

MARKER CODE



Pacific
Community
Communauté
du Pacifique



Student Personal Identification Number

South Pacific Form Seven Certificate

MATHEMATICS WITH CALCULUS

2017

QUESTION and ANSWER BOOKLET

Time allowed: Three hours

(An extra 10 minutes is allowed for reading this paper.)

INSTRUCTIONS

Write your **Student Personal Identification Number (SPIN)** in the space provided on the top right hand corner of this page.

Answer **ALL QUESTIONS**. Write your answers in the spaces provided in this booklet.

Show all working. Unless otherwise stated, numerical answers correct to **three significant figures** will be adequate.

If you need more space for answers, ask the Supervisor for extra paper. Write your SPIN on all extra sheets used and clearly number the questions. Attach the extra sheets at the appropriate places in this booklet.

Major Learning Outcomes (Achievement Standards)	Skill Level & Number of Questions				Weight/ Time
	Level 1 <i>Uni- structural</i>	Level 2 <i>Multi- structural</i>	Level 3 <i>Relational</i>	Level 4 <i>Extended Abstract</i>	
Strand 1: Algebra Apply algebraic techniques to real and complex numbers	17	-	1	-	20% 52 min
Strand 2: Trigonometry Use and manipulate trigonometric functions and expressions	-	2	2	-	10% 24 min
Strand 3: Differentiation Demonstrate knowledge of advanced concepts and techniques of differentiation	-	3	2	2	20% 52 min
Strand 4: Integration Demonstrate knowledge of advanced concepts and techniques of integration.	-	3	2	2	20% 52 min
TOTAL	17	8	7	4	70% 180 min

Check that this booklet contains pages 2-24 in the correct order and that none of these pages is blank.
A 4-page booklet (No. 108/2) containing mathematical formulae and tables is provided.

HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

1.1	<p>Solve the inequation: $\frac{x-3}{2} - 4 \leq \frac{1-x}{3}$</p> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/>	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr style="background-color: #cccccc;"> <th colspan="2">Unistructural</th> </tr> </thead> <tbody> <tr> <td style="text-align: center; width: 10%;">1</td><td style="width: 10%;"></td></tr> <tr> <td style="text-align: center;">0</td><td></td></tr> <tr> <td style="text-align: center;">NR</td><td></td></tr> </tbody> </table>	Unistructural		1		0		NR	
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1.2	<p>The focal length of a lens is given by $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$. Make ‘v’ the subject of the formula.</p> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/>	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr style="background-color: #cccccc;"> <th colspan="2">Unistructural</th> </tr> </thead> <tbody> <tr> <td style="text-align: center; width: 10%;">1</td><td style="width: 10%;"></td></tr> <tr> <td style="text-align: center;">0</td><td></td></tr> <tr> <td style="text-align: center;">NR</td><td></td></tr> </tbody> </table>	Unistructural		1		0		NR	
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Assessor's use only

If $x^2 - 6x + 4 + b = (x + a)^2$, where 'a' and 'b' are constants, find the value of 'b'.

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Simplify $\frac{(2\sqrt{x})^6 y^5}{8(xy^2)^2}$

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1.5	<div>Find the third term in the binomial expansion of $(2 - 3x)^7$</div> 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Factorize the expression $f(x) = 4x^2 + 25$.

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Unistructural	
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Simplify the following surd expression: $3\sqrt{2} + (1 - \sqrt{2})^2 - 2\sqrt{8}$.

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Unistructural	
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Solve the equation $\frac{x+3}{2} - \frac{4x-1}{3} = 1$

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Unistructural	
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Divide the polynomial $f(x) = 2x^3 - x^2 + 3x - 5$ by $(x - 1)$

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Unistructural	
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Unistructural	
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Assessor's use only

1.16 Two straight lines $2y + x = 4$ and $y - 3x = 1$ meet at point B. Find the coordinates of B

[illegible]

Unistructural	
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NR	

1.17 Find x if $8^x = 32^{x+2}$

[illegible]

Unistructural	
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0	
NR	

1.18

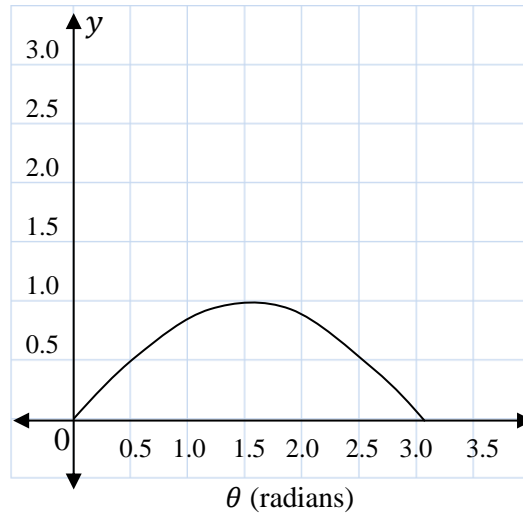
Use de Moivre's Theorem to find the three roots of the complex number equation $\mathbf{z^3 = w}$, where $\mathbf{w = 8i}$.

