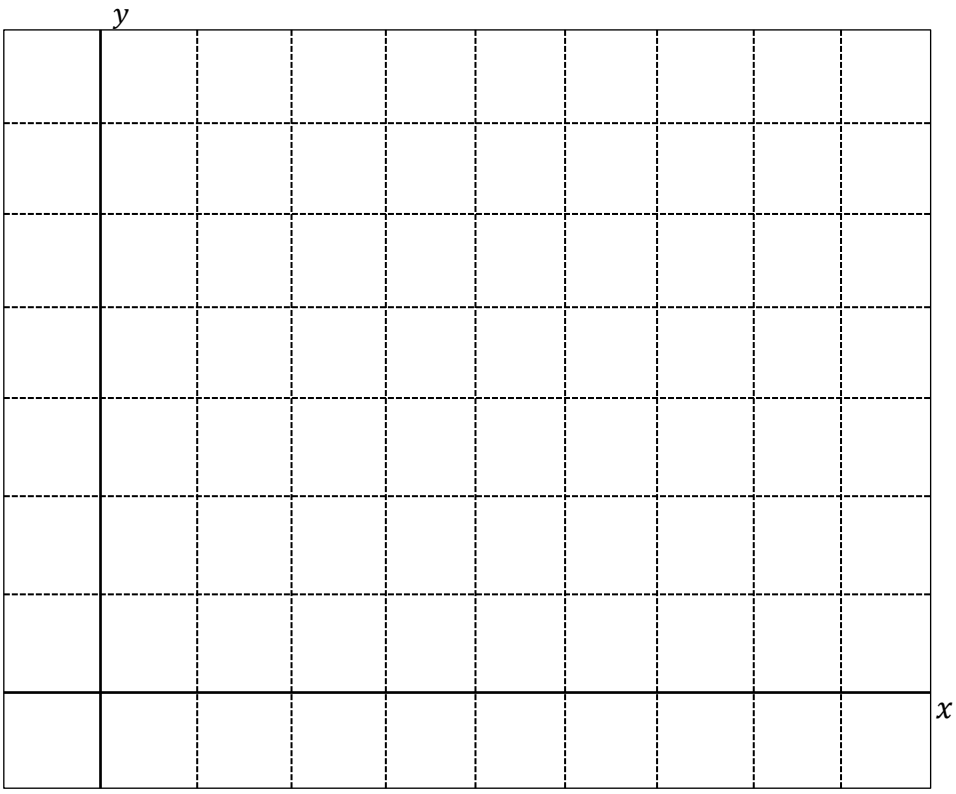
Assessor's use only

Use the following information to answer questions 2.7a and 2.7b.

2.7a Consider optimising the objective function $f(x, y) = 4x + 5y$, subject to the following constraints:

$$\begin{cases} x \geq 1 \\ y \geq 1 \\ x + y \leq 6 \end{cases}$$

Graph the inequations and shade the **feasible region** (shading in or shading out). You must clearly label the lines with relevant intercepts.



Relational	
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2.7b Using the graph in 2.7a above, find the points where the function $f(x, y)$ may attain the optimal value.

DO NOT ATTEMPT TO FIND ANY OPTIMAL VALUE OF $f(x, y)$.

Multistructural	
2	
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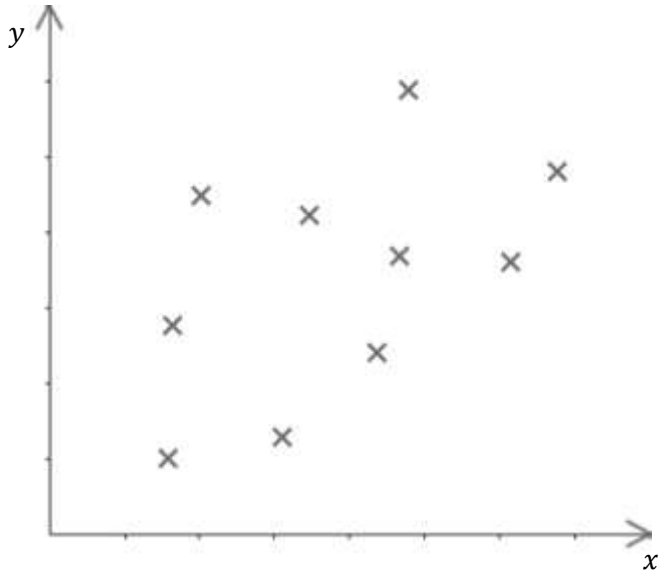
STRAND 3: STATISTICAL INVESTIGATIONS

Assessor's use only

3.1 Consider the following scatter plot, which displays the relationship between variables x (horizontal) and y (vertical).



State the strength and nature of the relationship between x and y.

Unistructural	
1	
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NR	

Assessor's use only

3.2	Describe the **stratified** method of sampling.	

Multistructural	
2	
1	
0	
NR	

3.3	The average score obtained by 10 students in a test is 25. The standard deviation for the sample is 8. Find the confidence interval at 95% confidence level.	

Relational	
3	
2	
1	
0	
NR	

STRAND 4: NUMERICAL AND ALGEBRAIC METHODS

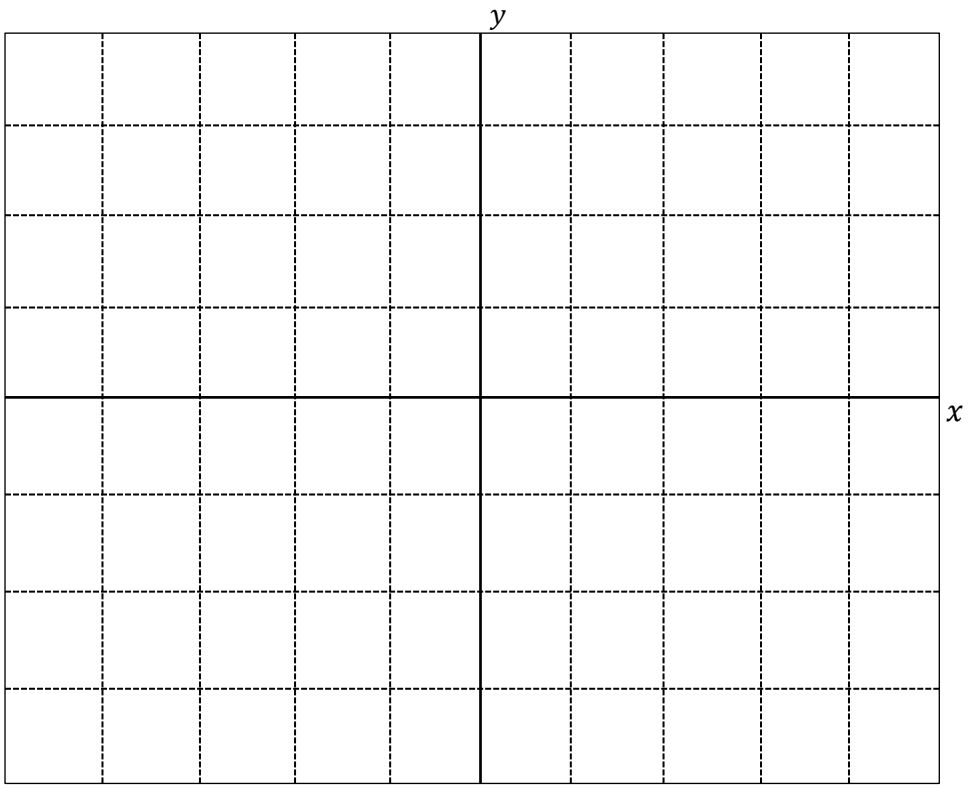
Assessor's use only

4.1	State **one** type of solution for a system of linear equations which has two variables. _____ _____ _____	**Unistructural** 1 ☐ 0 ☐ NR ☐
4.2	State **one** advantage of Bisection Method to approximate a root of $f(x)$. _____ _____ _____ _____	**Unistructural** 1 ☐ 0 ☐ NR ☐
4.3	Give **one** advantage of using the Newton-Raphson method to approximate a root of $f(x)$. _____ _____ _____ _____	**Unistructural** 1 ☐ 0 ☐ NR ☐

Assessor's use only

4.4 Interpret the nature of the solution to the following equations.

$x + y = 3$ and $x - y = 1$



Multistructural	
2	
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Assessor's use only

4.5 A retailer is selling hot dogs and juice. Each hot dog costs \$1.50 and each glass of juice costs \$0.50. At the end of the night the retailer made a total of \$78.50 from selling a total of 87 hot dogs and glasses of juices combined.

x = number of hot dog sold y = number of glasses of juice sold

Write down a system of simultaneous equations that represents this information.

DO NOT ATTEMPT TO SOLVE YOUR SYSTEM.

Multistructural	
2	
1	
0	
NR	

4.6 Solve the following system of linear equations. (Use of matrices is also accepted.)

$$x + y + z = 6; \quad 2y + 5z = -4; \quad 2x + 5y - z = 27$$

Relational	
3	
2	
1	
0	
NR	

Assessor's use only

4.7 Use Newton-Raphson method to determine $x_2 \; for \; f(x) = x^3 - 7x^2 + 8x - 3$

$if \; x_0 = 5$

Extended Abstract	
4	
3	
2	
1	
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NR	

THE END