Relational	
3	
2	
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STRAND 2: TRIGONOMETRY
Answer All Questions

Assessor's use only

The diagram shows part of the graph of $f(\theta) = \sin \theta$ for $0 \leq \theta \leq \pi$ radians.



- 2.1 Sketch in the grid above the graph of $f^{-1}(\theta)$ and hence write the domain of $f(\theta)$ for which $f^{-1}(\theta)$ is a function

- 2.2 Prove that $1 + \cot^2 \theta = \csc^2 \theta$

- 2.3 Use a compound angle formula to find the values of x which satisfy the equation: $2 \cos 45 \cos x - 2 \sin 45 \sin x = 1$, where $0 \leq x \leq 90^\circ$

Multistructural	
2	
1	
0	
NR	

Multistructural	
2	
1	
0	
NR	

Relational	
3	
2	
1	
0	
NR	

The diagram shows a cross-section of a coastal profile. A horizontal dashed line represents the high tide level, labeled "high tide (4.30 am)". Another horizontal dashed line represents the low tide level, labeled "low tide (10 am)". The vertical distance between these two lines is marked as "5.5 m". The low tide level is also marked as "1.1 m" above a solid horizontal line at the bottom, which represents the datum.

[illegible]

Relational	
3	
2	
1	
0	
NR	

3.1 A particle has a distance-time relation given by $d = 3t^2 - 2t + 5$. How far does it move before coming to a stop?

[illegible]

Multistructural	
2	
1	
0	
NR	

3.2 Find the equation of the tangent to the curve $y = \frac{1}{2x-1}$ at the point $(2, \frac{1}{3})$.

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Multistructural	
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Find the values of x at which $f(x) = \frac{1}{1-\sin^2 x}$ is discontinuous.

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Multistructural	
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Use implicit differentiation to find $\frac{dy}{dx}$ given that $xe^{x+y} = 5 \sin y$

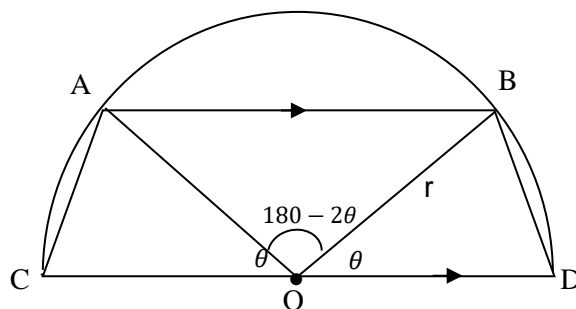
[illegible]

Relational	
3	
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3.6 The surface area S of a spherical balloon rising in the atmosphere increases at a rate of 20 cm^2 per second. At what rate does the volume of the balloon increase when its radius is 4 cm ? [These formulae are needed: $S = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$]

Extended Abstract	
4	
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NR	

A trapezium has the diameter of a semi-circle as its longer side CD. The centre of the circle is O and the radius is 'r'. The diagram shows three triangles that will help in finding the area of the trapezium.

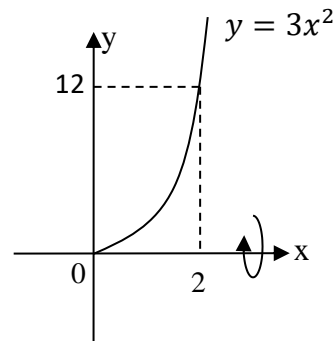


Find the value of the angle θ that makes the area of the trapezium a maximum.

[illegible]

Extended Abstract	
4	
3	
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4.1

[illegible]

Multistructural	
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4.2

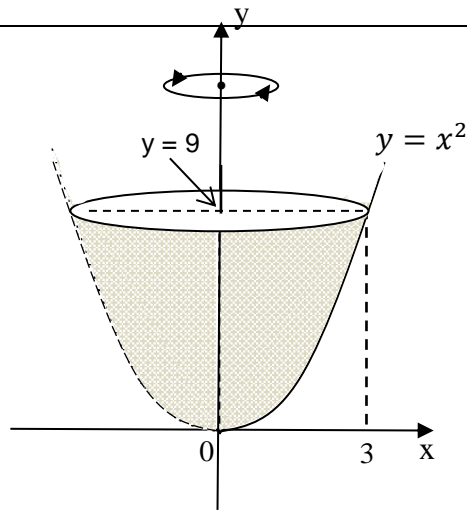
Multistructural	
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4.3

The diagram at right shows the bowl formed when the graph of $y = x^2$ between $y = 0$ and $y = 9$ is rotated 360° about the y -axis.

Calculate the volume of the bowl.

[Hint: $V = \pi \int_a^b x^2 dy$]



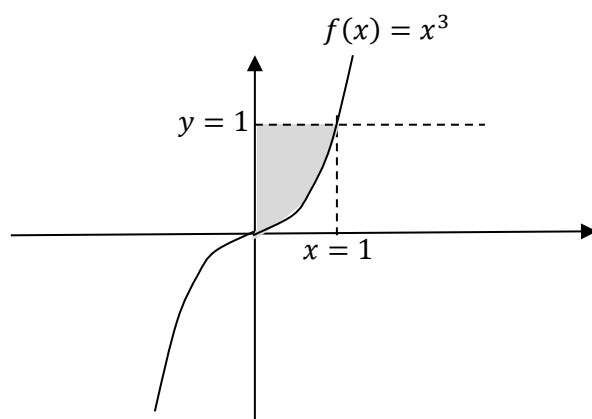
Multistructural	
2	
1	
0	
NR	

4.4

The rate of change in volume V of water in a leaking urn is directly proportional to the square root of the volume of water in the urn. Before the leak there was 40 L of water but after 2 hours, there was 16 L in the urn. If the leak is not stopped, how long will it take for the urn to be empty?

Relational	
3	
2	
1	
0	
NR	

The diagram shows the graph of $f(x) = x^3$ and the region bounded by the line $y = 1$, $x = 0$ and the graph of $f(x)$.

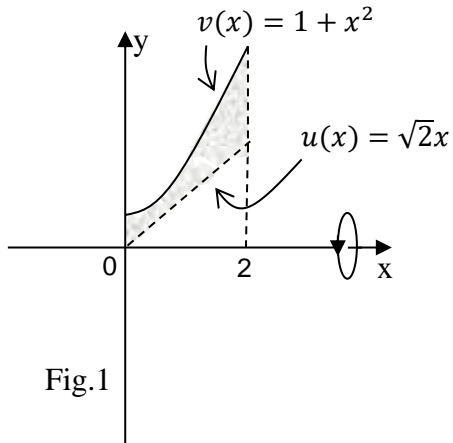


Find the volume of the solid formed when the shaded region is rotated 360° about the line $y = 1$.

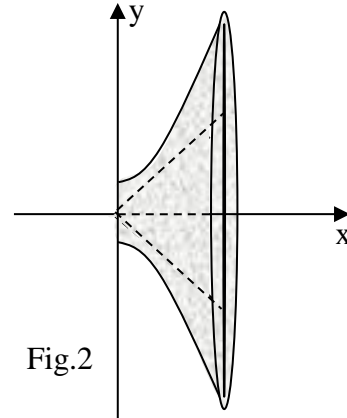
Relational	
3	
2	
1	
0	
NR	

4.6

The diagram below (Fig.1) shows the shaded area between the graphs of the functions $v(x) = 1 + x^2$ and the line $u(x) = \sqrt{2}x$ and between the lines $x = 0$ and $x = 2$. When this area is rotated 360° around the x-axis the solid in Fig.2 is formed. Calculate the volume of the solid so formed in Fig.2.



Rotate 360°



Extended
Abstract

4

3

2

1

0

NR

4.7

The rate of cooling of an object, $\frac{dT}{dt}$, is proportional to the difference between the temperature T of the object and the temperature T_m of the surrounding medium. The original temperature of the object at $t = 0 \text{ min}$ is 80°C , and the surrounding medium is at a constant temperature of 28°C . It is also observed that after 10 minutes the object has cooled down to a temperature of 64°C .

Show that the temperature of the cooling object at any time t minutes is given by

$$T(t) = 52e^{-0.04t} + 28$$

**Extended
Abstract**

4	
3	
2	
1	
0	
NR	

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THE END